UNIT 7 Trigonometry

EXERCISE 7.1

1. Find the signs of the following:

(i)	$\sin 55^\circ$	(ii)	$\cos 145^\circ$	(iii)	$\tan 111^\circ$
(iv)	$\sec 179^\circ$	(v)	$\text{cosec}\, 88^\circ$	(vi)	$\cot 14^\circ$

Solution

1. **(i)** Positive **(ii)** Negative **(iii)** Negative **(iv)** Negative **(v)** Positive **(vi)** Positive

2. Fill in the blanks:

Solution

(i)	$\tan(180^\circ - \theta) = \ldots - \ldots \tan\theta$	(ii)	$\sin(180^\circ - \theta) = \ldots + \ldots \sin\theta$
(iii)	$\tan(90^\circ + \theta) = \ldots - \ldots \cot\theta$	(iv)	$\cos(90^\circ + \theta) = \ldots - \ldots \sin\theta$
(v)	$\text{cosec}(180^\circ - \theta) = \ldots + \ldots \text{cosec}\,\theta$	(vi)	$\sec(90^\circ - \theta) = \ldots + \ldots \text{cosec}\,\theta$

3. Without using calculator, find the exact values of the following trigonometric functions:

(i)	$\sin 150^\circ$	(ii)	$\tan 150^\circ$	(iii)	$\sec 150^\circ$
(iv)	$\text{cosec}\, 120^\circ$	(v)	$\cos 120^\circ$	(vi)	$\cot 120^\circ$
(vii)	$\sin 135^\circ$	(viii)	$\sec 135^\circ$	(ix)	$\cot 135^\circ$

Solution

$\boldsymbol{sin150}^\circ = sin(1 \times 90^\circ + 60^\circ) = cos60^\circ = \frac{1}{2}$	$\boldsymbol{tan150}^\circ = tan(1 \times 90^\circ + 60^\circ) = -cot60^\circ = -\frac{1}{\sqrt{3}}$
$\boldsymbol{sec150}^\circ = sec(1 \times 90^\circ + 60^\circ) = -cosec60^\circ = -\frac{2}{\sqrt{3}}$	$\boldsymbol{cosec120}^\circ = cosec(1 \times 90^\circ + 30^\circ) = sec30^\circ = \frac{2}{\sqrt{3}}$
$\boldsymbol{cos120}^\circ = cos(1 \times 90^\circ + 30^\circ) = -sin30^\circ = -\frac{1}{2}$	$\boldsymbol{cot120}^\circ = cot(1 \times 90^\circ + 30^\circ) = -tan30^\circ = -\frac{1}{\sqrt{3}}$
$\boldsymbol{sin135}^\circ = sin(1 \times 90^\circ + 45^\circ) = cos45^\circ = \frac{1}{\sqrt{2}}$	$\boldsymbol{sec135}^\circ = sec(1 \times 90^\circ + 45^\circ) = -cosec45^\circ = -\sqrt{2}$
$\boldsymbol{cot135}^\circ = cot(1 \times 90^\circ + 45^\circ) = -tan45^\circ - 1$	

EXERCISE 7.2

1. Solve the triangle ABC, in which

(i) $a = 6.1$ cm, $b = 8.4$ cm, $\alpha = 42°$

(ii) $a = 12.2$ cm, $c = 15.8$ cm, $\gamma = 50°$

(iii) $b = 5.2$ cm, $c = 5$ cm, $\gamma = 48°$

(iv) $b = 4.8$ cm, $a = 4$ cm, $\beta = 71°$

(v) $\beta = 70°$, $b = 8$cm, $\alpha = 100°$

(vi) $a = 6$ cm, $\alpha = 55°$, $\gamma = 60°$

(vii) $c = 7$ cm, $\beta = 34°$, $\gamma = 64°$

(viii) $b = 12$ cm, $\alpha = 92°$, $\beta = 77°$

Solution

(i) $a = 6.1$, $b = 8.4$, $\alpha = 42°$

Law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}.$$

$$\sin\beta = \frac{b\sin\alpha}{a} = \frac{8.4\sin 42°}{6.1} = 0.921.$$

$$\beta = \sin^{-1}(0.921) = 67.1°.$$

$$\gamma = 180° - \alpha - \beta = 180° - 42° - 67.1° = 70.9$$

$$c = \frac{a\sin\gamma}{\sin\alpha} = \frac{6.1\sin 70.9°}{\sin 42°} = 8.8.$$

(ii) $a = 12.2$, $c = 15.8$, $\gamma = 50°$

Law of sines:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}.$$

$$\sin\alpha = \frac{a\sin\gamma}{c} = \frac{12.2\sin 50°}{15.8} = 0.591.$$

$$\alpha = \sin^{-1}(0.591) = 36.2°.$$

$$\beta = 180° - \alpha - \gamma = 180° - 36.2° - 50° = 93.8$$

$$b = \frac{c\sin\beta}{\sin\gamma} = \frac{15.8\sin 93.8°}{\sin 50°} = 20.6.$$

(iii) $b = 5.2$, $c = 5$, $\gamma = 48°$

Law of sines:

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$

$$\sin\beta = \frac{b\sin\gamma}{c} = \frac{5.2\sin 48°}{5} = 0.773.$$

$$\beta = \sin^{-1}(0.773) = 50.6°.$$

$$\alpha = 180° - \beta - \gamma = 180° - 50.6° - 48° = 81.4$$

$$a = \frac{b\sin\alpha}{\sin\beta} = \frac{5.2\sin 81.4°}{\sin 50.6°} = 6.61.$$

(iv) $b = 4.8$, $a = 4$, $\beta = 71°$

Law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}.$$

$$\sin\alpha = \frac{a\sin\beta}{b} = \frac{4\sin 71°}{4.8} = 0.788.$$

$$\alpha = \sin^{-1}(0.788) = 52.0°.$$

$$\gamma = 180° - \alpha - \beta = 180° - 52.0° - 71° = 57.0$$

$$c = \frac{a\sin\gamma}{\sin\alpha} = \frac{4\sin 57.0°}{\sin 52.0°} = 4.20.$$

(v) $\beta = 70^\circ$, $b = 8$, $\alpha = 100^\circ$ $\gamma = 180^\circ - \alpha - \beta = 180^\circ - 100^\circ - 70^\circ = 10^\circ$. Law of sines: $a = \frac{b\sin\alpha}{\sin\beta} = \frac{8\sin 100^\circ}{\sin 70^\circ} = 8.39.$ $c = \frac{b\sin\gamma}{\sin\beta} = \frac{8\sin 10^\circ}{\sin 70^\circ} = 1.48.$	**(vi)** $a = 6$, $\alpha = 55^\circ$, $\gamma = 60^\circ$ $\beta = 180^\circ - \alpha - \gamma = 180^\circ - 55^\circ - 60^\circ = 65^\circ$. Law of sines: $b = \frac{a\sin\beta}{\sin\alpha} = \frac{6\sin 65^\circ}{\sin 55^\circ} = 6.64.$ $c = \frac{a\sin\gamma}{\sin\alpha} = \frac{6\sin 60^\circ}{\sin 55^\circ} = 6.34.$
(vii) $c = 7$, $\beta = 34^\circ$, $\gamma = 64^\circ$ $\alpha = 180^\circ - \beta - \gamma = 180^\circ - 34^\circ - 64^\circ = 82^\circ$. Law of sines: $a = \frac{c\sin\alpha}{\sin\gamma} = \frac{7\sin 82^\circ}{\sin 64^\circ} = 7.72.$ $b = \frac{c\sin\beta}{\sin\gamma} = \frac{7\sin 34^\circ}{\sin 64^\circ} = 4.35.$	**(viii)** $b = 12$, $\alpha = 92^\circ$, $\beta = 77^\circ$ $\gamma = 180^\circ - \alpha - \beta = 180^\circ - 92^\circ - 77^\circ = 11^\circ$. Law of sines: $a = \frac{b\sin\alpha}{\sin\beta} = \frac{12\sin 92^\circ}{\sin 77^\circ} = 12.3.$ $c = \frac{b\sin\gamma}{\sin\beta} = \frac{12\sin 11^\circ}{\sin 77^\circ} = 2.29.$

2. Calculate area of each triangle ABC.

(i) $a = 7$ cm, $b = 8$ cm, $\gamma = 38^\circ$ (ii) $a = 11$ cm, $c = 14$ cm, $\beta = 51^\circ$

(iii) $b = 3$ cm, $c = 9$ cm, $\alpha = 78^\circ$ (iv) $a = 10$ cm, $\alpha = 62^\circ$, $\beta = 69^\circ$

(v) $c = 4$ cm, $\beta = 36^\circ$, $\gamma = 80^\circ$ (vi) $c = 6.6$ cm, $\alpha = 23^\circ$, $\gamma = 89^\circ$

(vii) $a = 5.3$ cm, $b = 4.7$ cm, $c = 8.2$ cm (viii) $a = 6.12$ cm, $b = 8.34$ cm, $c = 7.12$ cm

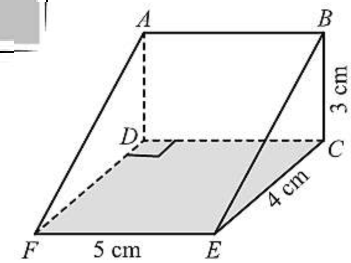
Solution

(i) $a = 7$, $b = 8$, $\gamma = 38^\circ$ Area formula: $\text{Area} = \frac{1}{2}ab\sin\gamma.$ $\text{Area} = \frac{1}{2}(7)(8)\sin 38^\circ = 17.24\text{ cm}^2$	**(ii)** $a = 11$, $c = 14$, $\beta = 51^\circ$ $\text{Area} = \frac{1}{2}ac\sin\beta.$ $\text{Area} = \frac{1}{2}(11)(14)\sin 51^\circ = 59.82\text{ cm}^2$
(iii) $b = 3$, $c = 4$, $\alpha = 78^\circ$ $\text{Area} = \frac{1}{2}bc\sin\alpha.$ $\text{Area} = \frac{1}{2}(3)(4)\sin 78^\circ = 13.21\text{ cm}^2$	**(iv)** $a = 10$, $\alpha = 62^\circ$, $\beta = 69^\circ$ $\gamma = 180^\circ - \alpha - \beta = 180^\circ - 62^\circ - 69^\circ = 49^\circ$. Area formula: $\text{Area} = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}.$ $\text{Area} = \frac{10^2\sin 69^\circ\sin 49^\circ}{2\sin 62^\circ} = 39.89\text{ cm}^2$

