# UNIT 5
# Algebraic Fractions

## EXERCISE 5.1

1. Reduce the following rational expressions to lowest forms:

(i) $\frac{3a^2-6ab}{2a^2b-4ab^2}$ (ii) $\frac{abx+bx^2}{acx+cx^2}$ (iii) $\frac{ac}{a^2x^2-ax}$

(iv) $\frac{15a^2b^2c}{100(a^2-a^2b)}$ (v) $\frac{4x^2-9y^2}{4x^2+6xy}$ (vi) $\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}$

(vii) $\frac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$ (viii) $\frac{x^2-5x}{x^2-4x-5}$ (ix) $\frac{3x^2+6x}{x^2+4x+4}$

(x) $\frac{x^2+xy-2y^2}{x^3-y^3}$ (xi) $\frac{2x^2+17x+21}{3x^2+26x+35}$

**Solution**

**(i)** $\frac{3a^2-6ab}{2a^2b-4ab^2}$

$= \frac{3a(a-2b)}{2ab(a+2b)}$

$= \frac{3}{2b}$

**(ii)** $\frac{abx+bx^2}{acx+cx^2}$

$= \frac{bx(a+x)}{cx(a+x)}$

$= \frac{b}{c}$

**(iii)** $\frac{ac}{a^2x^2-ax}$

$= \frac{ac}{ax(ax-1)}$

$= \frac{c}{x(ax-1)}$

**(iv)** $\frac{15a^2b^2c}{100(a^2-a^2b)}$

$= \frac{15a^2b^2c}{100a^2(1-b)}$

$= \frac{3b^2c}{20(1-b)}$

(v) $\frac{4x^2-9y^2}{4x^2+6xy}$

$= \frac{(2x)^2-(3y)^2}{2x(2x+3y)}$

$= \frac{(2x-3y)(2x+3y)}{2x(2x+3y)}$

$= \frac{2x-3y}{2x}$

(vi) $\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}$

$= \frac{20(x-y)(x^2+xy+y^2)}{5(x^2+xy+y^2)}$

$= 4(x-y)$

(vii) $\frac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$

$= \frac{ax(2a-3x)}{ax(4a^2-9x^2)}$

$= \frac{(2a-3x)}{(2a)^2-(3x)^2}$

$= \frac{(2a-3x)}{(2a-3x)(2a+3x)}$

$= \frac{1}{(2a+3x)}$

(viii) $\frac{x^2-5x}{x^2-4x-5}$

$= \frac{x(x-5)}{x^2-5x+x-5}$

$= \frac{x(x-5)}{x(x-5)+1(x-5)}$

$= \frac{x(x-5)}{(x-5)(x+1)}$

$= \frac{x}{(x+1)}$

(ix) $\frac{3x^2+6x}{x^2+4x+4}$

$= \frac{3x(x+2)}{x^2+2x+2x+4}$

$= \frac{3x(x+2)}{x(x+2)+2(x+2)}$

$= \frac{3x(x+2)}{(x+2)(x+2)}$

$= \frac{3x}{(x+2)}$

(x) $\frac{x^2+xy-2y^2}{x^3-y^3}$

$= \frac{x^2+2xy-xy-2y^2}{(x-y)(x^2+xy+y^2)}$

$= \frac{x(x+2y)-y(x+2y)}{(x-y)(x^2+xy+y^2)}$

$= \frac{(x+2y)(x-y)}{(x-y)(x^2+xy+y^2)}$

$= \frac{x+2y}{x^2+xy+y^2}$

(xi) $\frac{2x^2+17x+21}{3x^2+26x+35} = \frac{2x^2+14x+3x+21}{3x^2+21x+5x+35} = \frac{2x(x+7)+3(x+7)}{3x(x+7)+5(x+7)} = \frac{(x+7)(2x+3)}{(x+7)(3x+5)}$

