UNIT 6

Vectors in Plane

EXERCISE 6.1

1. Name the quadrant in which each point lies.

(i) (4, 3) (ii) (5, –4) (iii) (–6,2) (iv) (–4, –4)

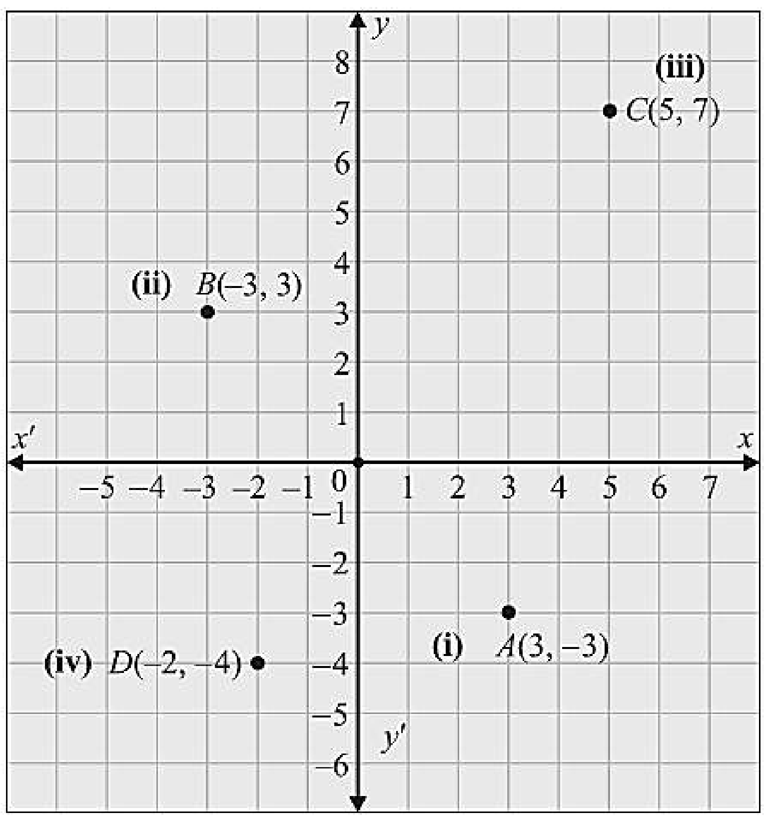
Solution

1. (i) I **(ii)** IV **(iii)** II **(iv)** III

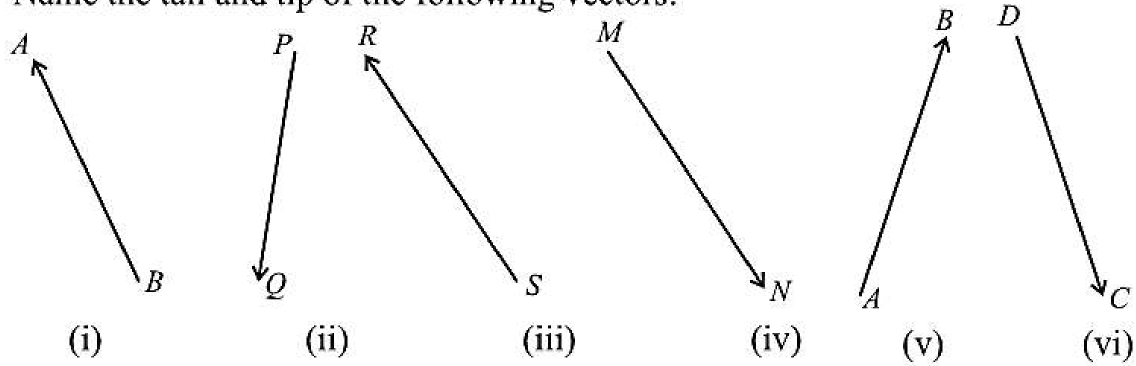
2. Plot the following points on the coordinate plane:

(i) $A(3, -3)$ (ii) $B(-3, 3)$ (iii) $C(5, 7)$ (iv) $D(-2, -4)$

Solution



3. Name the tail and tip of the following vectors:



Solution

(i) Tail $= B$, Tip $= A$

(ii) Tail $= P$, Tip $= Q$

(iii) Tail $= S$, Tip $= R$

(iv) Tail $= M$, Tip $= N$

(v) Tail $= A$, Tip $= B$

(vi) Tail $= D$, Tip $= C$

4. Write the vector AB in the form of $x\underline{i} + y\underline{j}$:

(i) $A(1, -7), B(-2, 4)$ (ii) $A(8, 9), B(12, 3)$

Solution

(i) A(1,-7),B(-2,4)

$$\overrightarrow{AB} = (x_2-x_1)\underline{i}+(y_2-y_1)\underline{j}$$
$$= (-2-1)\underline{i}+(4+7)\underline{j}$$
$$= -3\underline{i}+11\underline{j}$$

(ii) A(8,9),B(12,3)

$$\overrightarrow{AB} = (x_2-x_1)\underline{i}+(y_2-y_1)\underline{j}$$
$$= (12-8)\underline{i}+(3-9)\underline{j}$$
$$= 4\underline{i}-6\underline{j}$$

5. Find the magnitude of the $\underline{a}$:

(i) $\underline{a} = -3\underline{i} + 2\underline{j}$ (ii) $\underline{a} = 4\underline{i} - 3\underline{j}$ (iii) $\underline{a} = \frac{1}{2}\underline{i} + \frac{3}{2}\underline{j}$

Solution

(i) $\underline{a} = -3\underline{i}+2\underline{j}$ $\lvert a\rvert = \sqrt{(-3)^2 + (2)^2}$ $= \sqrt{9+4}$ $= \sqrt{13}$	(ii) $\underline{a} = 4\underline{i}-3\underline{j}$ $\lvert a\rvert = \sqrt{(4)^2 + (-3)^2}$ $= \sqrt{16+9}$ $= \sqrt{25} = 5$

