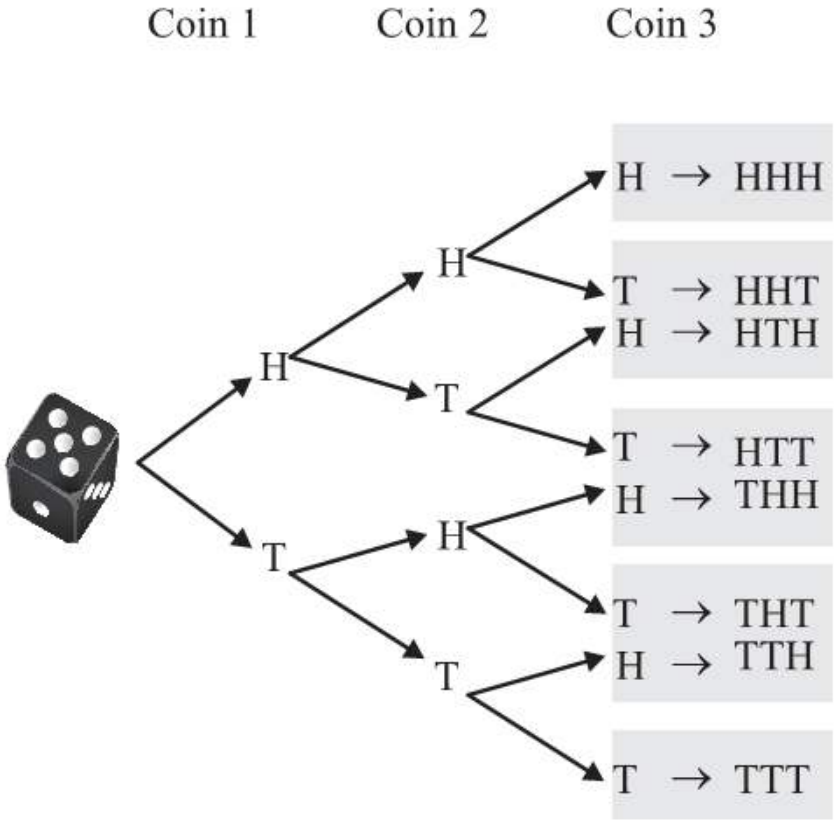
UNIT 12 Probability

EXERCISE 12.1

1. Find the sample space for tossing three coins using tree diagram.

Solution

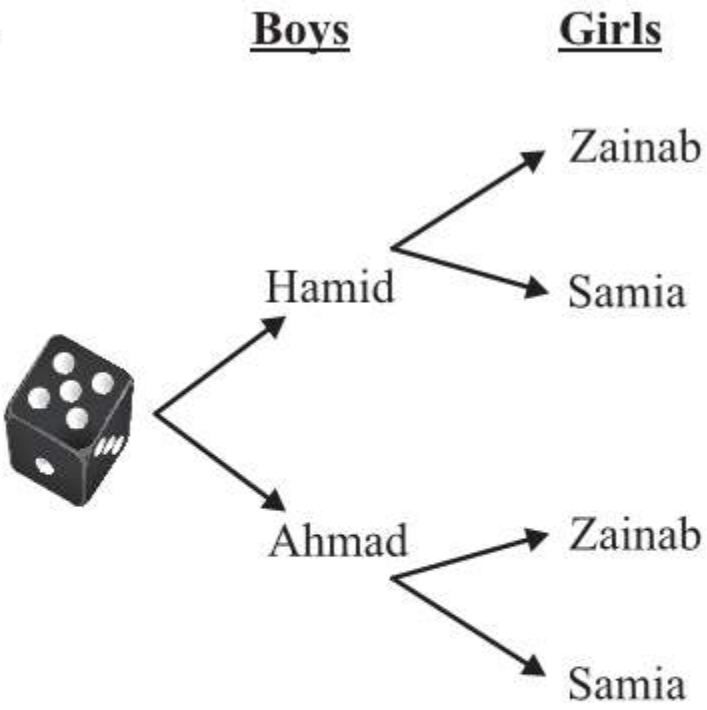


Sample space = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

Total outcomes: n(S) = 8

2. A teacher randomly selects one boy and one girl from a group of 2 boys (Hamid, Ahmad) and 2 girls (Zainab, Samia). Draw a tree diagram and list the sample space for all possible outcomes.

