

UNIT 9

Tangent and Angles of a Circle

EXERCISE 9.1

1. Find the value of x .

Solution

Given:

- Tangent AB
- Radius OB
- $\angle BAO = 38^\circ$

Since radius is perpendicular to tangent, so:

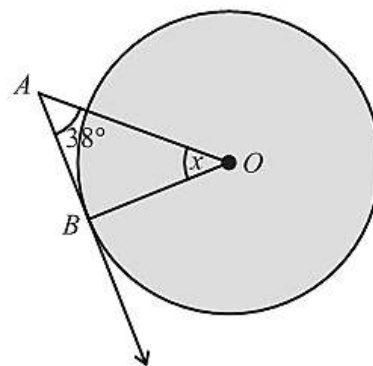
$$\angle ABO = 90^\circ$$

In triangle AOB

$$x + 38^\circ + 90^\circ = 180^\circ$$

$$x = 180^\circ - 128^\circ$$

$$x = 52^\circ$$



2. Find the angle ABC .

Solution

Concept:

- Tangent drawn from an external point to a circle are equal in length making $\triangle ABC$ an isosceles triangle
- In an isosceles triangle the angles opposite to equal sides are equal ($\angle ABC = \angle ACB$)
- The sum of angles in a triangle is 180°

Given:

$$\angle BAC = 65^\circ$$

Calculation:

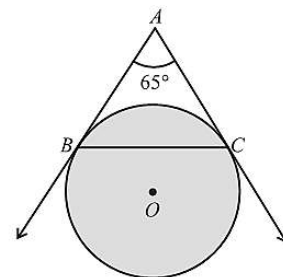
Let $\angle ABC = y$, since $\angle ABC = \angle ACB$

$$y + y + 65^\circ = 180^\circ$$

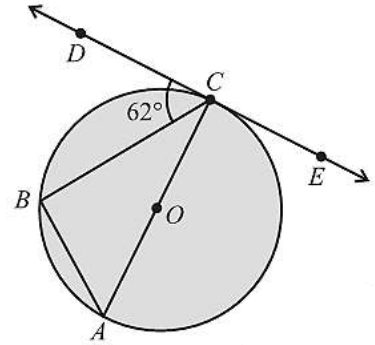
$$2y = 180^\circ - 65^\circ$$

$$y = \frac{115^\circ}{2}$$

$$y = 57.5^\circ$$



3. A , B and C are points on the circumference of a circle with centre at O . \overline{AC} is the diameter of the circle and \overleftrightarrow{DE} is the tangent to the circle at the point C and $m\angle BCD = 62^\circ$. Find
 (i) $m\angle BCA$ (ii) $m\angle BAC$



Solution

Given: \overleftrightarrow{DE} is a tangent at point c and $m\angle BCD = 62^\circ$, AC the diameter.

- (i) Find $m\angle BCD$

Concept:

- A tangent is perpendicular to the radius (or diameter) at the point of tangency, therefore $m\angle ACD = 90^\circ$

Calculation:

$$\angle BCD = \angle ACD - \angle BCD = 90^\circ - 62^\circ = 28^\circ$$

- (ii) Find $m\angle BAC$

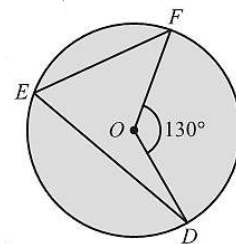
Concept:

- The angle in a semicircle is a right angle $\angle ABC = 90^\circ$
- Use the alternate segment theorem OR tangent – chord theorem: the angle between a tangent and a chord ($\angle BCD$) is equal to the angle in the alternate segment ($\angle BAC$)

Calculation:

By the alternate theorem: $\angle BAC = \angle BCD = 62^\circ$
 $m\angle BAC = 62^\circ$

4. D , E and F are points on the circumference of a circle with centre at O and $m\angle DOF = 130^\circ$. Find $m\angle DEF$.



Solution

Concept:

- The angle subtended an arc at the center of a circle is double the angle subtended by the same arc at any point on the remaining circumference.

