UNIT 8
Chords and Arcs of a Circle

EXERCISE 8.1

1. Calculate the length of a chord which stands at a distance of 5 cm from the centre of a circle whose radius is 13 cm.

Solution

$$\begin{aligned}\text{Chord} &= 2\sqrt{r^2 - d^2} \\ &= 2\sqrt{13^2 - 5^2} \\ &= 2\sqrt{169 - 25} \\ &= 2\sqrt{144} \\ &= 24 \text{ cm}\end{aligned}$$

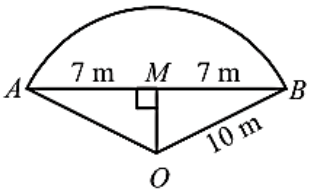
2. In construction, three steel rods are fixed at points A, B, and C (not in a straight line). A circular hoop needs to pass through all three. How many hoops can be used?

Solution

Three non-collinear points determine a unique circle passing through them.

$$\text{Number of hoops} = 1.$$

3. In a park, lamp posts are 14 m apart on the edge of a circular part of radius 10 m as shown in figure. Find the distance of the chord from the centre of the park.



Solution

$$\begin{aligned}d &= \sqrt{r^2 - (\frac{\text{chord}}{2})^2} \\ &= \sqrt{10^2 - 7^2} \\ &= \sqrt{100 - 49} \\ &= \sqrt{51} \\ &\approx 7.14 \text{ m}\end{aligned}$$

4. In a circle, chords AB and CD both have length 10 cm.
If the distance from the centre to $\overline{AB}$ is 6 cm, what is the distance from centre to $\overline{CD}$?

Solution

Chords AB and CD have equal length
they are equidistant from the centre.

$$d_{AB} = 6$$
$$d_{CD} = d_{AB} = 6 \text{ cm}$$

5. Two holes A and B are drilled 12 cm apart on a circular tabletop of radius 10 cm. Find the perpendicular distance from the centre to AB.

Solution

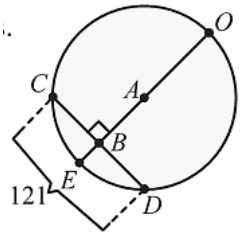
$$\begin{aligned} d &= \sqrt{r^2 - (\frac{\text{chord}}{2})^2} \\ &= \sqrt{10^2 - 6^2} \\ &= \sqrt{100 - 36} \\ &= \sqrt{64} \\ &= 8 \text{ cm} \end{aligned}$$

6. A chord 8 cm long is at a distance of 3 cm from the centre. Calculate the radius of the circle.

Solution

$$\begin{aligned} r &= \sqrt{d^2 + (\frac{\text{chord}}{2})^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \text{ cm} \end{aligned}$$

7. In the given figure, $m\overline{CD} = 121$ units and $m\overline{BC} = 3x$ units. Find the value of x.



Solution

Since $\overline{CD}$ and $\overline{CB}$ are chords in the circle with a diameter $\overline{OCD}$, and angles in the same segment are equal, we have

$m\widehat{BC} = \frac{1}{2}m\widehat{CD}$

$3x = \frac{1}{2}(121°)$

$x = \frac{121}{6} = 20.17\text{units}$

8. In a circle with centre at O, the perpendicular distance of the each chord PQ and RS from the centre is 6 cm. If the length of chord PQ is 18 cm, find the length of the other chord.

Solution

$d = 6, \quad PQ = 18.$

$r^2 = d^2 + \left(\frac{PQ}{2}\right)^2.$

$r^2 = 6^2 + 9^2.$

$r^2 = 36 + 81 = 117.$

$RS = 2\sqrt{r^2 - d^2}.$

$RS = 2\sqrt{117 - 6^2}.$

$RS = 2\sqrt{117 - 36}.$

$RS = 2\sqrt{81}.$

$RS = 18 \text{ cm}$

9. A line from the centre of a circle cuts a 10 cm chord at right angle where radius of the circle is 6cm. What is the length from the centre to the chord?

Solution

$r = 6, \quad \text{chord} = 10.$

$d^2 = r^2 - \left(\frac{10}{2}\right)^2.$

$d^2 = 6^2 - 5^2.$

$d^2 = 36 - 25.$

$d = \sqrt{11}.$ = 3.32 cm

10. In a circle, a perpendicular is drawn from the centre to chord AB. If $m\overline{AB} = 12\,\text{cm}$, what is the length of each segment after bisecting?