(v) $c=4$, $\beta=36^\circ$, $\gamma=80^\circ$ $\alpha=180^\circ-\beta-\gamma=180^\circ-36^\circ-80^\circ=64^\circ$ $\text{Area}=\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}.$ $\text{Area}=\frac{4^2\sin 64^\circ\sin 36^\circ}{2\sin 80^\circ}=4.29\ \text{cm}^2$	**(vi)** $c=6.6$, $\alpha=23^\circ$, $\gamma=89^\circ$ $\beta=180^\circ-\alpha-\gamma=180^\circ-23^\circ-89^\circ=68^\circ.$ $\text{Area}=\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}.$ $\text{Area}=\frac{6.6^2\sin 23^\circ\sin 68^\circ}{2\sin 89^\circ}=7.89\ \text{cm}^2$
(vii) $a=5.3$ cm, $b=4.7$ cm, $c=8.2$ cm $S=\frac{a+b+c}{2}=\frac{5.3+4.7+8.2}{2}=9.1$ $\Delta=\sqrt{S(S-a)(S-b)(S-c)}$ $\Delta=\sqrt{9.1(9.1-5.3)(9.1-4.7)(9.1-8.2)}$ $\Delta=\sqrt{9.1(3.8)(4.4)(0.9)}$ $\Delta=\sqrt{136.94}$ $\Delta=11.70\text{cm}^2$	(viii) $a=6.12$ cm, $b=8.34$ cm, $c=7.12$ cm $S=\frac{a+b+c}{2}=\frac{6.12+8.34+7.12}{2}=10.79$ $\Delta=\sqrt{S(S-a)(S-b)(S-c)}$ $\Delta=$ $\sqrt{10.79(10.79-6.12)(10.79-8.34)(10.79-7.12)}$ $\Delta=\sqrt{10.79(4.67)(2.45)(3.67)}$ $\Delta=\sqrt{453.1}$ $\Delta=21.29\text{cm}^2$

EXERCISE 7.3

1. In the triangular prism, find
   (i) the length $\overline{CF}$.
   (ii) the length $\overline{BF}$.
   (iii) the angle BFC, correct to one decimal place.



Solution

(i) In right ΔFEC,

$$CF^2 = FE^2 + EC^2 = 5^2 + 4^2 = 25 + 16 = 41$$

$$CF = \sqrt{41} \approx \boxed{6.4\text{ cm}}$$

(ii) In right ΔBCF,

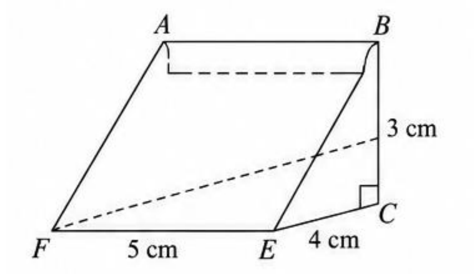
$$BF^2 = BC^2 + CF^2 = 3^2 + 41 = 9 + 41 = 50$$

$$BF = \sqrt{50} = 5\sqrt{2} \approx \boxed{7.1\text{ cm}}$$

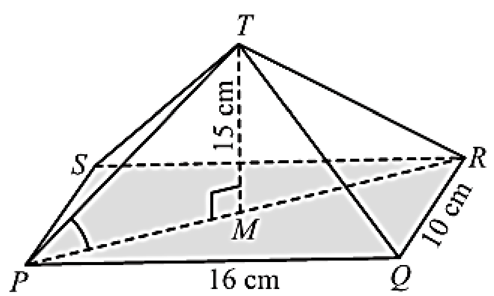
(iii) In right ΔBCF,

$$\tan \angle BFC = \frac{BC}{CF} = \frac{3}{6.4} = 0.46875$$

$$\angle BFC = \tan^{-1}(0.46875) \approx \boxed{25.1^\circ}$$



2. The diagram shows a triangular pyramid with a horizontal rectangular base $PQRS$, in which $m\overline{PQ} = 16$ cm, $m\overline{QR} = 10$ cm. M is the midpoint of the line PR. The vertex, T, is vertically above M and $m\overline{MT} = 15$ cm. Calculate the size of the angle between TP and the base $PQRS$. Give your answer correct to 1 decimal place.