$= \frac{2x+3}{3x+5}$

# EXERCISE 5.2

(i) $\frac{3}{x-y}+\frac{1}{y-x}$

$=\frac{3}{(x-y)}+\frac{1}{-(x-y)}$

$=\frac{3}{(x-y)}-\frac{1}{(x-y)}$

$=\frac{3-1}{(x-y)}$

$=\frac{2}{(x-y)}$

(ii) $\frac{4x}{x^2-3x+2}-\frac{4}{1-x}-\frac{5}{x-2}$

$=\frac{4x}{x^2-2x-x+2}-\frac{4}{1-x}-\frac{5}{x-2}$

$=\frac{4x}{x(x-2)-1(x-2)}-\frac{4}{1-x}-\frac{5}{x-2}$

$=\frac{4x}{(x-2)(x-1)}+\frac{4}{(x-1)}-\frac{5}{(x-2)}$

$=\frac{4x+4(x-2)-5(x-1)}{(x-2)(x-1)}$

$=\frac{4x+4x-8-5x+5}{(x-2)(x-1)}$

$=\frac{4x+4x-5x-8+5}{(x-2)(x-1)}$

$=\frac{3x-3}{(x-2)(x-1)}$

$=\frac{3(x-1)}{(x-2)(x-1)}$

$=\frac{3}{(x-2)}$

(iii) $\frac{x+y}{12x-6y}+\frac{x-y}{18x-9y}$

$=\frac{x+y}{6(2x-y)}+\frac{x-y}{9(2x-y)}$

$=\frac{9(x+y)+6(x-y)}{54(2x-y)}$

$=\frac{9x+9y+6x-6y}{54(2x-y)}$

$=\frac{15x+3y}{54(2x-y)}$

$=\frac{3(5x+y)}{54(2x-y)}$

$=\frac{5x+y}{18(2x-y)}$

(iv) $\frac{2}{x+2y}-\frac{x-6y}{x^2-4y^2}$

$=\frac{2}{x+2y}-\frac{x-6y}{(x)^2-(2y)^2}$

$=\frac{2}{x+2y}-\frac{x-6y}{(x-2y)(x+2y)}$

$=\frac{2(x-2y)-(x-6y)}{(x+2y)(x-2y)}$

$=\frac{2x-4y-x+6y}{(x+2y)(x-2y)}$

$=\frac{(x+2y)}{(x+2y)(x-2y)}$

$=\frac{1}{x-2y}$

(v) $\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x^2+3x+2}$

$=\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x^2+2x+x+2}$

$=\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{x(x+2)+1(x+2)}$

$=\frac{1}{x+1}-\frac{2}{x+2}-\frac{2x+3}{(x+2)(x+1)}$

$=\frac{(x+2)-2(x+1)-(2x+3)}{(x+2)(x+1)}$

$=\frac{x+\cancel{2}-2x-\cancel{2}-2x-3}{(x+2)(x+1)}$

$=\frac{-3x-3}{(x+2)(x+1)}$

$=\frac{-3(x+\cancel{1})}{(x+2)(x\cancel{+}1)}$

$=\frac{-3}{x+2}$

(vi) $\frac{5}{x^2+x-6}-\frac{1}{2x^2-7x+6}$

$=\frac{5}{x^2+3x-2x-6}-\frac{1}{2x^2-3x-4x+6}$

$=\frac{5}{x(x+3)-2(x+3)}-\frac{1}{x(2x-3)-2(2x-3)}$

$=\frac{5}{(x+3)(x-2)}-\frac{1}{(2x-3)(x-2)}$