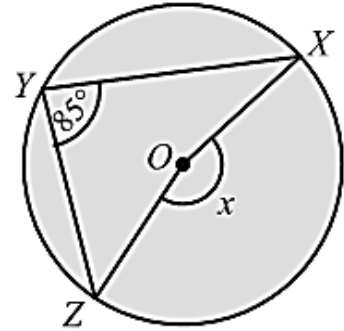
Given: Central angle $m\angle DOF = 130^\circ$

Calculation:

The angle of the circumference is half of the central angle ($m\angle DOF$)

$$m\angle DEF = \frac{m\angle DOF}{2} = \frac{130^\circ}{2} = 65^\circ$$

5. X , Y and Z are points on the circumference of a circle with centre at O and $m\angle XYZ = 85^\circ$. Find x .



Solution

Concept:

- **Using the standard theorem:** the central angle $x(m\angle XOZ)$ subtended by the major arc is twice the angle at the circumference $m\angle XYZ$.

Given: $m\angle XYZ = 85^\circ$

Calculation:

$$x = 2 \times m\angle XYZ$$

$$= 2 \times 85^\circ = 170^\circ$$

6. In a historical monument, a circular fountain with a radius of 3 m is built. A flagpole is erected 7 m away from the centre of the fountain. Two ropes from the pole are tied to the edge of the fountain, just touching it. Find the length of each rope.

Solution

Concept:

- A line touching a circle at just one point is a tangent
- The radius of a circle is perpendicular to the tangent at the point of creating a right-angled triangle
- We can use the Pythagorean theorem

Given:

- Radius of the fountain(r) = 3 m (perpendicular)
- Distance from the center to the flag pole (d) = 7 m (hypotenuse)
- Length of the rope (L)= tangent (base)

Calculation:

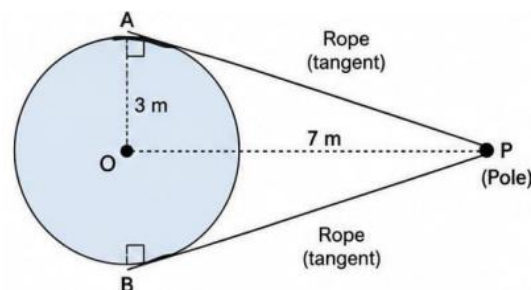
By using Pythagorean Theorem

$$d^2 = r^2 + L^2$$

$$7^2 = 3^2 + L^2$$

$$L^2 = 49 - 9$$

$$L^2 = 40 \text{ , } L = 6.32\text{ m}$$



7. Two circular gears touch each other externally for proper rotation in a machine. The radii of the two circular gears are 5cm and 7cm. What is the distance between their centres if they touch externally?

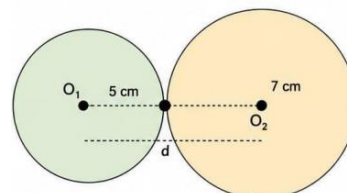
Solution**Concept:**

- When two circles touch each other externally, the distance between their centers is equal to the sum of their radii.

Given:

- Radius of the first gear (r_1) = 5cm
- Radius of the second gear (r_2) = 7cm

$$D = r_1 + r_2 = 5 + 7 = 12\text{cm}$$



8. A small sensor lies inside a satellite dish and touches its wall internally. If the dish has radius 15 cm and the sensor has radius 2.5 cm, find the distance between their centres.

Solution**Concept:**

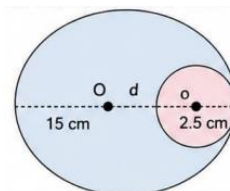
- When one circle lies inside the another and touches it internally, the distance between their centers is equal to the difference between the radii.

Given:

- Radius of the satellite dish (R) = 15cm
- Radius of the sensor (r) = 2.5 cm

Calculation:

$$\text{Distance} = R - r = 15\text{cm} - 2.5\text{cm} = 12.5\text{cm}$$



9. An inner holder touches the outer cylindrical container internally. If the outer container has radius 8 cm and the inner holder has radius 6 cm, find the distance between their centres.