Solution

In rectangle $PQRS$,

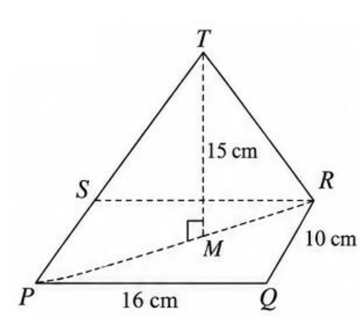
$$PR = \sqrt{PQ^2 + QR^2} = \sqrt{16^2 + 10^2} = \sqrt{356} \approx 18.9\text{ cm}$$

Since M is midpoint of PR, $PM = \frac{PR}{2} \approx \frac{18.9}{2} = 9.45$ cm

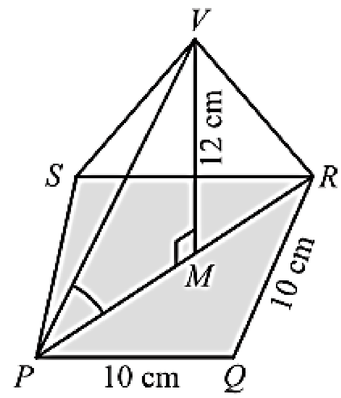
In right ΔTPM,

$$\tan\theta = \frac{MT}{PM} = \frac{15}{9.45} = 1.587$$

$$\theta = \tan^{-1}(1.587) \approx \boxed{57.8^\circ}$$



3. The diagram shows a pyramid. The base, *PQRS*, is a horizontal square of side 10 cm. The vertex, *V*, is vertically above the midpoint, *M* and $m\overline{VM} = 12$ cm. Calculate the size of angle *VPM*.



Solution

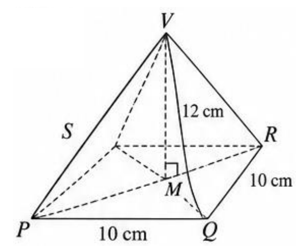
In square *PQRS*, diagonal $PR = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14$ cm

Since *M* is midpoint of *PR*, $PM = \dfrac{PR}{2} = \dfrac{14.14}{2} = 7.07$ cm

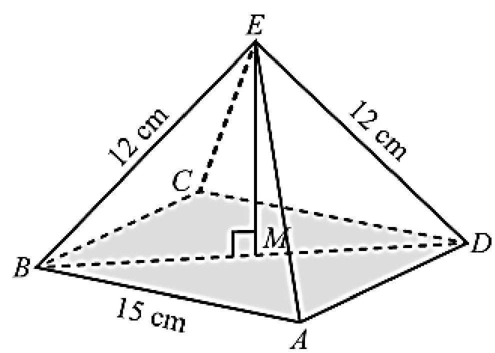
In right $\triangle VPM$,

$$\tan\theta = \frac{VM}{PM} = \frac{12}{7.07} = 1.697$$

$$\theta = \tan^{-1}(1.697) \approx \boxed{59.5^\circ}$$



4. *ABCDE* is a square based pyramid, in which $m\overline{AE} = m\overline{BE} = m\overline{CE} = m\overline{DE} = 12$ cm and $m\overline{AB} = 15$ cm. Calculate the size of angle *DEB*. Give your answer in degree (whole numbers).



Solution

In square base *ABCD*, diagonal $BD = \sqrt{15^2 + 15^2} = \sqrt{450} = 15\sqrt{2} \approx 21.21$ cm

In $\triangle DEB$, $DE = BE = 12$ cm and $BD \approx 21.21$ cm

By cosine rule,

$$BD^2 = DE^2 + BE^2 - 2(DE)(BE)\cos\angle DEB$$

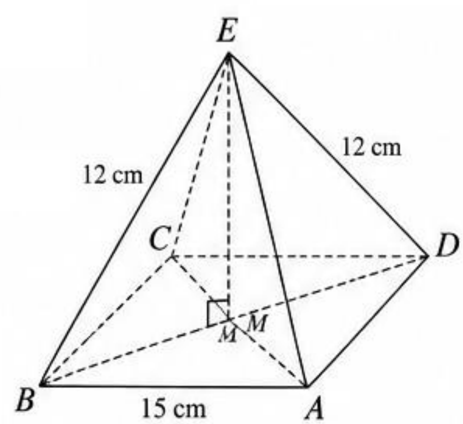
$$(21.21)^2 = 12^2 + 12^2 - 2(12)(12)\cos\angle DEB$$

$$450 = 288 - 288\cos\angle DEB$$

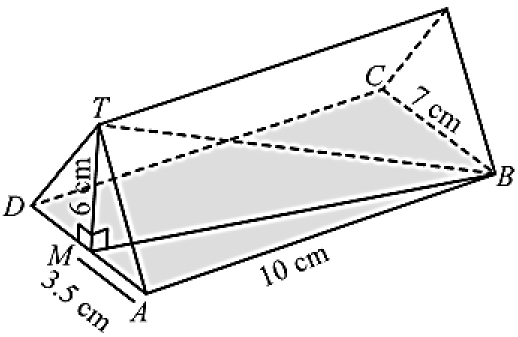
$$288\cos\angle DEB = 288 - 450 = -162$$

$$\cos\angle DEB = -\frac{162}{288} = -0.5625$$

$$\angle DEB = \cos^{-1}(-0.5625) \approx \boxed{124^\circ}$$



5. The diagram shows a triangular prism with a horizontal rectangular base $ABCD$. $m\overline{AB} = 10$ cm, $m\overline{BC} = 7$ cm, M is the midpoint of AD. The vertex T is vertically above M and $m\overline{MT} = 6$ cm. Calculate the size of the angle between $\overline{TB}$ and the base.