Solution



Sample Space = {(Hamid, Zainab),
(Hamid, Samia),
(Ahmad, Zainab),
(Ahmad, Samia)}

Total outcomes: n(S) = 4

3. If A is an event of a random experiment such that $P(A) : P(\overline{A}) = 17:15$ and $n(S) = 640$, then find (i) $P(\overline{A})$ (ii) $n(A)$

Solution

P(A) : P($\bar{A}$) = 17 : 15 and n(S) = 640 and Let P(A) = 17x , P($\bar{A}$) = 15x.

Since P(A) + P($\bar{A}$) = 1 ⇒ 17x + 15x = 1 ⇒ 32x = 1 ⇒ x = $\frac{1}{32}$

(i) P($\bar{A}$) = 15x ⇒ P($\bar{A}$) = $\frac{15}{32}$

(ii) P(A) = P(A) × n(S) = $\frac{17}{32} \times 640 = 340$

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution

Sample Space (S): {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} ⇒ n(S) = 8

Event E (two consecutive tails): {HTT, TTH, TTT} ⇒ n(E) = 3

P(E) = $\frac{3}{8}$

5. A dice is rolled and coin is tossed together.

(i) Find sample space by drawing possibility diagram.

(ii) Find sample space by sketching tree diagram.

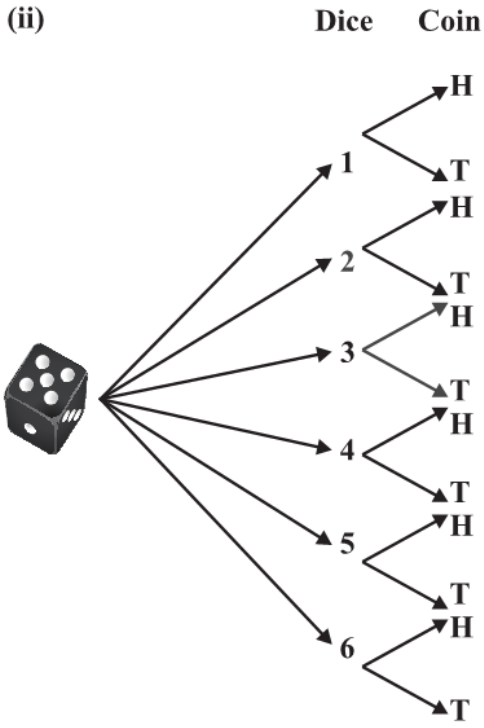
(iii) What is a probability of getting a tail and an even number?

Solution

(i)

Dice \ Coin	*H*	*T*
1	(1, H)	(1, T)
2	(2, H)	(2, T)
3	(3, H)	(3, T)
4	(4, H)	(4, T)
5	(5, H)	(5, T)
6	(6, H)	(6, T)

(ii)



(iii) Probability of getting a tail and an even number:

Total outcomes: n(S) = 12

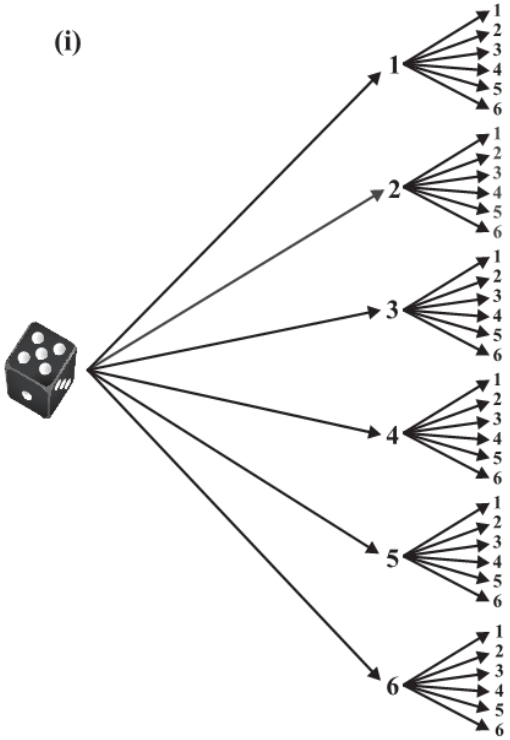
Event E = {(2,T), (4,T), (6,T)} ⇒ n(E) = 3

P(E) = $\frac{3}{12} = \frac{1}{4}$

6. Two unbiased dice are rolled once.
   (i) Find sample space by sketching tree diagram.
   (ii) Find sample space by drawing possibility diagram.
   (iii) Find the probability of getting
      (a) same number on both dice.
      (b) the product as a prime number.
      (c) the sum as an even number.
      (d) the sum as 13.

Solution

(i)



(ii)

Dice 1 \ Dice 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Total outcomes: n(S) = 36

(iii) Find the probability of getting:

(a) Same number on both dice:

E = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} ⇒ n(E) = 6 ⇒ P(E) = $\frac{6}{36} = \frac{1}{6}$

(b) The product as a prime number: Products must be 2, 3, or 5.

E = {(1,2), (2,1), (1,3), (3,1), (1,5), (5,1)} ⇒ n(E) = 6 ⇒ P(E) = $\frac{6}{36} = \frac{1}{6}$

(c) The sum as an even number:

n(E) = 18 (half of the total outcomes) ⇒ P(E) = $\frac{18}{36} = \frac{1}{2}$

(d) The sum as 13:

Maximum possible sum is 6 + 6 = 12 ⇒ n(E) = 0 ⇒ P(E) = $\frac{0}{36} = 0$

7. Three fair coins are tossed together. Find the probability of getting

(i)	all tails	(ii)	at least one head
(iii)	at most two tails	(iv)	2 heads
(v)	at most 2 heads	(vi)	no head

Solution

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} ⇒ n(S) = 8

(i) All tails: E = {TTT} ⇒ P(E) = $\frac{1}{8}$

(ii) At least one head: E = S − {TTT} ⇒ n(E) = 7 ⇒ P(E) = $\frac{7}{8}$

(iii) At most two tails: Everything except TTT ⇒ n(E) = 7 ⇒ P(E) = $\frac{7}{8}$

(iv) 2 heads: E = {HHT, HTH, THH} ⇒ P(E) = $\frac{3}{8}$

(v) At most 2 heads: Everything except HHH ⇒ n(E) = 7 ⇒ P(E) = $\frac{7}{8}$

(vi) No head: E = {TTT} ⇒ P(E) = $\frac{1}{8}$

8. A bag contains 4 red balls, 5 white balls, 6 green balls and 3 black balls. Ali draws a ball at random from the bag. Find the probability that the ball drawn is

(i)	white	(ii)	red
(iii)	not white	(iv)	not black

Solution

Total balls = 4 Red + 5 White + 6 Green + 3 Black ⇒ n(S) = 18

(i) White: P(White) = $\frac{5}{18}$ (ii) Red: P(Red) = $\frac{4}{18} = \frac{2}{9}$

(iii) Not white: P(Not White) = $1 - \frac{5}{18} = \frac{13}{18}$

(iv) Not black: P(Not Black) = $\frac{18-3}{18} = \frac{15}{18} = \frac{5}{6}$

9. A number is selected at random from the set of whole numbers 1 to 15, both inclusive. Find the probability that the number selected is:

(i)	odd	(ii)	a multiple of 5	(iii)	the square of 2
(iv)	prime	(v)	20		

Solution

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} ⇒ n(S) = 15

(i) Odd: E = {1, 3, 5, 7, 9, 11, 13, 15} ⇒ P(E) = $\frac{8}{15}$

(ii) A multiple of 5: E = {5, 10, 15} ⇒ P(E) = $\frac{3}{15} = \frac{1}{5}$

(iii) The square of 2: $2^2 = 4$ ⇒ E = {4} ⇒ P(E) = $\frac{1}{15}$

(iv) Prime: E = {2, 3, 5, 7, 11, 13} ⇒ P(E) = $\frac{6}{15} = \frac{2}{5}$

(v) 20: No such element ⇒ P(E) = $\frac{0}{15} = 0$

10. If the probability of an event A is $\frac{7}{10}$, then find the probability of the event "not A".

Solution

Given P(A) = $\frac{7}{10}$

P(not A) = P($\bar{A}$) = 1 - P(A) = 1 - $\frac{7}{10} = \frac{3}{10}$

11. A dice is rolled twice. Find the probability of having a number greater than 4 on each roll.

Solution

A die is rolled twice. n(S) = 36

Numbers greater than 4 on a single die are {5, 6\}.

Event E (number > 4 on both rolls):

E = {(5,5), (5,6), (6,5), (6,6)}

⇒ n(E) = 4

P(E) = $\frac{4}{36} = \frac{1}{9}$

EXERCISE 12.2

1. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(A \cup B)$.

Solution

$P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{8}$

2. In an apartment, selecting a house from door numbers 1 to 50 randomly, find the probability of getting the door number of the house to be an even number or a perfect square number.

Solution

$S = \{1, 2, 3, \ldots, 50\} \Rightarrow n(S) = 50$

Let $A = \{\text{Even numbers}\} \Rightarrow n(A) = 25 \Rightarrow P(A) = \frac{25}{50}$

Let $B = \{\text{Perfect squares}\} = \{1, 4, 9, 16, 25, 36, 49\} \Rightarrow n(B) = 7 \Rightarrow P(B) = \frac{7}{50}$

$A \cap B = \{\text{Even perfect squares}\} = \{4, 16, 36\} \Rightarrow n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{3}{50}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{25}{50} + \frac{7}{50} - \frac{3}{50} = \frac{29}{50}$

3. The probability of a team winning any match is $\frac{3}{10}$ and the probability of losing any match is $\frac{2}{10}$. What is the probability that

(i) the team wins or loses a particular match.

(ii) the team neither wins nor loses a match.

Solution

$P(\text{Win}) = \frac{3}{10}$, $P(\text{Lose}) = \frac{2}{10}$, $P(A \cap B) = \frac{1}{8}$

(i) $P(\text{Win or Lose}) = \frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$

(ii) $P(\text{neither Win nor Lose}) = P(\text{Draw}) = 1 - \frac{5}{10} = \frac{5}{10} = \frac{1}{2}$

4. In a single throw of two dice, find the probability of having sum of 7 or 11.

Solution: $n(S) = 36$

Let $A = \{\text{Sum of 7}\} = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\} \Rightarrow n(A) = 6$

Let $B = \{\text{Sum of 11}\} = \{(5,6), (6,5)\} \Rightarrow n(B) = 2$

Mutually exclusive events: $\Rightarrow n(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$

5. Find the probability of getting a sum of 5 or 7 in a throw of two dice.

Solution

n(S) = 36

Let A = {Sum of 5} = {(1,4), (2,3), (3,2), (4,1)} ⇒ n(A) = 4

Let B = {Sum of 7} = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)} ⇒ n(B) = 6

$P(A \cup B) = P(A) + P(B) = \frac{4}{36} + \frac{6}{36} = \frac{10}{36} = \frac{5}{18}$

6. A card is taken out at random from a standard pack of 52 cards. Find the probability of taking out.

(i) A king or a Jack. (ii) Neither a king nor a Jack.