Solution

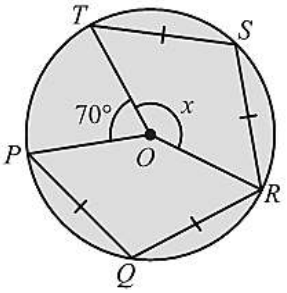
$m\overline{AB} = 12.$

$\text{segment length} = \frac{12}{2}.$

$= 6\ \text{cm}$

EXERCISE 8.2

1. In the given figure, if $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ and $m\angle POT = 70°$, then find the value of x.

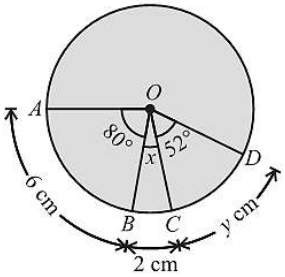


Solution

Since $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$,
the corresponding central angles are equal.
Let each of them be $x°$.
$m\angle POT + 4x = 360°$
$70° + 4x = 360°$
$4x = 290°$
$x = 145°$

$x = 145°$

2. In the given figure, find the values of x and y.



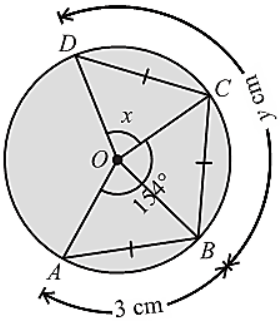
Solution

Let the radius be r.
Arc AB = 6 cm corresponds to 80°.
$\frac{6}{80} = \frac{2}{x} \Rightarrow x = \frac{80 \times 2}{6} = 26.67° \approx 77°$
Also, $\angle AOB + \angle BOC + \angle COD = 180°$
$80° + x + 52° = 180°$
$x = 180° - 132° = 77°$

Arc AB = 6 cm corresponds to 80°.
Arc CD = y cm corresponds to 52°.
$\frac{6}{80} = \frac{y}{52} \Rightarrow y = \frac{6 \times 52}{80} = 3.9 \text{ cm} \approx 6 \text{ cm}$

$x = 77°$, $y = 6$ cm

3. Find the values of x and y in the given figure, such that $m\overset{\frown}{AB} = m\overset{\frown}{BD}$



Solution

Given $m\overset{\frown}{AB} = m\overset{\frown}{BD}$
$\Rightarrow \angle AOB = \angle BOD$
From the figure, $\angle AOB = 140°$
$\Rightarrow \angle BOD = 140°$
Now, $\angle AOD + \angle DOB + \angle BOA = 360°$
$x + 140° + 140° = 360°$
$x = 360° - 280° = 80°$
For arcs, equal central angles subtend equal arcs, so arc BD = arc AB = 3 cm.
Hence, y = 4 cm.

$x = 28°$, $y = 4$ cm

4. Two congruent arcs in a circular track subtend angles of 60° each at the centre. If the length of one chord of the circle is 10 metres, what is the length of other chord?

Solution

Congruent arcs subtend equal chords.

Both arcs subtend $60^\circ \Rightarrow$ their chords are equal.

So, the other chord also has length 10 metres.

Length of other chord = 10 metres

5. In a circular fountain, two water jets are installed such that they spray water along arcs of equal length. If one jet sprays between points P and Q and the straight-line distance (chord PQ) is 12 metres, what is the straight-line distance (chord RS) covered by the second jet spraying along arc RS, which is congruent to arc PQ?

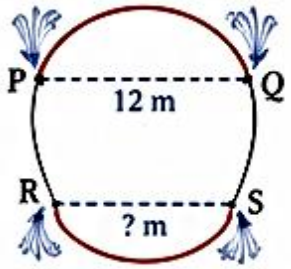
Solution:

Equal arcs subtend equal chords.

Since arc $\overline{RS} \cong$ arc $PQ \Rightarrow$ chord RS = chord PQ

Given chord PQ = 12 metres.

Chord RS = 12 metres



6. In a circular park, two walkways $\overline{AB}$ and $\overline{CD}$ are both straight paths (chords) of length 14 metres. What can be said about the minor arcs subtended by these walkways?