Solution**Concept:**

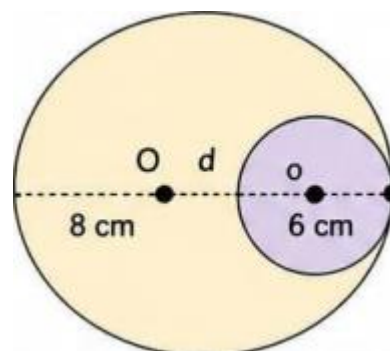
- When one circle lies inside another and touches it internally, the distance between their centers is equal to the difference between the radii.

Given:

- Radius of the outer container (R) = 8cm
- Radius of the inner container (r) = 6cm

Calculation:

$$\begin{aligned} \text{Distance} &= R - r \\ &= 8\text{cm} - 6\text{cm} = 2\text{cm} \end{aligned}$$



10. A pyramid-shaped sculpture is placed in the center of a circular plaza with a radius of 10 m . A decorative pole stands 26 m from the center of the circle. Two guide wires are attached from the pole to the plaza edge, just touching the circle. Find the length of each wire.

Solution

Concept:

- A wire that just touches the edge of a circular plaza acts as a tangent to the circle.
- The radius connecting the center of the plaza to the point of tangency from a right angle with the wire.
- This creates a right angle triangle where we can apply the Pythagorean Theorem.

Given:

- Radius of the circular plaza (r) = 10 m (perpendicular)
- Distance from the pole to the center (d) = 26 m (hypotenuse)
- Length of each guide wire (W) = tangent (base)

Calculation:

Using the Pythagorean Theorem

$$d^2 = r^2 + w^2$$

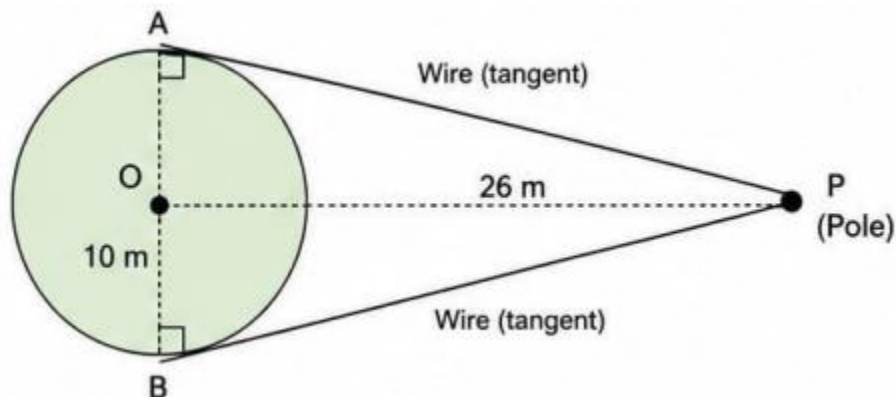
$$26^2 = 10^2 + w^2$$

$$676 = 100 + w^2$$

$$w^2 = 676 - 100$$

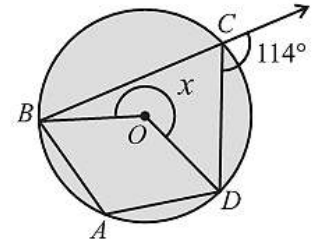
$$w^2 = 576$$

$$w = 24\text{ m}$$



EXERCISE 9.2

1. A, B, C and D are points on circumference of a circle with centre O as shown in figure.
 Find the angle x .



Solution

114° is the exterior angle at C .

Interior angle at C :

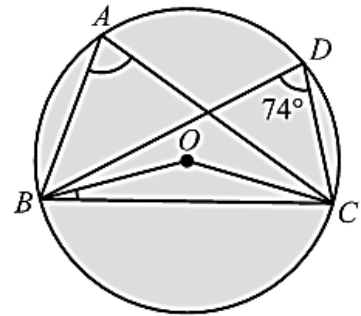
$$\angle BCD = 180^\circ - 114^\circ = 66^\circ$$

Angle at centre is twice the angle at circumference on the same arc:

$$x = 2 \times 66^\circ = 132^\circ$$

$$\text{Or } x = 360^\circ - 132^\circ = 228^\circ$$

2. In the adjoining figure $m\angle BDC = 74^\circ$,
 find $m\angle BAC$, $m\angle BOC$ and $m\angle OBC$.



Solution

Given:

$$\angle BDC = 74^\circ$$

Angles standing on the same chord are equal: $\angle BAC = 74^\circ$

Central angle:

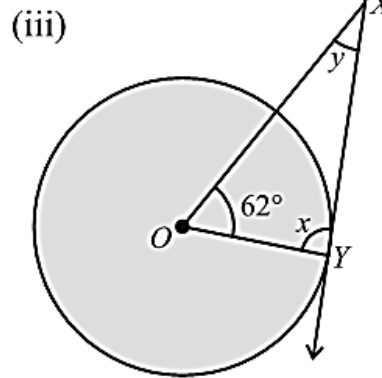
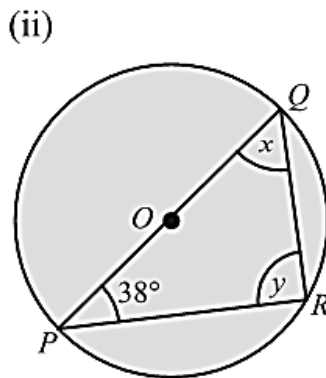
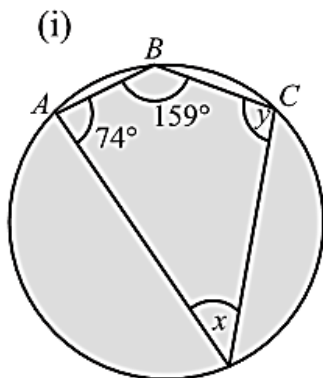
$$\angle BOC = 2 \times 74^\circ = 148^\circ$$

In triangle BOC :

$$OB = OC$$

$$\angle OBC = (180^\circ - 148^\circ) / 2 = 16^\circ$$

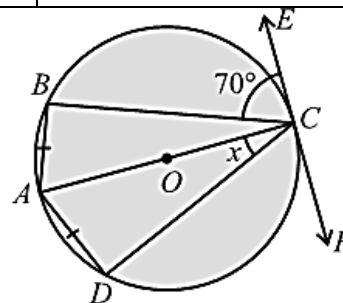
3. Find the angles x and y in the following figures.



Solution

<p>(i) Opposite angles of cyclic quadrilateral are supplementary. $x + 74^\circ = 180^\circ$ $x = 106^\circ$ $y + 159^\circ = 180^\circ$ $y = 21^\circ$</p>	<p>(ii) PQ is diameter. Angle in semicircle = 90° $y = 90^\circ$ In triangle PQR: $x + 90^\circ + 38^\circ = 180^\circ$ $x = 52^\circ$</p>	<p>(iii) Radius is perpendicular to tangent. $x = 90^\circ$ $62^\circ + 90^\circ + y = 180^\circ$ $y = 28^\circ$</p>
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4. Find the angle x in the given figure.



Solution: using alternate segment theorem

Angle between tangent and chord equals angle in alternate segment.

$\angle BCE = \angle BAC = 70^\circ$ and $\angle OCE = 90^\circ$

$\angle OCB = \angle OBC = 90^\circ - 70^\circ = 20^\circ$ also $\angle BOC = 180^\circ - 20^\circ - 20^\circ = 140^\circ$

$\angle ACD = \angle ABD = x = 20^\circ$ since $\widehat{AB} = \widehat{AD}$

5. In a circular arch over a monument entrance, two spotlight fixtures are placed such that they each shine from two different points on the arch to the same chord PQ on the base of the arch. If the angle formed at one light is 55° , what is the angle at the second light on the same side of chord PQ ?

Solution

Angles standing on the same chord and same side are equal.

Second angle = 55°

6. A circular garden has a walking path forming a quadrilateral inscribed in it. If one angle is 87° , what is the opposite angle?

Solution

Opposite angles of cyclic quadrilateral are supplementary.

Opposite angle = $180^\circ - 87^\circ = 93^\circ$

7. A ferris wheel has a radius of 12 m. If a passenger travels a distance of 18 m along the circumference of the ferris wheel, what is the angle (in radians) swept by the passenger's position from the starting point?