Solution

n(S) = 52, Kings = 4, Jacks = 4

(i) A king or a Jack:

$P(K \cup J) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

(ii) Neither a king nor a Jack:

$P(\text{Neither}) = 1 - \frac{2}{13} = \frac{11}{13}$

7. A dice is thrown twice. What is the probability that at least one of the two throws comes up with number 3.

Solution

n(S) = 36

Event E (at least one 3):

E = {(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(1,3),(2,3),(4,3),(5,3),(6,3)}

⇒ n(E) = 11 ⇒ $P(E) = \frac{11}{36}$

8. There are 15 cards in a bag marked as 1, 2, 3, …., 15. Find the probability of picking a card at random, the number written on which is a multiple of 5 or of 7.

Solution

S = {1, 2, 3,…, 15} ⇒ n(S) = 15

Let A = {Multiple of 5} = {5, 10, 15} ⇒ n(A) = 3

Let B = {Multiple of 7} = {7, 14} ⇒ n(B) = 2

⇒ $n(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) = \frac{3}{15} + \frac{2}{15} = \frac{5}{15} = \frac{1}{3}$

9. Two fair coins are tossed once. What is the probability of getting at least one head or two heads.

Solution

S = {HH, HT, TH, TT} ⇒ n(S) = 4
Let A = {At least one head} = {HH, HT, TH} ⇒ n(A) = 3
Let B = {Two heads} = {HH} ⇒ n(B) = 1
Since B is a subset of A, "at least one head or two heads" is just event A.
$P(A \cup B) = \frac{3}{4}$

10. At a busy intersection, 50% of vehicles turn right, 30% turn left and 20% go straight. What is the probability that a randomly selected vehicle turn left or right?

Solution

P(Right) = 50%, P(Left) = 30%, P(Straight) = 20%
$P(\text{Left} \cup \text{Right}) = 30\% + 50\% = 80\% = \frac{80}{100} = \frac{4}{5}$ (or 0.8)

11. Two fair coins are tossed. What is the probability of getting either two heads or two tails?

Solution

S = {HH, HT, TH, TT} ⇒ n(S) = 4
Event E (two heads or two tails) = {HH, TT} ⇒ n(E) = 2
$P(E) = \frac{2}{4} = \frac{1}{2}$

12. If A and B are two mutually exclusive events of a random experiment and $P(\text{not } A) = 0.45$, $P(A \cup B) = 0.65$, then find $P(B)$.

Solution

Mutually exclusive ⇒ $P(A \cap B) = 0$
P(not A) = 0.45, P(A ∪ B) = 0.65
P(A) = 1 - 0.45 = 0.55
P(A ∪ B) = P(A) + P(B)
0.65 = 0.55 + P(B)
P(B) = 0.65 - 0.55 = 0.10

13. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$ then find $P(A \cap B)$.

Solution

$P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{11}{15}$

14. A and B are two events such that, $P(A) = 0.42$, $P(B) = 0.48$, and $P(A \cap B) = 0.16$.

Find: (i) $P(\bar{A})$ (ii) $P(\bar{B})$ (iii) $P(A \cup B)$

Solution

$P(A) = 0.42$, $P(B) = 0.48$, $P(A \cap B) = 0.16$

(i) $P(\bar{A}) = 1 - 0.42 = 0.58$

(ii) $P(\bar{B}) = 1 - 0.48 = 0.52$

(iii) $P(A \cup B) = 0.42 + 0.48 - 0.16 = 0.74$

EXERCISE 12.3

1. Two numbers are randomly chosen from 1 to 10 with replacement. Find the probability that:
   (i) both are prime (ii) their product is even