$=\frac{5(2x-3)-(x+3)}{(2x-3)(x+3)(x-2)}$

$=\frac{10x-15-x-3}{(2x-3)(x+3)(x-2)}$S

$=\frac{9x-18}{(2x-3)(x+3)(x-2)}$

$=\frac{9(x+\cancel{2})}{(2x-3)(x+3)(x\cancel{-}2)}$

$=\frac{9}{(2x-3)(x+3)}$

(vii) $\frac{2x}{x+y}-\frac{y}{x-y}-\frac{2y^2}{x^2-y^2}$

$=\frac{2x}{x+y}-\frac{y}{x-y}-\frac{2y^2}{(x-y)(x+y)}$

$=\frac{2x(x-y)-y(x+y)-2y^2}{(x-y)(x+y)}$

$=\frac{2x^2-2xy-xy-y^2-2y^2}{(x-y)(x+y)}$

$=\frac{2x^2-3xy-3y^2}{(x-y)(x+y)}$

(viii) $\frac{7}{2x^2-x-6}-\frac{8}{3x^2-4x-4}$

$=\frac{7}{2x^2-4x+3x-6}-\frac{8}{3x^2-6x+2x-4}$

$=\frac{7}{2x(x-2)+3(x-2)}-\frac{8}{3x(x-2)+2(x-2)}$

$=\frac{7}{(x-2)(2x+3)}-\frac{8}{(x-2)(3x+2)}$

$=\frac{7(3x+2)-8(2x+3)}{(3x+2)(2x+3)(x-2)}$

$=\frac{21x+14-16x-24}{(3x+2)(2x+3)(x-2)}$

$=\frac{5x-10}{(3x+2)(2x+3)(x-2)}$

$=\frac{5(x+\cancel{2})}{(3x+2)(2x+3)(x\cancel{+}2)}$

$=\frac{5}{(3x+2)(2x+3)}$

(ix) $\frac{x+2}{x^2-x-12}-\frac{x}{x^2+6x+9}$

$=\frac{x+2}{x^2-4x+3x-12}-\frac{x}{x^2+3x+3x+9}$

$=\frac{x+2}{x(x-4)+3(x-4)}-\frac{x}{x(x+3)+3(x+3)}$

$=\frac{x+2}{(x-4)(x+3)}-\frac{x}{(x+3)(x+3)}$

$=\frac{(x+2)(x+3)-x(x-4)}{(x-4)(x+3)^2}$

$=\frac{x^2+3x+2x+6-x^2+4x}{(x-4)(x+3)^2}$

$=\frac{9x+6}{(x-4)(x+3)^2}$

$=\frac{3(3x+2)}{(x+3)^2(x-4)}$

(x) $\frac{1}{x^2-4x+3}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-5x+6}$

$=\frac{1}{x^2-3x-x+3}+\frac{1}{x^2-2x-x+2}-\frac{1}{x^2-3x-2x+6}$

$=\frac{1}{x(x-3)-1(x-3)}+\frac{1}{x(x-2)-1(x-2)}-\frac{1}{x(x-3)-2(x-3)}$

$=\frac{1}{(x-3)(x-1)}+\frac{1}{(x-2)(x-1)}-\frac{1}{(x-3)(x-2)}$

$=\frac{(x-2)+(x-3)-(x-1)}{(x-1)(x-2)(x-3)}$

$=\frac{x-2+x-3-x+1}{(x-1)(x-2)(x-3)}$

$=\frac{x-4}{(x-1)(x-2)(x-3)}$

(xi) $\frac{x^3}{x-y}+\frac{y^3}{y-x}=\frac{x^3}{x-y}+\frac{y^3}{-(x-y)}=\frac{x^3}{x-y}-\frac{y^3}{x-y}=\frac{x^3-y^3}{(x-y)}$