Solution

$$l = r\theta \Rightarrow \theta = \frac{l}{r}$$

$$\theta = 18 / 12 = 1.5 \text{ radian}$$

8. Find θ , when

(i) $l = 3 \text{ cm}, r = 2.2 \text{ cm}$

(ii) $l = 5.6 \text{ cm}, r = 2 \text{ cm}$

Solution

<p>(i)</p> $l = r\theta \Rightarrow \theta = \frac{l}{r}$ $\theta = 3 / 2.2 = 1.36 \text{ rad}$	<p>(ii)</p> $l = r\theta \Rightarrow \theta = \frac{l}{r}$ $\theta = 5.6 / 2 = 2.8 \text{ rad}$
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9. Find r , when

(i) $l = 5.5 \text{ cm}, \theta = 40^\circ 20'$

(ii) $l = 13 \text{ cm}, \theta = 70^\circ$

Solution

<p>(i)</p> $\theta = 40^\circ 20' = 0.704 \text{ rad}$ $l = r\theta \Rightarrow r = \frac{l}{\theta}$ $r = 5.5 / 0.704 = 7.81 \text{ cm}$	<p>(ii)</p> $\theta = 70^\circ = 1.222 \text{ rad}$ $l = r\theta \Rightarrow r = \frac{l}{\theta}$ $r = 13 / 1.222 = 10.64 \text{ cm}$
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10. Find l and area of sector, when

(i) $r = 1.7 \text{ cm}, \theta = 0.25 \text{ radian}$

(ii) $r = 3 \text{ cm}, \theta = 45^\circ$

Solution

<p>(i)</p> $l = r\theta$ $l = 1.7 \times 0.25 = 0.425 \text{ cm}$ $A = \frac{1}{2} r^2 \theta$ $A = \frac{1}{2} \times (1.7)^2 \times 0.25 = 0.361 \text{ cm}^2$	<p>(ii)</p> $l = r\theta$ $l = 3\pi / 4 = 2.36 \text{ cm}$ $A = \frac{1}{2} r^2 \theta$ $A = 9\pi / 8 \text{ cm}^2 = 3.53 \text{ cm}^2$
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11. Uzma cut a pizza of radius 14 cm into 8 equal slices. What is the area of one slice (sector).

Solution

Radius of pizza: $r = 14$ cm

Area of whole pizza = πr^2

$$A = \pi \times (14)^2 = \pi \times 196$$

$$A = (22/7) \times 196 = 616 \text{ cm}^2 \qquad \text{Using } \pi = 22/7$$

Since the pizza is divided into 8 equal slices:

$$\text{Area of one slice} = 616 \div 8 = \mathbf{77 \text{ cm}^2}$$

12. The perimeter and area of a sector are 14 cm and 10 cm^2 respectively. Find the radius of the circle and the central angle of the sector.

Solution

Perimeter of sector = $2r + l$

$$14 = 2r + l$$

$$l = 14 - 2r$$

Area of sector = $1/2 \times r \times l$

$$10 = 1/2 \times r \times (14 - 2r)$$

$$20 = r(14 - 2r)$$

$$20 = 14r - 2r^2$$

$$2r^2 - 14r + 20 = 0$$

$$r^2 - 7r + 10 = 0$$

$$(r - 5)(r - 2) = 0$$

Therefore, $r = 5$ cm or $r = 2$ cm

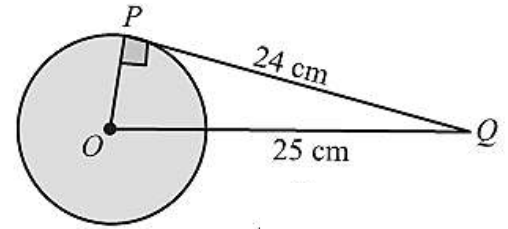
Case I: $r = \mathbf{5 \text{ cm}}$ $l = 14 - 2(5) = 4$ $\theta = \frac{l}{r} = 4/5 = \mathbf{0.8 \text{ radian}}$	Case II: $r = \mathbf{2 \text{ cm}}$ $l = 14 - 2(2) = 10$ $\theta = \frac{l}{r} = 10/2 = \mathbf{5 \text{ radians}}$
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REVIEW EXERCISE