Solution

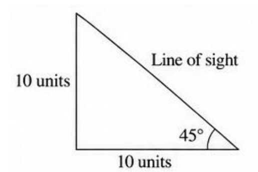
In right ΔTMB

$MB = \sqrt{AM^2 + AB^2} = \sqrt{3.5^2 + 10^2} = 10.60cm$

$tan\theta = \frac{MT}{MB} = \frac{6}{10.60} = 0.5668$

$\theta = tan^{-1}(0.5668) = 29.5°$

6. In an isometric game, the camera is placed at a 45° angle from the ground. If the player is 10 units in front and 10 units above, what is the direct line of sight distance?



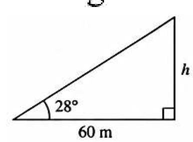
Solution

Using Pythagoras theorem,

Distance $= \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2} \approx$ **14.14 units**

7. A surveyor spots the top of a tower at an elevation angle of 28°. He is standing 60 metres from the base. Find the height of the tower.

Solution



$\tan 28° = \frac{height}{60}$

height $= 60 \tan 28° = 60 \times 0.5317 \approx$ **31.90 m**

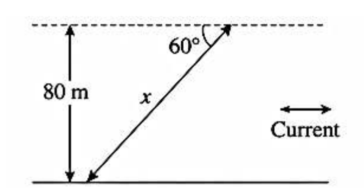
8. A boat crosses a river 80 m wide, at a 60° angle to the current. What distance does the boat actually travel?

Solution

$cos60° = \frac{80}{x}$

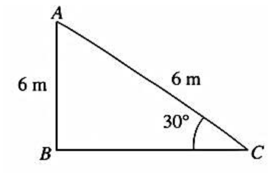
$x = \frac{80}{cos60°}$

$x = \frac{80}{\frac{1}{2}} = 160m$



9. A listener hears a sound from two speakers. One is 6 m directly ahead and the other is at 30° to the side, 6 m away. Find the distance between the speakers.

Solution



Using cosine rule,

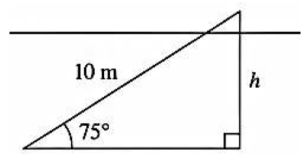
$$d^2 = 6^2 + 6^2 - 2(6)(6)\cos 30°$$

$$= 72 - 72\left(\frac{\sqrt{3}}{2}\right) = 72\left(1 - \frac{\sqrt{3}}{2}\right)$$

$d^2 = 9.65 \Rightarrow d = \sqrt{9.65} = 3.1m$

10. A 10 m ladder leans against a wall making an angle of 75° with the ground. How high does it reach up the wall?

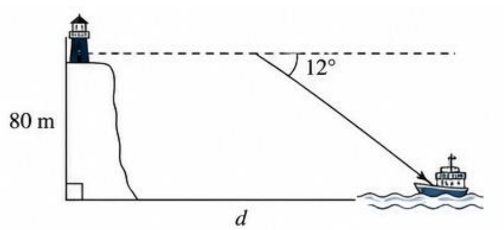
Solution



$$\text{Height} = 10 \sin 75° = 10 \times 0.9659 \approx \boxed{9.66 \text{ m}}$$

11. A lighthouse is located on a cliff 80 m above sea level. A ship is spotted at an angle of depression of 12°. How far is the ship from the base of the cliff?

Solution



$$\tan 12° = \frac{80}{d}$$

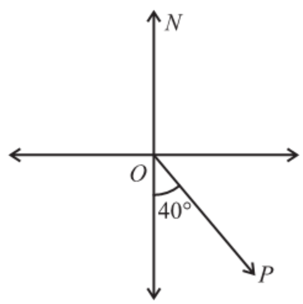
$$d = \frac{80}{\tan 12°} = \frac{80}{0.2126} \approx \boxed{376.5 \text{ m}}$$

EXERCISE 7.4

1. Find bearing of point P in each of the following:

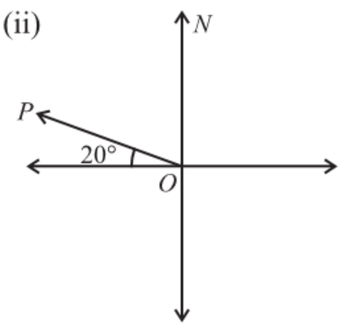
Solution

(i)



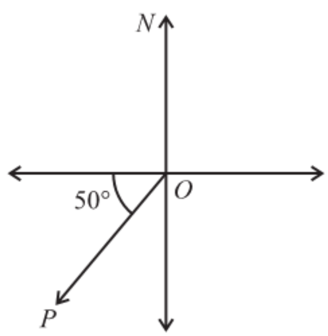
Bearing of $\overrightarrow{OP} = 180° - 40° = 140°$

(ii)