(iii) $\underline{a} = \frac{1}{2}\underline{i} + \frac{3}{2}\underline{j}$

$$|a| = \sqrt{(\frac{1}{2})^2 + (\frac{3}{2})^2}$$

$$= \sqrt{\frac{1}{4} + \frac{9}{4}}$$

$$= \sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{2}$$

6. Find a unit vector in the direction of the vector given below:

(i) $\underline{a} = -4\underline{i} + 5\underline{j}$ (ii) $\underline{a} = 6\underline{i} + 8\underline{j}$

(iii) $\underline{a} = \frac{1}{\sqrt{6}}\underline{i} + \frac{1}{\sqrt{6}}\underline{j}$ (iv) $\underline{a} = \frac{1}{2}\underline{i} - \frac{3}{4}\underline{j}$

Solution

(i) $\underline{a} = -4\underline{i}+5\underline{j}$

$$\hat{a} = \frac{\underline{a}}{|a|} = \frac{-4\underline{i}+5\underline{j}}{\sqrt{41}} = \frac{-4}{\sqrt{41}}\underline{i} + \frac{5}{\sqrt{41}}\underline{j}$$

$$|a| = \sqrt{(-4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$$

(ii) $\underline{a} = 6\underline{i} + 8\underline{j}$

$$\hat{a} = \frac{\underline{a}}{|a|} = \frac{6\underline{i}+8\underline{j}}{\sqrt{(6)^2+(8)^2}} = \frac{6\underline{i}+8\underline{j}}{\sqrt{36+64}} = \frac{6\underline{i}+8\underline{j}}{\sqrt{100}} = \frac{6\underline{i}+8\underline{j}}{10} = \frac{6}{10}\underline{i} + \frac{8}{10}\underline{j} = \frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$$

(iii) $\underline{a} = \frac{1}{\sqrt{6}}\underline{i} + \frac{1}{\sqrt{6}}\underline{j}$

$$\hat{a} = \frac{\underline{a}}{|a|} = \frac{\frac{1}{\sqrt{6}}\underline{i}+\frac{1}{\sqrt{6}}\underline{j}}{\sqrt{(\frac{1}{\sqrt{6}})^2+(\frac{1}{\sqrt{6}})^2}} = \frac{\frac{1}{\sqrt{6}}\underline{i}+\frac{1}{\sqrt{6}}\underline{j}}{\sqrt{\frac{1}{6}+\frac{1}{6}}} = \frac{\frac{1}{\sqrt{6}}\underline{i}+\frac{1}{\sqrt{6}}\underline{j}}{\sqrt{\frac{1}{3}}} = \frac{\frac{1}{\sqrt{6}}}{\sqrt{\frac{1}{3}}}\underline{i} + \frac{\frac{1}{\sqrt{6}}}{\sqrt{\frac{1}{3}}}\underline{j} = \frac{\sqrt{3}}{\sqrt{6}}\underline{i} + \frac{\sqrt{3}}{\sqrt{6}}\underline{j} = \frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}$$

(Iv) $\underline{a} = \frac{1}{2}\underline{i} - \frac{3}{4}\underline{j}$

$$\hat{a} = \frac{\underline{a}}{|a|} = \frac{\frac{1}{2}\underline{i}-\frac{3}{4}\underline{j}}{\sqrt{(\frac{1}{2})^2+(\frac{3}{4})^2}} = \frac{\frac{1}{2}\underline{i}-\frac{3}{4}\underline{j}}{\sqrt{\frac{1}{4}+\frac{9}{16}}} = \frac{\frac{1}{2}\underline{i}-\frac{3}{4}\underline{j}}{\sqrt{\frac{13}{16}}} = \frac{\frac{1}{2}\underline{i}-\frac{3}{4}\underline{j}}{\frac{\sqrt{13}}{4}} = \frac{1}{2}\times\frac{4}{\sqrt{13}}\underline{i} - \frac{3}{4}\times\frac{4}{\sqrt{13}}\underline{j} = \frac{2}{\sqrt{13}}\underline{i} - \frac{3}{\sqrt{13}}\underline{j}$$

7. If $\underline{a} = 5\underline{i} - 7\underline{j}$, $\underline{b} = -\underline{i} - \underline{j}$ and $\underline{c} = 2\underline{i} + 3\underline{j}$, then find unit vector parallel to $\underline{a} + \underline{b} - 3\underline{c}$.

Solution

$$\underline{a} + \underline{b} - 3\underline{c} = \left(5\underline{i} - 7\underline{j}\right) + \left(-\underline{i} - \underline{j}\right) - 3\left(2\underline{i} + 3\underline{j}\right)$$
$$= 5\underline{i} - 7\underline{j} - \underline{i} - \underline{j} - 6\underline{i} - 9\underline{j}$$
$$= -2\underline{i} - 17\underline{j}$$

Let

$$\vec{v} = \underline{a} + \underline{b} - 3\underline{c} = -2\underline{i} - 17\underline{j}$$
$$\vec{v} = -2\underline{i} - 17\underline{j}$$

Now we show that unit vector is parallel to $\underline{a} + \underline{b} - 3\underline{c}$

$$\hat{v} = \frac{v}{|v|}$$
$$= \frac{-2\underline{i} - 17\underline{j}}{\sqrt{(-2)^2 + (-17)^2}}$$
$$= \frac{-2\underline{i} - 17\underline{j}}{\sqrt{4 + 289}}$$
$$= \frac{-2\underline{i} - 17\underline{j}}{\sqrt{293}}$$
$$= \frac{-2}{\sqrt{293}}\underline{i} - \frac{17}{\sqrt{293}}\underline{j}$$

NOTE:-

When a vector is divided by its magnitude its direction remains the same but its magnitude becomes the 1. Therefore the original vector and the unit vector are parallel .