$=\frac{(x-y)(x^2+xy+y^2)}{(x-y)}=x^2+xy+y^2$

2. Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

$=\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}$

$=\frac{(2x^3+x^2+3)-(x^2+2)}{(x^2+2)^2}$

$=\frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2}$

$=\frac{2x^3+1}{(x^2+2)^2}$

# EXERCISE 5.3

**(i)** $\frac{24lm}{5l+10m} \times \frac{5l^2-20m^2}{16mn}$

$= \frac{24lm}{5(l+2m)} \times \frac{5(l^2-4m^2)}{16mn}$

$= \frac{24lm}{5(l+2m)} \times \frac{5[(l)^2-(2m)^2]}{16mn}$

$= \frac{3l}{(l+2m)} \times \frac{(l-2m)(l+2m)}{2n}$

$= \frac{3l(l-2m)}{2n}$

**(ii)** $\frac{x^2-3x+2}{x^2+3x-4} \times \frac{2x^2+8x}{3x+6}$

$= \frac{x^2-2x-x+2}{x^2+4x-x-4} \times \frac{2x(x+4)}{3(x+2)}$

$= \frac{x(x-2)-1(x-2)}{x(x+4)-1(x+4)} \times \frac{2x(x+4)}{3(x+2)}$

$= \frac{(x-2)(x-1)}{(x+4)(x-1)} \times \frac{2x(x+4)}{3(x+2)}$

$= \frac{2x(x-2)}{3(x+2)}$

**(iii)** $\frac{a^2-4b^2}{a^2+2ba} \times \frac{2a^2+10ab}{a^2+3ab-10b^2}$

$= \frac{(a)^2-(2b)^2}{a(a+2b)} \times \frac{2a(a+5b)}{a^2-2ab+5ab-10b^2}$

$= \frac{(a-2b)(a+2b)}{a(a+2b)} \times \frac{2a(a+5b)}{a(a-2b)+5b(a-2b)}$

$= \frac{(a-2b)(a+2b)}{a(a+2b)} \times \frac{2a(a+5b)}{(a-2b)(a+5b)}$

$= 2$

**(iv)** $\frac{(a+b)^2-c^2}{(a+c)^2-b^2} \times \frac{a^2-(b-c)^2}{(a+c)^2-c^2}$

$= \frac{(a+b-c)(a+b+c)}{(a+c-b)(a+b+c)} \times \frac{(a-b+c)(a+b-c)}{(a+c+c)(a+c-c)}$

$= \frac{(a+b-c)^2}{a(a+2c)}$

**(v)** $\frac{x^3-8}{x^2-4} \div \frac{x^2+2x+4}{x^2+4x+4}$

$= \frac{(x)^3-(2)^3}{(x)^2-(2)^2} \div \frac{x^2+2x+4}{x^2+2x+2x+4}$

$= \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} \div \frac{x^2+2x+4}{x(x+2)+2(x+2)}$

$= \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} \div \frac{x^2+2x+4}{(x+2)(x+2)}$

$= \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} \times \frac{(x+2)(x+2)}{(x^2+2x+4)}$

$= x + 2$

**(vi)** $\frac{(a^2+ab)^2}{(a^2-ab)^2} \div \left(\frac{a+b}{a-b}\right)^2$

$= \frac{[a(a+b)]^2}{[a(a-b)]^2} \div \frac{(a+b)^2}{(a-b)^2}$

$= \frac{a^2(a+b)^2}{a^2(a-b)^2} \times \frac{(a-b)^2}{(a+b)^2}$

$= 1$

(vii) $\frac{x^2-x-6}{x^2-x-20} \div \left\{\frac{x^3-3x^2}{x^2+4x} \times \frac{x^3-5x^2-14x}{x^2-12x+35}\right\}$

$= \frac{x^2-3x+2x-6}{x^2-5x+4x-20} \div \left\{\frac{x^2(x-3)}{x(x+4)} \times \frac{x(x^2-5x-14)}{x^2-7x-5x+35}\right\}$

$= \frac{x(x-3)+2(x-3)}{x(x-5)+4(x-5)} \div \left\{\frac{x^2(x-3)}{(x+4)} \times \frac{(x^2-7x+2x-14}{x(x-7)-5(x-7)}\right\}$

$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \left\{\frac{x^2(x-3)}{(x+4)} \times \frac{x(x-7)+2(x-7)}{(x-7)(x-5)}\right\}$

$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \left\{\frac{x^2(x-3)}{(x+4)} \times \frac{(x-7)(x+2)}{(x-7)(x-5)}\right\}$

$= \frac{(x-3)(x+2)}{(x-5)(x+4)} \div \left\{\frac{x^2(x-3)(x+2)}{(x+4)(x-5)}\right\} = \frac{(x-3)(x+2)}{(x-5)(x+4)} \times \frac{(x+4)(x-5)}{x^2(x-3)(x+2)} = \frac{1}{x^2}$