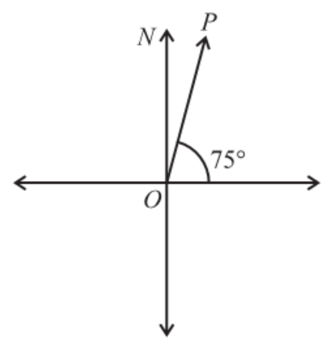
Bearing of $\overrightarrow{OP} = 270° + 20° = 290°$

(iii)



Bearing of $\overrightarrow{OP} = 180° + 40° = 220°$

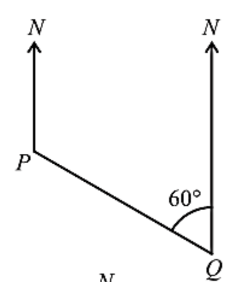
(iv)



Bearing of $\overrightarrow{OP} = 090° - 75° = 015°$

Or Bearing of $\overrightarrow{OP} = 270° - 50° = 220°$

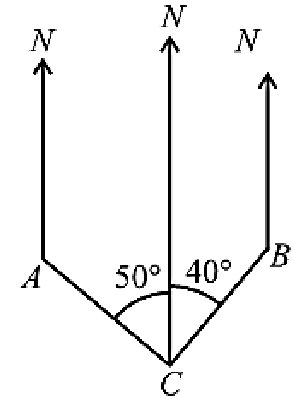
2. The diagram shows the positions of two ships P and Q.
   (i) What is the bearing of ship P from ship Q?
   (ii) What is bearing of ship Q from ship P?



Solution

i. Bearing of ship $\vec{P}$ from ship $\vec{Q} = 360° - 60° = 300°$
ii. Bearing of ship $\vec{Q}$ from ship $\vec{P} = 300° - 180° = 120°$

3. The diagram shows 3 places A, B and C. Find the bearing of:
   (i) A from C
   (ii) C from A
   (iii) C from B
   (iv) B from C

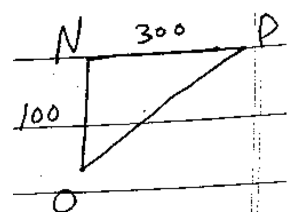


Solution

i. Bearing of $\vec{A}$ from $\vec{C} = 360° - 50° = 310°$
ii. Bearing of $\vec{C}$ from $\vec{A} = 180° - 50° = 130°$
iii. Bearing of $\vec{C}$ from $\vec{B} = 180° + 40° = 220°$
iv. Bearing of $\vec{B}$ from $\vec{C} = 040°$

4. Abdul Hadi walks 100 m North and then 300 m East.
   (i) How far is he from his starting position?
   (ii) On what bearing should he walk to get back to his starting position?

Solution



(i) Distance from starting position:

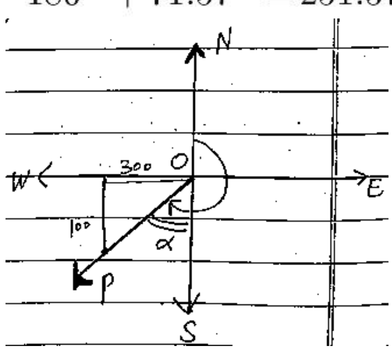
$$\sqrt{100^2 + 300^2} = \sqrt{10000 + 90000} = 316.2 \text{ m}$$

(ii) Bearing to return to start:

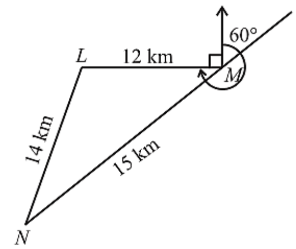
$$\tan\theta = \frac{300}{100} \implies \theta = \tan^{-1}(3) = 71.57°.$$

Bearing is **S** $71.57°$ **W** (or $252°$

$180° + 71.57° = 251.57°$).



5. Three ships L, M, N are in the position shown in the diagram. Ship M is North East of ship N. Find the bearing of L from M.



Solution

Bearing of $\vec{L}$ from $\vec{M} = 270°$

Or another solution

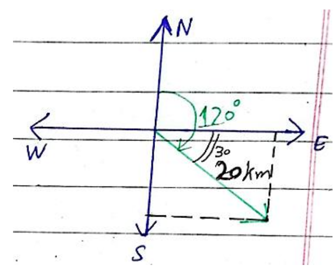
At M, MN is on a bearing 225° (from M to N).

ML is 90° to the left of MN.

Bearing of L from M

$= 225° - 90° = 135°$

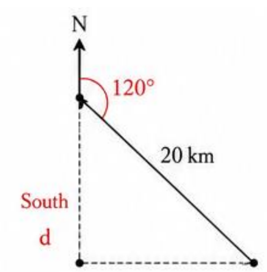
6. A ship sails 20 km on a bearing of 120°. How far to the South has the ship moved from its original position?