8. If $\underline{a}=3\underline{i}-\underline{j}$, $\underline{b}=-2\underline{i}+4\underline{j}$ and $\underline{c}=\underline{i}+2\underline{j}$, then find unit vector parallel to $3\underline{a}-2\underline{c}+4\underline{b}$.

Solution

$$3\underline{a}-2\underline{c}+4\underline{b}=3\left(3\underline{i}-\underline{j}\right)-2\left(-2\underline{i}+4\underline{j}\right)+4\left(\underline{i}+2\underline{j}\right)$$
$$=9\underline{i}-3\underline{j}+4\underline{i}-8\underline{j}+4\underline{i}+8\underline{j}$$
$$=-\underline{i}+9\underline{j}$$

Let

$\vec{v}=-\underline{i}+9\underline{j}$

Now find $\vec{v}$

$$\hat{v}=\frac{\underline{v}}{|v|}=\frac{-\underline{i}+9\underline{j}}{\sqrt{(-1)^2+(9)^2}}=\frac{-\underline{i}+9\underline{j}}{\sqrt{82}}=\frac{-1}{\sqrt{82}}\underline{i}+\frac{9}{\sqrt{82}}\underline{j}$$

NOTE:-

When a vector is divided by its magnitude its direction remains the same but its magnitude becomes the 1. Therefore the original vector and the unit vector are parallel .

9. Which of the following vectors are parallel?

(i) $\underline{a}=6\underline{i}+\underline{j}$, $\underline{b}=12\underline{i}+2\underline{j}$ (ii) $\underline{a}=-2\underline{i}+3\underline{j}$, $\underline{b}=6\underline{i}-9\underline{j}$

(iii) $\underline{a}=5\underline{i}-4\underline{j}$, $\underline{b}=6\underline{i}-3\underline{j}$ (iv) $\underline{a}=3\underline{i}-7\underline{j}$, $\underline{b}=6\underline{i}-14\underline{j}$

Solution

(i) $\underline{a}=6\underline{i}+\underline{j}\,,\ \underline{b}=12\underline{i}+2\underline{j}$

$$\underline{b}=2(6\underline{i}+\underline{j})$$

$$\underline{b}=2\underline{a}\quad \text{(parallel)}$$

(ii) $\underline{a} = -2\underline{i} + 3\underline{j}$, $\underline{b} = 6\underline{i} - 9\underline{j}$

$$b = -3(-2\underline{i} + 3\underline{j})$$

$$\underline{b} = -3\underline{a} \text{ (parallel)}$$

(Iii) $\underline{a} = 5\underline{i} - 4\underline{j}$, $\underline{b} = 6\underline{i} - 3\underline{j}$

$$b = 3(2\underline{i} - \underline{j}) \quad \therefore a \neq 2\underline{i} - \underline{j} \text{ so it is not parallel}$$

(Iv) $\underline{a} = 3\underline{i} - 7\underline{j}$, $\underline{b} = 6\underline{i} - 14\underline{j}$

$$\underline{b} = 2(3\underline{i} - 7\underline{j})$$

$$\underline{b} = 2\underline{a} \text{ (parallel)}$$

10. Find a vector thrice in length of $3\underline{i} - 2\underline{j}$, but opposite in direction.

Solution

Given vector = $3\underline{i} - 2\underline{j}$

Three time in length = $3(3\underline{i} - 2\underline{j})$

$$= 9\underline{i} - 6\underline{j}$$

Opposite direction = $-9\underline{i} + 6\underline{j}$

11. Find two vectors that are double in magnitude of $3\underline{i} - 5\underline{j}$, one in the same direction of it and other in its opposite direction.

Solution

Given vector = $3\underline{i} - 5\underline{j}$

Double magnitude = $2(3\underline{i} - 5\underline{j})$

$$= 6\underline{i} - 10\underline{j}$$

Opposite direction = $-2\left(3\underline{i} - 5\underline{j}\right)$

$$= -6\underline{i} + 10\underline{j}$$

EXERCISE 6.2

1. If $\underline{a}=7\underline{i}-3\underline{j}$ and $\underline{b}=\underline{i}+5\underline{j}$, then find the following vectors:

(i) $\underline{a}+\underline{b}$ (ii) $\underline{a}+3\underline{b}$ (iii) $3\underline{a}+\frac{1}{2}\underline{b}$

(iv) $\underline{b}-\underline{a}$ (v) $4\underline{b}-5\underline{a}$ (vi) $\frac{3}{2}\underline{a}-\underline{b}$

Solution

(i) $\underline{a}+\underline{b}$ $= 7\underline{i} - 3\underline{j} + \underline{i} + 5\underline{j}$ $= 8\underline{i} + 2\underline{j}$	(ii) $\underline{a}+3\underline{b}$ $= 7\underline{i} - 3\underline{j} + 3(\underline{i} + 5\underline{j})$ $= 10\underline{i} + 12\underline{j}$
(iii) $3\underline{a} + \frac{1}{2}\underline{b}$ $= 3\left(7\underline{i} - 3\underline{j}\right) + \frac{1}{2}(\underline{i} + 5\underline{j})$ $= 21i - 9j + \frac{1}{2}i + \frac{5}{2}j$ $= \frac{43}{2}i - \frac{13}{2}j$	(iv) $\underline{b} - \underline{a}$ $= \left(\underline{i} + 5\underline{j}\right) - (7\underline{i} - 3\underline{j})$ $= \underline{i} + 5\underline{j} - 7\underline{i} + 3\underline{j}$ $= -6\underline{i} + 8\underline{j}$
(v) $4\underline{b} - 5\underline{a}$ $= 4\left(\underline{i} + 5\underline{j}\right) - 5(7\underline{i} - 3\underline{j})$ $= 4\underline{i} + 20\underline{j} - 35\underline{i} + 15\underline{j}$ $= -31\underline{i} + 35\underline{j}$	(vi) $\frac{3}{2}\underline{a} - \underline{b}$ $= \frac{3}{2}\left(7\underline{i} - 3\underline{j}\right) - (\underline{i} + 5\underline{j})$ $= \frac{21}{2}\underline{i} - \frac{9}{2}\underline{j} - \underline{i} - 5\underline{j}$ $= \frac{19}{2}\underline{i} - \frac{19}{2}\underline{j}$