9

1. Four possible answers are given for the following questions. Choose the correct answer.
- (i) Tangent is a line that touches the circumference of the circle at :
 (a) many points (b) three points
 (c) two points (d) one point
- (ii) _____ tangents can be drawn to a circle from a point outside the circle.
 (a) one (b) two (c) three (d) many
- (iii) _____ tangents can be drawn to a circle from centre of the circle.
 (a) zero (b) one (c) two (d) three
- (iv) In circle, radius and tangent are:
 (a) parallel (b) equal
 (c) perpendicular (d) zero
- (v) An angle in a segment greater than a semicircle is _____ angle:
 (a) an acute (b) right (c) an obtuse (d) straight
- (vi) The angle subtended by the arc at the centre of the circle is called:
 (a) acute angle (b) central angle
 (c) right angle (d) complete angle
- (vii) What is the measure of an angle inscribed in a semi-circle?
 (a) 45° (b) 60°
 (c) 90° (d) 120°
- (viii) Any two angles in the same segment of the circle are:
 (a) supplementary (b) complementary
 (c) equal (d) zero
- (ix) Area of the sector of the circle = _____
 (a) $\frac{1}{2}r\theta$ (b) $\frac{3}{2}r^2\theta$ (c) $\frac{1}{2}r^2\theta$ (d) $\frac{3}{2}r\theta$
- (x) If $r = 6\text{cm}$ and $\theta = 2$ radians, then arc length is:
 (a) 12 cm (b) 8 cm (c) 6 cm (d) 3 cm

2. In the adjoining figure, find the radius of the circle.



Solution

Given Data from Figure:

- Hypotenuse (OQ) = 25 cm
- Tangent segment (PQ) = 24 cm
- $\angle OPQ = 90^\circ$ (Since a radius is always perpendicular to a tangent at its point of contact).

Step-by-step Solution:

Applying the Pythagorean theorem in the right-angled triangle ΔOPQ :

$$OQ^2 = OP^2 + PQ^2$$

$$(25)^2 = OP^2 + (24)^2$$

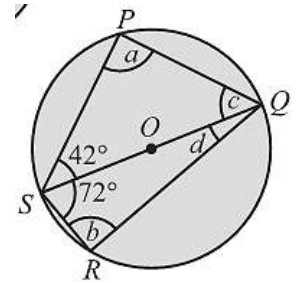
$$625 = OP^2 + 576$$

$$OP^2 = 625 - 576$$

$$OP^2 = 49$$

$$OP = \sqrt{49} = 7 \text{ cm}$$

3. Find the angles a , b , c and d in the given figure, if O is the centre of the circle.



Solution

Angle at the center = 180° because it's a straight line across the circle.

Angle at the circumference = $1/2 \times$ angle at the center.

$$a = 1/2 \times 180^\circ = 90^\circ$$

$$b = 1/2 \times 180^\circ = 90^\circ$$

c is the angle at the circumference for arc PS.

Angle at the center for arc PS = 96° .

$$c = 1/2 \times 96^\circ = 48^\circ$$

$$\text{Arc PS} = 2 \times c = 2 \times 48^\circ$$

$$\text{Arc PS} = 96^\circ$$

$$\angle POQ = 180^\circ - \text{Arc PS}$$

$$\angle POQ = 180^\circ - 96^\circ = 96^\circ$$

$$d + d + \angle POQ = 180^\circ$$

$$2d + 96^\circ = 180^\circ$$

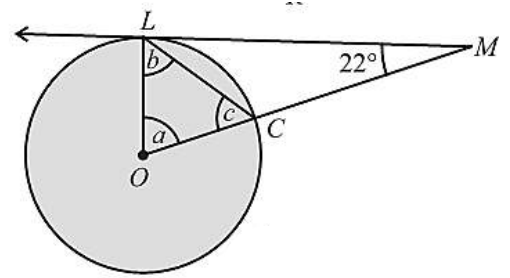
$$2d = 180^\circ - 96^\circ$$

$$2d = 84^\circ$$

$$d = 84^\circ / 2$$

$$d = 18^\circ$$

4. Find the angles a , b and c , if \overrightarrow{ML} is a tangent line and O is the centre of the circle.



Solution

Using Alternate Segment Theorem:

Alternate Segment Theorem: Angle between tangent and chord = angle in alternate segment.