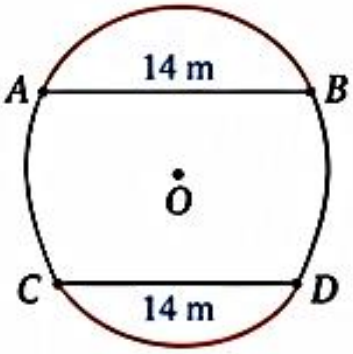
Solution:

Equal chords of a circle subtend equal (minor) arcs.

Since $\overline{AB} = \overline{CD}$ = 14 metres,

the minor arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are congruent.

Minor arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are congruent.



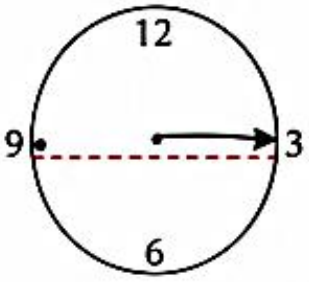
7. In two congruent circular clocks, the minute hand points from the centre to the 3 on both. A decorative string connects 3 to 9 on both clocks. Are the arcs from 3 to 9 on both clocks congruent?

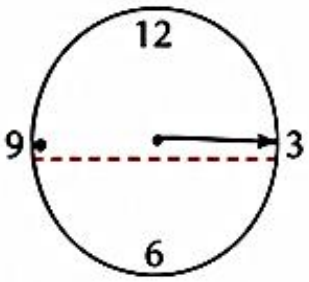
Solution:

Both clocks are congruent circles, and points 3 and 9 are in corresponding positions.

Therefore, arcs from 3 to 9 on both clocks are congruent.

Yes, the arcs from 3 to 9 on both clocks are congruent.





REVIEW EXERCISE 8

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) Distance of a point on the circumference to the centre of the circle is called:
(a) radius (b) arc (c) chord (d) tangent

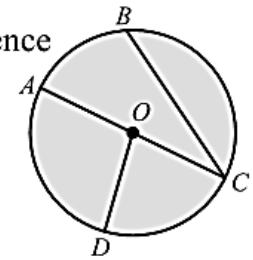
(ii) Radii of same circles are:
(a) all unequal (b) all equal
(c) half of each chord (d) double of the diameter

(iii) The boundary of the circle is called:
(a) chord (b) segment (c) circumference (d) diameter

(iv) Any part of a circumference is called:
(a) chord (b) diameter (c) radius (d) arc

(v) A chord passing through the centre of the circle is called:
(a) radius (b) diameter (c) secant (d) circumference

(vi) In the given figure, what is AB?
(a) diameter (b) tangent (c) chord (d) arc



(vii) In the given figure, major arc is:
(a) $m\overset{\frown}{AB}$ (b) $m\overset{\frown}{BC}$ (c) $m\overset{\frown}{BDC}$ (d) $m\overset{\frown}{AD}$

(viii) _____ circle(s) can pass through the three non-collinear points.
(a) one (b) two (c) three (d) many

(ix) Perpendicular bisector of a chord always passes through the ____of circle.
(a) arc (b) radius (c) centre (d) circumference

(x) The sum of the measures of central angles of a circle is:
(a) 90° (b) 180° (c) 270° (d) 360°

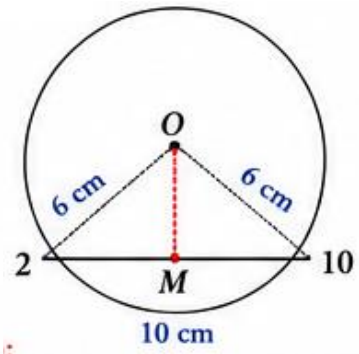
2. On a circular clock with a radius of 6cm, the points from 2 to 10 form a chord that is 10cm long. Find the perpendicular distance from the centre of clock to the chord.

Solution

$OM \perp$ chord and M is midpoint.
So, $PM = 5$ cm, $OP = 6$ cm.

$$OP^2 = OM^2 + PM^2$$
$$6^2 = OM^2 + 5^2$$
$$OM^2 = 36 - 25 = 11$$
$$OM = \sqrt{11} \text{ cm}$$

Perpendicular distance $= \sqrt{11}$ cm



3. A chord 6 cm long is at a distance of 4 cm from the centre. Calculate the radius of the circle.

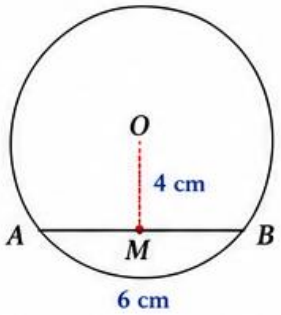
Solution:

$OM \perp$ chord and M is midpoint.