2. If $\underline{a} = 6\underline{i} - \underline{j}$, $\underline{b} = \underline{i} + 5\underline{j}$ and $\underline{c} = 3\underline{i} + 5\underline{j}$, then find the magnitudes of the following vectors:

(i) $\underline{b} - \underline{c}$ (ii) $\underline{a} - 2\underline{b} + \underline{c}$ (iii) $\underline{c} - \underline{b} - \underline{a}$

Solution

<table>
<tr>
<td>

(i) $\underline{b}$-$\underline{c}$

$$= \underline{i} + 5\underline{j} - (3\underline{i} + 5\underline{j})$$
$$= \underline{i} + 5\underline{j} - 3\underline{i} - 5\underline{j}$$
$$\underline{b}\text{-}\underline{c} = -2\underline{i} + 0\underline{j}$$
$$|\underline{b} - \underline{c}| = \sqrt{(-2)^2 + (0)^2} = 2$$

</td>
<td>

(ii) $\underline{a} - 2\underline{b} + \underline{c}$

$$\underline{a} - 2\underline{b} + \underline{c} = \left(6\underline{i} - \underline{j}\right) - 2\left(\underline{i} + 5\underline{j}\right) + (3\underline{i} + 5\underline{j})$$
$$= 6\underline{i} - \underline{j} - 2\underline{i} - 10\underline{j} + 3\underline{i} + 5\underline{j}$$
$$= 7\underline{i} - 6\underline{j}$$

Let $v = \underline{a} - 2\underline{b} + \underline{c} = 7\underline{i} - 6\underline{j}$

$$|\vec{v}| = \sqrt{(7)^2 + (-6)^2}$$
$$= \sqrt{49 + 36}$$
$$= \sqrt{85}$$

</td>
</tr>
<tr>
<td colspan="2">

(iii) $\underline{c} - \underline{b} - \underline{a}$

$$\underline{c} - \underline{b} - \underline{a} = \left(3\underline{i} + 5\underline{j}\right) - \left(\underline{i} + 5\underline{j}\right) - \left(6\underline{i} - \underline{j}\right)$$
$$= 3\underline{i} + 5\underline{j} - \underline{i} - 5\underline{j} - 6\underline{i} + \underline{j}$$
$$\underline{c} - \underline{b} - \underline{a} = -4\underline{i} + \underline{j}$$

Let $v = \underline{c} - \underline{b} - \underline{a} = -4\underline{i} + \underline{j}$

$$|\vec{v}| = \sqrt{(-4)^2 + (1)^2}$$
$$= \sqrt{16 + 1} = \sqrt{17}$$

</td>
</tr>
</table>

3. Find the values of x and y in each of the following equations:

(i) $(x\underline{i}+y\underline{j})+(2\underline{i}+3\underline{j})=7\underline{i}+6\underline{j}$ (ii) $(x\underline{i}-5\underline{j})+(3\underline{i}+5\underline{j})=-8\underline{i}+y\underline{j}$

(iii) $(y\underline{i}+3\underline{j})+(-5\underline{i}+2x\underline{j})=9\underline{i}+7\underline{j}$

Solution

(i)

$$\left(x\underline{i}+y\underline{j}\right)+\left(2\underline{i}+3\underline{j}\right)=(7\underline{i}+6\underline{j})$$
$$x\underline{i}+y\underline{j}+2\underline{i}+3\underline{j}=7\underline{i}+6\underline{j}$$
$$(x+2)\underline{i}+(y+3)\underline{j}=7\underline{i}+6\underline{j}$$

Comparing the coefficient

$$x+2=7 \quad , \quad y+3=6$$
$$x=7-2 \quad , \quad y=6-3$$
$$x=5 \qquad y=3$$

(ii)

$$\left(x\underline{i}-5\underline{j}\right)+\left(3\underline{i}+5\underline{j}\right)=-8\underline{i}+y\underline{j}$$
$$x\underline{i}-5\underline{j}+3\underline{i}+5\underline{j}=-8\underline{i}+y\underline{j}$$
$$(x+3)\underline{i}+(-5+5)\underline{j}=-8\underline{i}+y\underline{j}$$
$$(x+3)\underline{i}+0\underline{j}=-8\underline{i}+y\underline{j}$$
$$x+3=-8 \quad , \quad 0=y$$
$$x=-11$$

(iii)

$$\left(y\underline{i}+3\underline{j}\right)+\left(-5\underline{i}+2x\underline{j}\right)=9\underline{i}+7\underline{j}$$
$$y\underline{i}+3\underline{j}-5\underline{i}+2x\underline{j}=9\underline{i}+7\underline{j}$$
$$(y-5)\underline{i}+(3+2x)\underline{j}=9\underline{i}+7\underline{j}$$

Comparing the coefficient

$$y-5=9 \quad , \quad 3+2x=7$$
$$y=9+5 \qquad 2x=7-3$$
$$y=14 \qquad 2x=4$$
$$x=2$$

4. If $\underline{a}=\underline{i}+3\underline{j}$, $\underline{c}=2\underline{i}+\underline{j}$ and $\underline{a}+2\underline{b}=\underline{c}$, then find $|\underline{b}|$.