Solution

Numbers chosen from 1 to 10 with replacement.
Total sample space outcomes: n(S) = 10 × 10 = 100
Prime numbers in range: {2, 3, 5, 7} ⇒ 4 numbers
Even numbers in range: {2, 4, 6, 8, 10} ⇒ 5 numbers
Odd numbers in range: {1, 3, 5, 7, 9} ⇒ 5 numbers

(i) Both are prime:

Outcomes: 4 × 4 = 16
P(Both Prime) = $\frac{16}{100} = \frac{4}{25}$

(ii) Their product is even:

Odd outcomes: 5 × 5 = 25
P(Product Even) = 1 - P(Both Odd)
P(Product Even) = $1 - \frac{25}{100} = 1 - \frac{1}{4} = \frac{3}{4}$

2. One letter is chosen from the word PUNJAB and another from LAHORE. What is the probability that:
   (i) both are vowels
   (ii) one is consonant and other is vowel

Solution

Words: PUNJAB (6 letters: Vowels = {U, A}, Consonants = {P, N, J, B}),
LAHORE (6 letters: Vowels = {A, O, E}, Consonants = {L, H, R})
Total sample space outcomes: n(S) = 6 × 6 = 36

(i) Both are vowels:

Outcomes: 2 × 3 = 6
P(Both Vowels) = $\frac{6}{36} = \frac{1}{6}$

(ii) One is consonant and other is vowel:

Case 1: Consonant from PUNJAB (4) × Vowel from LAHORE (3) = 12
Case 2: Vowel from PUNJAB (2) × Consonant from LAHORE (3) = 6
Total favorable outcomes: 12 + 6 = 18
P(One Consonant, One Vowel) = $\frac{18}{36} = \frac{1}{2}$

3. A single dice is rolled twice. Find the probability that one roll is a multiple of 3 and the other is a 5.

Solution

A single die is rolled twice ⇒ n(S) = 36

Let M_3 = Multiple of 3 = {3, 6} and 5 = {5}.

Favorable outcomes: (M_3, 5) or (5, M_3)

E = {(3,5), (6,5), (5,3), (5,6)} ⇒ n(E) = 4

P(E) = $\frac{4}{36} = \frac{1}{9}$

4. Two dice are rolled. Find the probability of getting an odd number on one and a multiple of 2 on other.

Solution

Two dice are rolled ⇒ n(S) = 36.

Let O = Odd = {1, 3, 5} and E_v = Multiple of 2 = {2, 4, 6}.

Case 1: First die is odd, second die is even ⇒ 3 × 3 = 9 outcomes

Case 2: First die is even, second die is odd ⇒ 3 × 3 = 9 outcomes

Total favorable outcomes: 9 + 9 = 18

P(E) = $\frac{18}{36} = \frac{1}{2}$

5. From a pack of well shuffled cards, two cards are drawn at random one by one with replacement. Find the probability that the first is heart and second is king.

Solution

Two cards drawn one by one "with replacement" ⇒ n(S) = 52 × 52.

Hearts = 13, Kings = 4

P(First Heart and Second King) = $\frac{13}{52} \times \frac{4}{52} = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$

6. If two cards are selected from a standard deck of 52 cards without replacement, find the probability that

(i) Both are black. (ii) Both are queens.
(iii) Both are spades. (iv) Both are diamonds.

Solution

Two cards selected "without replacement".
Total cards = 52.

(i) Both are black:

Black cards = 26

P(Both Black) = $\frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

(ii) Both are queens:

Queens = 4

P(Both Queens) = $\frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

(iii) Both are spades:

Spades = 13

P(Both Spades) = $\frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$

(iv) Both are diamonds:

Diamonds = 13

P(Both Diamonds) = $\frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$

7. Saleem draws two cards one by one without replacement from a well shuffled pack of 52 playing cards. What is the probability that first card is jack and the second card is queen.