Solution

Ship movement from its original position from the south $= 20Sin30° = 10\text{km}$

Or another solution



Southward component

$= 20 \times \cos(120° - 180°)$

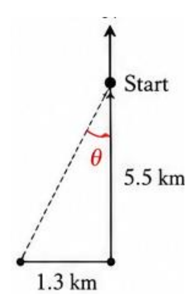
$= 20 \times \cos 60°$

$= 20 \times \frac{1}{2}$

$= 10$ km

7. Fatima walked South for 5.5 km and then turned West for 1.3 km. Calculate Huria's bearing from her starting point.

Solution



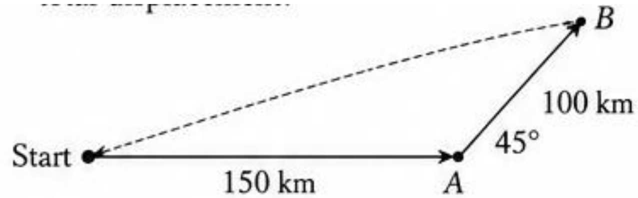
In ΔOPS

$tan \propto = \frac{1.3}{5.5} \Rightarrow \propto = tan^{-1}(0.236) \Rightarrow \propto = 13.29°$

Horia's bearing from Start $= 180° + 13.29° = 193.2°$

8. An aircraft flies 150 km east, then 100 km northeast (45° from East). What is the total displacement?

Solution



Resultant components:

East $= 150 + 100\cos 45° = 220.71$ km

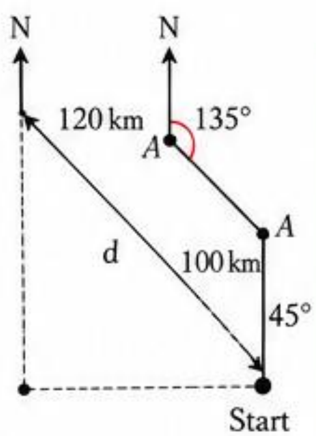
North $= 100\sin 45° = 70.71$ km

Displacement $= \sqrt{(220.71)^2 + (70.71)^2}$

$= 231.91$ km

9. A ship sails 100 km on a bearing of 045°, then changes course and sails 120 km on a bearing of 135°. Find the distance between the starting point and the final position.

Solution



Angle between the two courses at $A = 135° - 45° = 90°$

Using cosine rule:

$d^2 = 100^2 + 120^2 - 2(100)(120)\cos 90°$

$= 10000 + 14400 - 0$

$d = \sqrt{24400} = 156.2$ km

REVIEW EXERCISE 7

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) What is the value of cot 60°?

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

(ii) All trigonometric ratios are positive in:

(a) I-quadrant (b) II-quadrant
(c) III-quadrant (d) IV-quadrant

(iii) cosec θ is positive in:

(a) I & III-quadrants (b) II & IV-quadrants
(c) I & II-quadrants (d) I & IV-quadrants

(iv) $\sin(90^\circ + \theta) =$

(a) $\sin\theta$ (b) $-\sin\theta$ (c) $\cos\theta$ (d) $-\cos\theta$

(v) $\tan(180^\circ - \theta) =$

(a) $\tan\theta$ (b) $\cot\theta$ (c) $-\cot\theta$ (d) $-\tan\theta$

(vi) The law of sines is:

(a) $\frac{b}{\sin\alpha} = \frac{a}{\sin\beta} = \frac{c}{\sin\gamma}$ (b) $\frac{a}{\sin\alpha} = \frac{c}{\sin\beta} = \frac{b}{\sin\gamma}$

(c) $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ (d) $\frac{a}{\sin\alpha} = \frac{a}{\sin\beta} = \frac{a}{\sin\gamma}$

c

(vii) The law of cosines is:

(a) $\cos\alpha = \frac{b^2 + a^2 - c^2}{2ab}$ (b) $\cos\beta = \frac{c^2 + b^2 - a^2}{2cb}$

(c) $\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$ (d) $\cos\gamma = \frac{ca^2 + b^2 - c^2}{2ab}$

(viii) Area of $\Delta ABC =$

(a) $\frac{1}{2}ab\sin\alpha$ (b) $\frac{1}{2}bc\sin\beta$ (c) $\frac{1}{2}\sin\gamma$ (d) $\frac{1}{2}ac\sin\beta$

(ix) Bearing is measured from:

(a) East (b) West (c) North (d) South

(x) Bearing is written as a:

(a) 1 figure (b) 2 figures (c) 3 figures (d) 4 figures

2. Calculate the area of ΔABC, in which

(i) $a = 4$ cm, $b = 6$ cm, $c = 8$ cm (ii) $b = 2.1$ cm, $c = 5$ cm, $\gamma = 45°$

(iii) $c = 3.1$ cm, $\gamma = 44°$, $\alpha = 36°$

Solution

Q2 (i)	Q2 (ii)
$S = \frac{a+b+c}{2} = \frac{4 + 6 + 8}{2} = 9$ $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$ Area = $\sqrt{[9(9-4)(9-6)(9-8)]}$ = $\sqrt{135}$ ≈ 11.62 cm^2	Area = ½ × 2.1 × 5 × sin45° = 5.25 × 0.707 ≈ 3.71 cm^2

Q2 (iii)