Solution

$$\underline{a}+2\underline{b}=\underline{c}$$
$$\underline{i}+3\underline{j}+2\underline{b}=2\underline{i}+\underline{j}$$
$$2\underline{b}=2\underline{i}+\underline{j}-\underline{i}-3\underline{j}$$
$$2\underline{b}=\underline{i}-2\underline{j}$$
$$\underline{b}=\frac{1}{2}\underline{i}-\underline{j}$$
$$|\underline{b}|=\sqrt{(\tfrac{1}{2})^2+(-1)^2}=\sqrt{\frac{1}{4}+1}=\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}$$

5. If $\underline{a} = -2\underline{i} + 7\underline{j}$ and $\underline{b} = 3\underline{i} - 5\underline{j}$, then find the vector components of $\underline{a} + 5\underline{b}$.

Solution

$$\underline{a} + 5\underline{b} = \left(-2\underline{i} + 7\underline{j}\right) + 5(3\underline{i} - 5\underline{j})$$
$$= -2\underline{i} + 7\underline{j} + 15\underline{i} - 25\underline{j}$$
$$= 13\underline{i} - 18\underline{j}$$

6. If $5\underline{i} - 3\underline{j} = m(\underline{i} - 10\underline{j}) + n(4\underline{i} - 3\underline{j})$, then find the values of m and n.

Solution

$$5\underline{i} - 3\underline{j} = m\underline{i} - 10m\underline{j} + 4n\underline{i} - 7n\underline{j}$$
$$5\underline{i} - 3\underline{j} = (m + 4n)\underline{i} - (10m + 7n)\underline{j}$$

comparing the coefficient

$$m + 4n = 5 \quad , \quad -3 = -(10m + 7n)$$
$$10m + 7n = 3$$

$$10m + 7n = 3$$
$$\pm 10m \pm 40n = \pm 50$$
$$-37n = -47$$
$$n = \frac{47}{37}$$

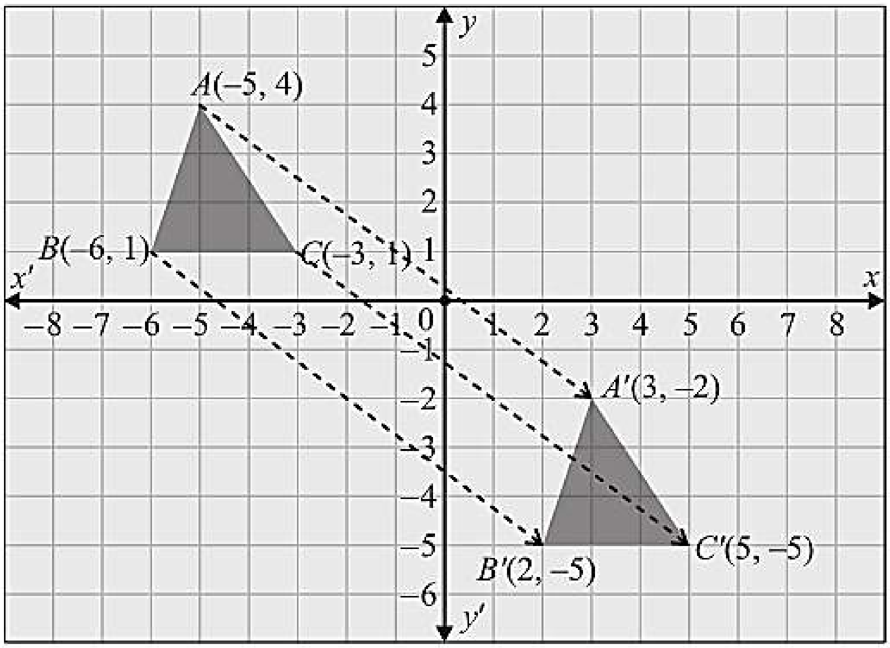
Now putting the value of n in $m + 4n = 5$

$$m + 4\left(\frac{47}{37}\right) = 5$$
$$m + \frac{188}{37} = 5$$
$$m = 5 - \frac{188}{37}$$
$$m = \frac{185 - 188}{37} = \frac{-3}{37}$$

EXERCISE 6.3

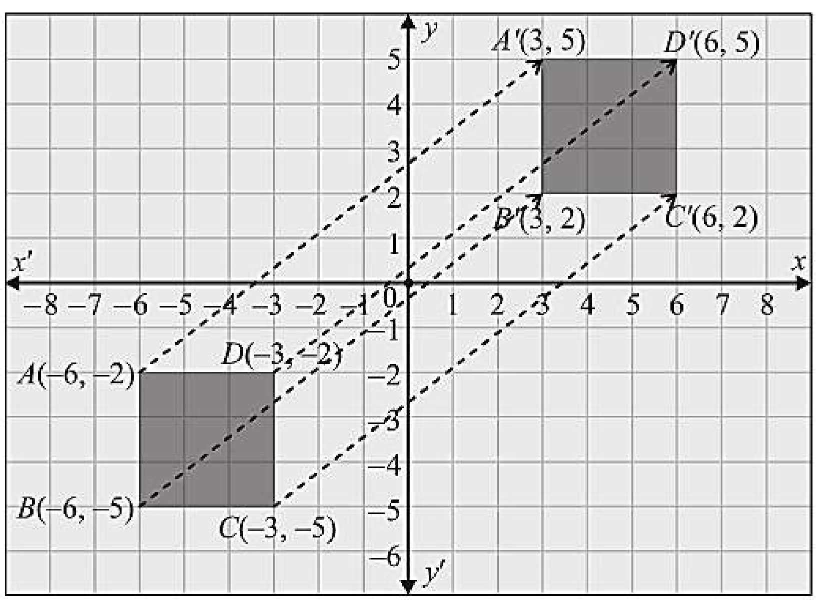
1. Plot $A(-5, 4)$, $B(-6, 1)$ and $C(-3, 1)$ to form a ΔABC. Also translate ΔABC to $\Delta A'B'C'$ by translation vector $8\underline{i} - 6\underline{j}$.