$AM = 3$ cm, $OM = 4$ cm.

$$OA^2 = OM^2 + AM^2$$
$$r^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$r = 5 \text{ cm}$$

Radius = 5 cm



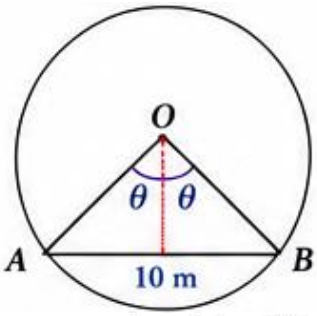
4. In a circular park, two benches are placed so that the chords formed by their positions are both 10 m long. What can you say about the angles subtended at the centre by each bench?

Solution:

Equal chords in a circle subtend equal angles at the centre.

Therefore, $\angle AOB = \angle BOA = \theta$.

The angles subtended at the centre by each bench are equal.



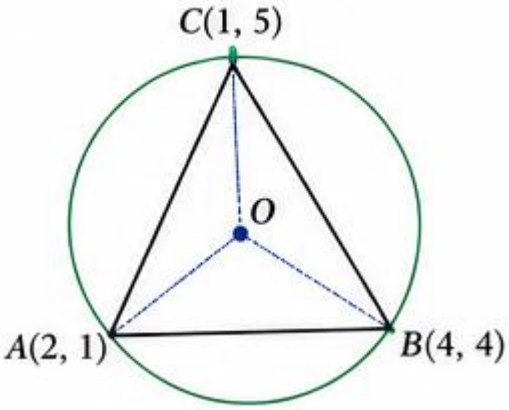
5. Jannat is designing a triangular garden with corners at points $A(2, 1)$, $B(4, 4)$ and $C(1, 5)$. Can she install a circular fountain that touches all three corners?

Solution:

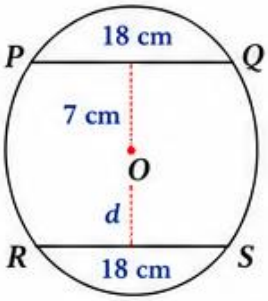
The perpendicular bisectors of all sides intersect at one point O (circumcentre).

Hence, a circle with centre O passes through A, B and C.

Yes, she can install the fountain.



6. Two chords, PQ and RS, each measure 18cm in length. If the distance of PQ from the centre of circle is 7cm. Find the distance of RS from the centre.



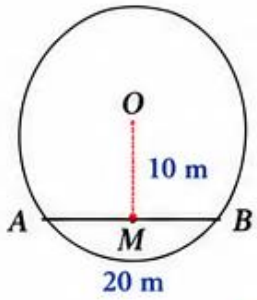
Solution:

Equal chords are equidistant from the centre.

So, $d = 7$ cm

Distance of RS from the centre $= 7$ cm

7. A tree branch lies across a circular pond, forming a 20 m chord. A measuring rod having length 10 m is perpendicular to chord from the centre. What is the radius of the pond?



Solution:

$AM = 10$ m (half of 20 m),

$OM = 10$ m.

$OA^2 = OM^2 + AM^2$

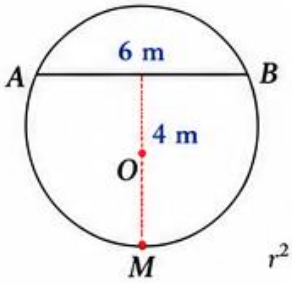
$r^2 = 10^2 + 10^2 = 100 + 100$

$r^2 = 200$

$r = 10\sqrt{2}$ m

Radius $= 10\sqrt{2}$ m

8. A steel bar 6 m long lies inside a circular structure with ends on the circle. It is bisected by a perpendicular rod from the centre. Find the radius of the circular structure if the perpendicular distance from the centre to the bar is 4 m.



Solution:

$AM = 3$ m (half of 6 m),

$OM = 4$ m.

$OA^2 = OM^2 + AM^2$

$r^2 = 4^2 + 3^2 = 16 + 9 = 25$

$r = 5$ m

Radius $= 5$ m