β = 180° − 44° − 36° = 100°

$$\boldsymbol{a = \frac{3.1 \times sin36°}{sin44°} = 2.623}$$

$$\boldsymbol{b = \frac{3.1 \times sin100°}{sin44°} = 4.394}$$

Area = $\frac{1}{2}\boldsymbol{absin44° = 4.01cm^2}$

3. Solve the triangle ABC, in which

(i) $a = 5.4$ cm, $b = 3.4$ cm, $\alpha = 49°$ (ii) $\alpha = 32°$, $\gamma = 48°$, $c = 81$cm

Solution

Q3 (i)	Q3 (ii)
(i) $a = 5.4$ cm, $b = 3.4$ cm, $\alpha = 49°$ $\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$ $\sin\beta = \frac{b\sin\alpha}{a} = \frac{3.4\sin 49°}{5.4} = 0.4755$ $\beta = 28.39°$ $\gamma = 180° - \alpha - \beta = 102.61°$ $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \Rightarrow c = \frac{a\sin\gamma}{\sin\alpha} = 6.48$ cm	β = 180° − 32° − 48° = 100° a / sin32° = 81 / sin48° a ≈ 57.8 cm b / sin100° = 81 / sin48° b ≈ 107.3 cm

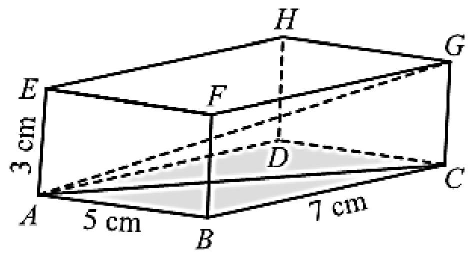
4. The diagram shows a cuboid $ABCDEFGH$ in which $m\overline{AB} = 5$ cm, $m\overline{BC} = 7$ cm and $m\overline{AE} = 3$ cm.

(i) Calculate the length of $\overline{AG}$.

Give your answer correct to 3 significant figures.

(ii) Calculate the size of the angle between $\overline{AG}$ and the plane $ABCD$.

Give your answer correct to 1 decimal place.



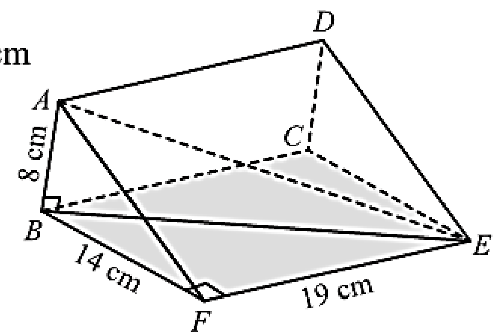
Solution

In a cuboid, $\overline{AG}$ is the space diagonal. $\overline{AG} = \sqrt{(AB)^2 + (BC)^2 + (AE)^2}$ $= \sqrt{5^2 + 7^2 + 3^2} = \sqrt{83} = 9.11$ cm	Let D be the foot of the perpendicular from G to the plane $ABCD$. In right triangle ADG, $\tan\theta = \frac{AE}{AD} = \frac{3}{\sqrt{5^2+7^2}} = \frac{3}{\sqrt{74}}$ $\theta = 20.9°$

5. The diagram shows a triangular prism $ABCDEF$ in which $m\overline{AB} = 8$ cm, $m\overline{BF} = 14$ cm and $m\overline{EF} = 19$ cm.

(i) Calculate the distance between A and F.

(ii) Calculate the angle between $\overline{AF}$ and the plane $BCEF$.



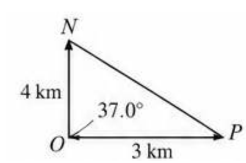
Solution

In right triangle ABF (right-angled at B), $AF = \sqrt{AB^2 + BF^2} = \sqrt{8^2 + 14^2} = \sqrt{260} = 16.1$ cm	In right triangle AFC (right-angled at F), $\sin\theta = \frac{AF}{AC} = \frac{AB}{AC} = \frac{8}{16.1}$ $\theta = 29.7°$

6. Hashim walks 4 km due North, then turns and walks 3km due East. What is the bearing from the starting point to his final position?

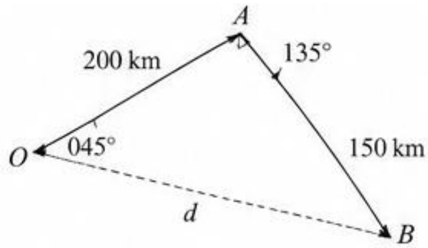
Solution

Let the starting point be O, final position be P. Then $OP = \sqrt{4^2 + 3^2} = 5$ km.
The direction of OP is 37.0° east of North.
Bearing = 037°



7. A pilot flies 200 km on a bearing of 045°, then turns and flies 150 km on a bearing of 135°. How far is the plane from its original position?

Solution



The angle between the two bearings is $135° - 45° = 90°$.

Using the cosine rule,

$d^2 = 200^2 + 150^2 - 2(200)(150)\cos 90° = 40000 + 22500 - 0 = 62500$

$d = \sqrt{62500} = 250 \text{ km}$