Solution



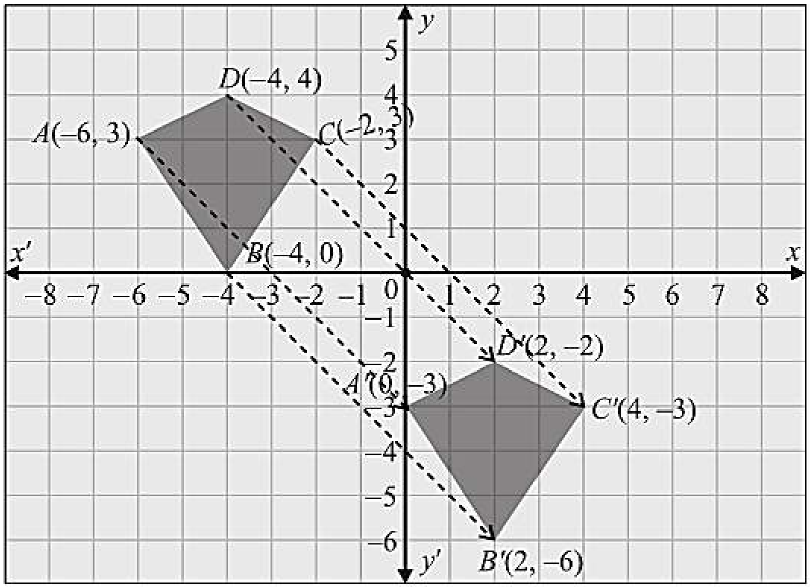
2. Plot $A(-6, -2)$, $B(-6, -5)$, $C(-3, -5)$ and $D(-3, -2)$ to form a square $ABCD$. Also translate square $ABCD$ to square $A'B'C'D'$ by translation vector $9\underline{i} + 7\underline{j}$.

Solution



3. Plot $A(-6, 3)$, $B(-4, 0)$, $C(-2, 3)$ and $D(-4, 4)$ to form a kite $ABCD$. Also translate kite $ABCD$ to kite $A'B'C'D'$ by translation vector $6\underline{i} - 6\underline{j}$.

Solution



4. The coordinates of A, B and D are (1,2), (6,3) and (2,8) respectively. Find the coordinates of C by using vector method if $ABCD$ is a parallelogram.

Solution

$$\overrightarrow{AB} = \overrightarrow{DC}.$$

$$\overrightarrow{AB} = (6-1, 3-2) = (5, 1).$$

$$\overrightarrow{DC} = (x_C - 2, y_C - 8).$$

$$x_C - 2 = 5$$
$$x_C = 5 + 2$$
$$x_C = 7.$$

$$y_C - 8 = 1$$
$$y_C = 1 + 8$$
$$y_C = 9.$$

$$x_C = 7,\ y_C = 9.$$

Answer is $C(7, 9)$

5. In parallelogram $ABCD$, the vectors representing two opposite sides are $\overrightarrow{AB} = 6\underline{i} + 2\underline{j}$, $\overrightarrow{DC} = -6\underline{i} - 2\underline{j}$. Show that the opposite sides are equal in magnitude and parallel.

Solution

$$|\overrightarrow{AB}| = \sqrt{6^2 + 2^2} = \sqrt{40},$$
$$|\overrightarrow{DC}| = \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}.$$

$$\overrightarrow{AB} = -\overrightarrow{DC}.$$

Opposite sides are equal in magnitude and parallel.

6. Points $A(1, 2)$, $B(4, 6)$ and $C(7, 2)$ form a triangle. Check whether triangle ABC is an isosceles by using vector magnitude.

Solution

$$\overrightarrow{AB} = (4 - 1, 6 - 2) = (3, 4),$$
$$\overrightarrow{AC} = (7 - 1, 2 - 2) = (6, 0),$$
$$\overrightarrow{BC} = (7 - 4, 2 - 6) = (3, -4).$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + 4^2} = 5,$$
$$|\overrightarrow{AC}| = 6,$$
$$|\overrightarrow{BC}| = \sqrt{3^2 + (-4)^2} = 5.$$

Since $|\overrightarrow{AB}| = |\overrightarrow{BC}|$, triangle ABC is **isosceles**.

7. Use vectors to show that $PQRS$ is a parallelogram, where the points P, Q, R and S have coordinates (1, 2), (5, 2), (7, 6) and (3, 6) respectively.

Solution

$$\overrightarrow{PQ} = (5-1, 2-2) = (4,0),$$
$$\overrightarrow{SR} = (7-3, 6-6) = (4,0).$$

$$\overrightarrow{PS} = (3-1, 6-2) = (2,4),$$
$$\overrightarrow{QR} = (7-5, 6-2) = (2,4).$$

Since $\overrightarrow{PQ} = \overrightarrow{SR}$ and $\overrightarrow{PS} = \overrightarrow{QR}$, $PQRS$ is a parallelogram.

8. Use vectors to show that triangle XYZ is an isosceles, where the points X, Y, Z have the coordinates (0, 0), (2, 0) and (1, 3) respectively.

Solution

$$\overrightarrow{XY} = (2-0, 0-0) = (2,0),$$
$$\overrightarrow{XZ} = (1-0, 3-0) = (1,3),$$
$$\overrightarrow{YZ} = (1-2, 3-0) = (-1,3).$$

$$|\overrightarrow{XY}| = 2,$$
$$|\overrightarrow{XZ}| = \sqrt{1^2+3^2} = \sqrt{10},$$
$$|\overrightarrow{YZ}| = \sqrt{(-1)^2+3^2} = \sqrt{10}.$$

Since $|\overrightarrow{XZ}| = |\overrightarrow{YZ}|$, triangle XYZ is **isosceles**.