Angle between tangent ML and chord LC = $\angle MLC = 56^\circ$

So $\angle OLC = \angle MLC = 56^\circ$

$b = 56^\circ$

$OC = OL = \text{radius}$, so triangle OLC is isosceles.

Therefore $\angle OCL = \angle OLC$

$c = b = 56^\circ$

Sum of angles in triangle OLC:

$\angle LOC + \angle OLC + \angle OCL = 180^\circ$

$a + 56^\circ + 56^\circ = 180^\circ$

$a + 112^\circ = 180^\circ$

$a = 180^\circ - 112^\circ$

$a = 68^\circ$

5. Find the angles marked in the given figure, where O is the centre of the circle.

Solution

In triangle ΔSOR , segments OS and OR are both radii of the circle ($OS = OR$). Since two sides are equal, it forms an isosceles triangle where its base angles are equivalent: $a = b$

Summing up the internal angles of ΔSOR :

$\angle SOR + a + b = 180^\circ$

$100^\circ + a + a = 180^\circ$ (Since $a = b$)

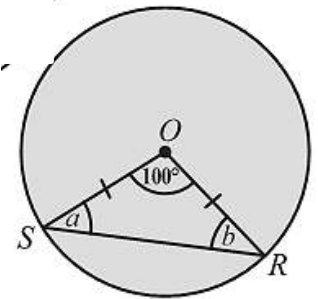
$100^\circ + 2a = 180^\circ$

$2a = 180^\circ - 100^\circ$

$2a = 80^\circ$

$a = 40^\circ$

Since $a = b$, then: $b = 40^\circ$



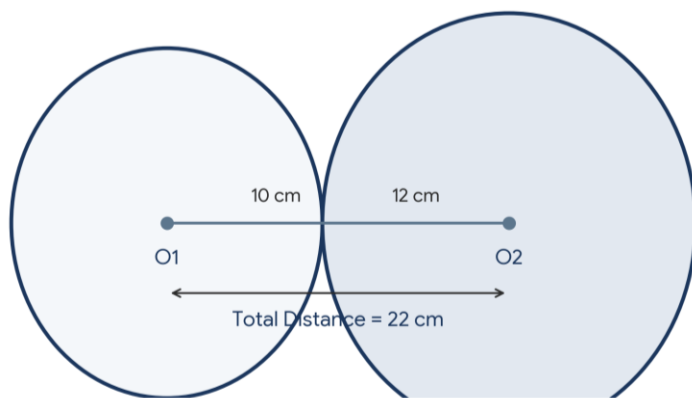
6. Two round dining tables are arranged to touch at their edges. The radii of the two round dining tables are 10 cm and 12 cm. What is the distance between their centres if they touch externally?

Solution

When two distinct circles touch each other externally, the straight-line distance (d) between their centers is exactly equal to the sum of their individual radii ($r_1 + r_2$).

$$d = r_1 + r_2$$

$$d = 10 \text{ cm} + 12 \text{ cm} = 22 \text{ cm.}$$



7. A circular mosaic pattern is laid on the floor beneath a modern pyramid structure. Two tiles are placed such that they both connect to the same chord EF and reach two points G and H on the arc above EF . If $m\angle EGF = 65^\circ$, find $m\angle EHF$.

Solution

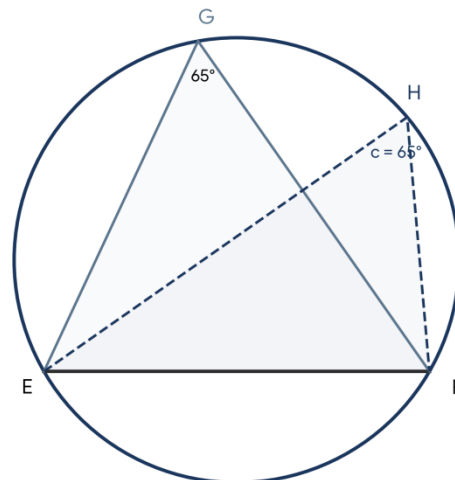
According to the properties of cyclic domains and the circle segment theorem, angles subtended by the same base chord across the same major segment region are perfectly uniform.