Solution

Two cards drawn one by one "without replacement".
Total cards = 52.
Jacks = 4, Queens = 4

P(First Jack and Second Queen) = $\frac{4}{52} \times \frac{4}{51} = \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$

REVIEW EXERCISE 12

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) The probability of impossible event is:
(a) 0 (b) 1 (c) 2 (d) –1

(ii) What is the probability of getting a head when a fair coin is tossed once?
(a) 0 (b) 0.25 (c) 0.5 (d) 1

(iii) The sum of probabilities of all possible outcomes of an experiment is:
(a) 0.5 (b) 0.25 (c) 1 (d) 0.4

(iv) If $P(A) = 0.6$, then the probability of event A not happening is:
(a) 0.4 (b) 0.6 (c) 1 (d) 1.6

(v) Two events are said to be mutually exclusive if:
(a) they can happen at the same time.
(b) one affects the other.
(c) they cannot happen together.
(d) they are always equal.

(vi) If one coin is tossed and one dice is rolled, then the number of sample point are:
(a) 3 (b) 6 (c) 2 (d) 12

(vii) What is the probability of sure event?
(a) 1 (b) 0 (c) $\frac{2}{3}$ (d) $\frac{4}{5}$

(viii) The probability of getting a number greater than 6 on dice is:
(a) 0.33 (b) 1 (c) 0 (d) 0.5

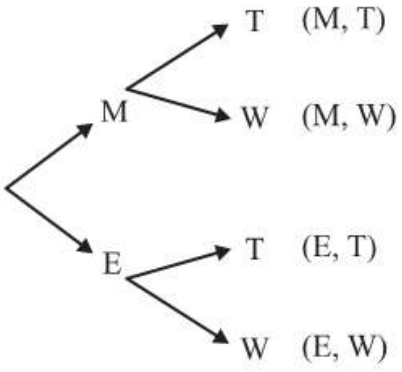
(ix) _____ outcomes are possible when we draw a card from deck of cards.
(a) 13 (b) 1 (c) 52 (d) 26

(x) If $P(E) = 0.07$, then the probability of 'not E' is:
(a) 0.95 (b) 0.89 (c) 0.93 (d) 0.90

2. Arshia selects a day from (Tuesday, Wednesday) and a time from (Morning, Evening). Draw a possibility diagram. Also sketch a tree diagram.

Solution

	Tuesday	Wednesday
Morning	(M, T)	(M, W)
Evening	(E, T)	(E, W)



3. A card is drawn from a deck of cards. Find the probability of getting an ace or a spade card.

Solution

Total cards: n(S) = 52

Let A = {Ace} ⇒ n(A) = 4

Let B = {Spade} ⇒ n(B) = 13

A ∩ B = {Ace of Spades} ⇒ n(A ∩ B) = 1

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$P(A \cup B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

4. Fatima dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. Find the probability that both cards are red.

Solution

Two cards dealt "without replacement".

Total cards = 52

Red cards = 26

$P(\text{Both Red}) = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

5. If two dice are rolled, then find the probability of getting the product of face values 6 or the difference of face values 5.

Solution

Two dice are rolled ⇒ n(S) = 36

Let A = {Product is 6} = {(1,6), (2,3), (3,2), (6,1)} ⇒ n(A) = 4

Let B = {Difference is 5} = {(1,6), (6,1)} ⇒ n(B) = 2

A ∩ B = {(1,6), (6,1)} ⇒ n(A ∩ B) = 2

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$P(A \cup B) = \frac{4}{36} + \frac{2}{36} - \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$

6. A bag contains 7 orange and 5 purple marbles. Two marbles are drawn one after the other without replacement. Find the probability that the first marble is orange and the second marble is also orange.

Solution

Total marbles = 7 Orange + 5 Purple = 12

Drawn "without replacement"

P(First Orange and Second Orange) $= \frac{7}{12} \times \frac{6}{11} = \frac{42}{132} = \frac{7}{22}$