9. A ball is projected with an initial velocity vector $\underline{v}_0 = 10\underline{i} + 20\underline{j}$. The horizontal component is in the x-direction and gravity is $\underline{g} = 0\underline{i} - 10\underline{j}$. Find the maximum height and horizontal range.

Solution

Given $\underline{v}_0 = 10\underline{i} + 20\underline{j}$ and $\underline{g} = -10\underline{j}$.

Maximum height $H = \frac{v_{0y}^2}{2g} = \frac{20^2}{2 \cdot 10} = 20.$

Horizontal range $R = \frac{2v_{0x}v_{0y}}{g} = \frac{2 \cdot 10 \cdot 20}{10} = 40.$

10. A car enters a loop with velocity vector $\underline{v} = 30\underline{j}$ and exits with velocity $\underline{v}' = 30\underline{i}$. What is the change in velocity vector?

Solution

Given $\underline{v} = 30\underline{j}$ and $\underline{v}' = 30\underline{i}$.

Change in velocity $\Delta\underline{v} = \underline{v}' - \underline{v} = 30\underline{i} - 30\underline{j}.$

Magnitude of change

$|\Delta\underline{v}| = \sqrt{30^2 + (-30)^2} = 30\sqrt{2}.$

11. An aeroplane has airspeed $\underline{v}_p = 20\underline{i}$ and there is a crosswind $\underline{v}_w = 50\,\underline{j}$. Find the resultant velocity and its magnitude.

Solution

Given $\underline{v}_p = 20\underline{i}$ and $\underline{v}_w = 50\underline{j}$.

Resultant velocity $\underline{v}_r = 20\underline{i} + 50\underline{j}.$

Magnitude $|\underline{v}_r| = \sqrt{20^2 + 50^2} = \sqrt{2900} = 53.85.$

REVIEW EXERCISE 6

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) x-axis and y-axis divide a coordinate plane into ________ parts.
(a) one (b) two (c) three ✓(d) four

(ii) $P(4,-4)$ lies in ________ quadrant.
(a) first (b) second (c) third ✓(d) fourth

(iii) A vector having magnitude 1, is called:
(a) equal vector (b) parallel vector
✓(c) unit vector (d) zero vector

(iv) What is the value of $|3\underline{i}+4\underline{j}|$?
(a) 3 (b) 4 ✓(c) 5 (d) 7

(v) If $\underline{a}=\lambda\underline{b}$, then $\underline{a}$ and $\underline{b}$ are:
(a) equal ✓(b) parallel (c) perpendicular (d) non-parallel

(vi) If $\overrightarrow{OA}=\underline{a}, \overrightarrow{OB}=\underline{b}$, then $\overrightarrow{AB}$ is:
✓(a) $\underline{b}-\underline{a}$ (b) $\underline{a}-\underline{b}$ (c) $\underline{a}+\underline{b}$ (d) $\underline{b}+\underline{a}$

(vii) Translation vector shows:
(a) deformation (b) rotation ✓(c) movement (d) enlargement

(viii) Sum of two vectors is:
(a) a triangle ✓(b) a vector (c) a scalar (d) a length

(ix) The position vector of point P(3, −2) with respect to O is:
(a) $3\underline{i}+2\underline{j}$ ✓(b) $3\underline{i}-2\underline{j}$ (c) $-3\underline{i}+2\underline{j}$ (d) $2\underline{i}-3\underline{j}$

(x) Vector from point P(3, 4) to origin is:
(a) $3\underline{i}+4\underline{j}$ (b) $-3\underline{i}+4\underline{j}$ ✓(c) $-3\underline{i}-4\underline{j}$ (d) $3\underline{i}-4\underline{j}$

2. Find magnitude of the $\overrightarrow{AB}$:

(i) $A(7, 7), B(-12, 0)$ (ii) $A(9, 3), B(2, 11)$

Solution

(i)	(ii)
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (-12, 0) - (7, 7)$ $= (-12 - 7, 0 - 7)$ $= (-19, -7).$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(-19)^2 + (-7)^2}$ $= \sqrt{361 + 49} = \sqrt{410}.$	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (2, 11) - (9, 3)$ $= (2 - 9, 11 - 3)$ $= (-7, 8).$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(-7)^2 + 8^2}$ $= \sqrt{49 + 64} = \sqrt{113}.$

3. Find a unit vector in the direction of $\underline{a} = \frac{5}{3}\underline{i} + \frac{1}{3}\underline{j}$.

Solution

$$\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|}$$

$$= \frac{\frac{5}{3}\underline{i} + \frac{1}{3}\underline{j}}{\sqrt{(\frac{5}{3})^2 + (\frac{1}{3})^2}}$$

$$= \frac{\frac{5}{3}\underline{i} + \frac{1}{3}\underline{j}}{\sqrt{\frac{25}{9} + \frac{1}{9}}}$$

$$= \frac{\frac{5}{3}\underline{i} + \frac{1}{3}\underline{j}}{\sqrt{\frac{26}{9}}}$$

$$= \frac{1}{\sqrt{26}}(5\underline{i} + \underline{j}).$$

$$= \frac{5}{\sqrt{26}}\underline{i} + \frac{1}{\sqrt{26}}\underline{j}$$

4. If $\underline{a} = 2\underline{i} - \underline{j}$, $\underline{b} = 3\underline{i} + \underline{j}$ and $\underline{c} = 4\underline{i} + \underline{j}$, then find the following vectors:

(i) $5\underline{b} - \underline{a} + \underline{c}$ (ii) $8\underline{a} + \underline{b} + 5\underline{c}$ (iii) $\underline{c} + \underline{b} - 4\underline{a}$

Solution

(i)

$$\begin{aligned} 5\underline{b} - \underline{a} + \underline{c} &= 5(3\underline{i} + \underline{j}) - (2\underline{i} - \underline{j}) + (4\underline{i} + \underline{j}) \\ &= 15\underline{i} + 5\underline{j} - 2\underline{i} + \underline{j} + 4\underline{i} + \underline{j} \\ &= 17\underline{i} + 7\underline{j}. \end{aligned}$$

(ii)

$$\begin{aligned} 8\underline{a} + \underline{b} + 5\underline{c} &= 8(2\underline{i} - \underline{j}) + (3\underline{i} + \underline{j}) + 5(4\underline{i} + \underline{j}) \\ &= 16\underline{i} - 8\underline{j} + 3\underline{i} + \underline{j} + 20\underline{i} + 5\underline{j} \\ &= 39\underline{i} - 2\underline{j}. \end{aligned}$$

(iii)

$$\begin{aligned} \underline{c} + \underline{b} - 4\underline{a} &= (4\underline{i} + \underline{j}) + (3\underline{i} + \underline{j}) - 4(2\underline{i} - \underline{j}) \\ &= 4\underline{i} + \underline{j} + 3\underline{i} + \underline{j} - 8\underline{i} + 4\underline{j} \\ &= -\underline{i} + 6\underline{j}. \end{aligned}$$

5. Find the values of x and y in the following equation.

$$(2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) = \frac{1}{4}\underline{i} - 8\underline{j}$$

Solution

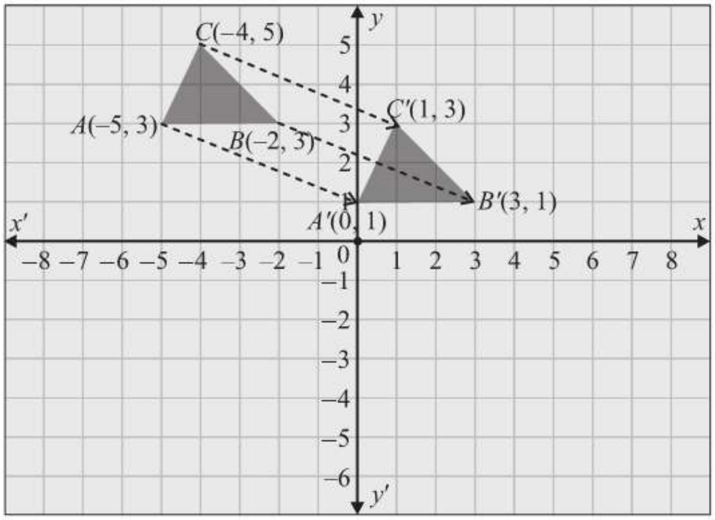
$$\begin{aligned} (2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) &= \frac{1}{4}\underline{i} - 8\underline{j} \\ (2x - 1)\underline{i} + (y + 5)\underline{j} &= \frac{1}{4}\underline{i} - 8\underline{j}. \end{aligned}$$

Equating components,

$$2x - 1 = \frac{1}{4} \quad \Rightarrow \quad 2x = \frac{5}{4} \quad \Rightarrow \quad x = \frac{5}{8},$$
$$y + 5 = -8 \quad \Rightarrow \quad y = -13.$$

6. Plot $A(-5, 3)$, $B(-2, 3)$ and $C(-4, 5)$ to form a triangle ABC. Also, translate ΔABC to $\Delta A'B'C'$ by translation vector $5\underline{i} - 2\underline{j}$.

Solution



7. Use vectors to show that $ABCD$ is a parallelogram, where the points are $A(2, 3)$, $B(6, 3)$, $C(7, 6)$ and $D(3, 6)$.

Solution

$$\overrightarrow{AB} = (6-2, 3-3) = (4, 0),$$
$$\overrightarrow{DC} = (7-3, 6-6) = (4, 0).$$

Since $\overrightarrow{AB} = \overrightarrow{DC}$, $ABCD$ is a parallelogram.

8. Use vectors to show that triangle ABC is an isosceles triangle, where the points A, B and C have coordinates $(1, 2)$, $(4, 6)$ and $(7, 2)$ respectively.

Solution

$$\overrightarrow{AB} = (4-1, 6-2) = (3, 4),$$
$$\overrightarrow{AC} = (7-1, 2-2) = (6, 0),$$
$$\overrightarrow{BC} = (7-4, 2-6) = (3, -4).$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + 4^2} = 5,$$
$$|\overrightarrow{AC}| = \sqrt{6^2 + 0^2} = 6,$$
$$|\overrightarrow{BC}| = \sqrt{3^2 + (-4)^2} = 5.$$

Since $|\overrightarrow{AB}| = |\overrightarrow{BC}|$, triangle ABC is isosceles.

9. A ball is projected with velocity vector $\underline{v} = 6\underline{i} + 8\underline{j}$. What is the magnitude of velocity?

Solution

$$\begin{aligned} |\underline{v}| &= |6\underline{i} + 8\underline{j}| \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} = \sqrt{100} = 10. \end{aligned}$$

10. An aircraft is flying due east with an airspeed of 200 km/h. There is a wind blowing due north at 60 km/h. Find the resultant velocity and its magnitude.

Solution

$$\begin{aligned} \underline{v}_{\text{resultant}} &= 200\underline{i} + 60\underline{j}, \\ |\underline{v}_{\text{resultant}}| &= \sqrt{200^2 + 60^2} \\ &= \sqrt{40000 + 3600} = \sqrt{43600} = 208.8 \text{ k} \end{aligned}$$