Since both vertex points G and H rely natively on the identical base chord baseline EF , it implies that:

$$m\angle EHF = m\angle EGF$$

Given: $m\angle EGF = 65^\circ$

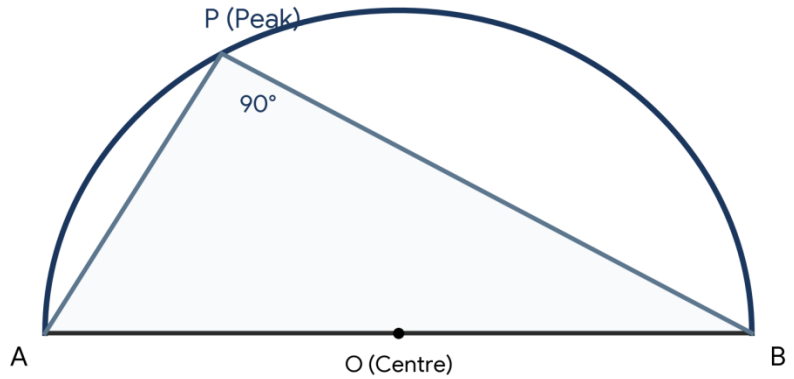
Therefore: $m\angle EHF = 65^\circ$.



8. A triangular frame within a semicircular gate joins the ends of the diameter to the peak. What is the angle formed at the peak?

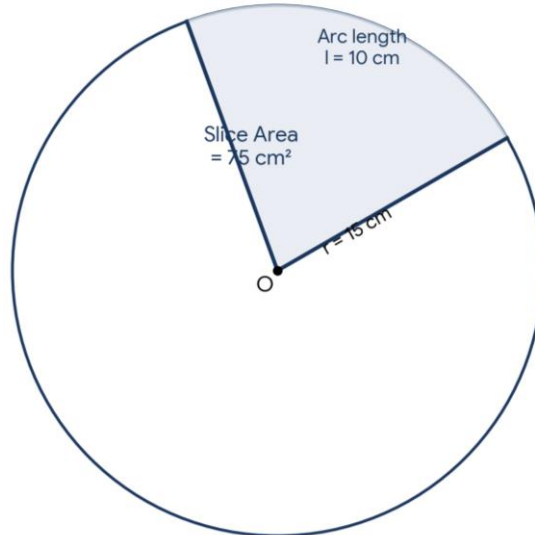
Solution

This system relies directly on Thales's Semicircular Apex Principle. The theorem demonstrates that whenever the diameter of a circle is projected as baseline chords matching an arbitrary node along its higher outer arc circumference; it produces a perfect right angle. Because both end elements of the diameter line extend up to the high peak terminal along the curved path of the semicircle, the interior vertex angle maps natively to a right angle (90°).



9. A circular birthday cake has a radius of 15cm. A slice has an arc length of 10cm. What is the area of this slice?

Solution



The physical area (A) of a sector slice cut out from a circle structure is derived smoothly using the active radius (r) along with its arc length span (l):

$$A = \frac{1}{2} \times l \times r$$

$$A = \frac{1}{2} \times 10 \times 15$$

$$A = 5 \times 15 = 75 \text{ cm}^2.$$

10. A sector cut from a circle of radius 5cm has a perimeter of 16cm. Find area of this sector.

Solution

Step 1: Isolate the sector's edge arc dimension (l).

The geometric perimeter boundaries (P) of any standard sector slice wrap two straight radial borders plus the curved perimeter arc length segment:

$$P = 2r + l$$

$$16 = 2(5) + l$$

$$16 = 10 + l$$

$$l = 6 \text{ cm}$$

Step 2: Solve the total area equation.

Using the arc area relationship:

$$A = \frac{1}{2} \times l \times r$$

$$A = \frac{1}{2} \times 6 \times 5$$

$$A = 3 \times 5 = 15 \text{ cm}^2.$$