(viii) $\frac{3x^2-3xy}{10x^2+10xy-20y^2} \times \frac{5xy+10y^2}{6x^2+6xy+6y^2} \div \frac{2x^2-2y^2}{4x^3-4y^3}$

$= \frac{3x(x-y)}{10(x^2+xy-2y^2)} \times \frac{5y(x+2y)}{6(x^2+xy+y^2)} \div \frac{2[(x)^2-(y)^2]}{4[(x)^3-(y)^3]}$

$= \frac{x(x-y)}{2(x^2+2xy-xy-2y^2)} \times \frac{y(x+2y)}{2(x^2+xy+y^2)} \div \frac{(x-y)(x+y)}{2(x-y)(x^2+xy+y^2)}$

$= \frac{x(x-y)}{2[x(x+2y)-y(x+2y)]} \times \frac{y(x+2y)}{2(x^2+xy+y^2)} \times \frac{2(x^2+xy+y^2)}{(x+y)}$

$= \frac{x(x-y)}{2(x+2y)(x-y)} \times \frac{y(x+2y)}{2(x^2+xy+y^2)} \times \frac{2(x^2+xy+y^2)}{(x+y)}$

$= \frac{xy}{2(x+y)}$

(ix) $\frac{2x^2-98}{x^3-125} \times \frac{x^2-3x-10}{3x-21} \div \frac{x^2+5x-14}{x^2+5x+25}$

$= \frac{2(x^2-49)}{(x)^3-(5)^3} \times \frac{x^2-5x+2x-10}{3(x-7)} \div \frac{x^2+7x-2x-14}{x^2+5x+25}$

$= \frac{2[(x)^2-(7)^2]}{(x-5)(x^2+5x+25)} \times \frac{x(x-5)+2(x-5)}{3(x-7)} \div \frac{x(x+7)-2(x+7)}{x^2+5x+25}$

$= \frac{2(x-7)(x+7)}{(x-5)(x^2+5x+25)} \times \frac{(x-5)(x+2)}{3(x-7)} \div \frac{(x+7)(x-2)}{x^2+5x+25}$

$= \frac{2(x-7)(x+7)}{(x-5)(x^2+5x+25)} \times \frac{(x-5)(x+2)}{3(x-7)} \times \frac{(x^2+5x+25)}{(x+7)(x-2)}$

$= \frac{2(x+2)}{3(x-2)}$

(x) $\frac{x^2-2x}{2x+6} \times \frac{x^2+x-6}{x^2-5x} \times \frac{6x-30}{x^2-2x}$

$= \frac{x(x-2)}{2(x+3)} \times \frac{x^2+3x-2x-6}{x(x-5)} \times \frac{6(x-5)}{x(x-2)}$

$= \frac{x(x-2)}{2(x+3)} \times \frac{x(x+3)-2(x+3)}{x(x-5)} \times \frac{6(x-5)}{x(x-2)}$

$= \frac{x(x-2)}{2(x+3)} \times \frac{(x+3)(x-2)}{x(x-5)} \times \frac{6(x-5)}{x(x-2)}$

$= \frac{3(x-2)}{x}$

# EXERCISE 5.4

1. Solve the following equations:

(i) $5x^4 - 19x^2 + 12 = 0$ (ii) $4x^4 - 27x^2 + 18 = 0$

(iii) $5x^4 - 22x^2 + 8 = 0$ (iv) $4\cdot 2^{2x+1} - 9\cdot 2^x + 1 = 0$

(v) $4^{1+x} + 4^{1-x} - 10 = 0$ (vi) $3^x + 3^{3-x} - 12 = 0$

(vii) $5^{1+3x} + 5^{2-3x} - 126 = 0$ (viii) $\left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) - 33 = 0$

(ix) $2\left(x^2 + \frac{1}{x^2}\right) - 25\left(x - \frac{1}{x}\right) - 17 = 0$ (x) $2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$

(xi) $(x+1)(x+2)(x-4)(x-5) + 8 = 0$ (xii) $(x+3)(x+4)(x+5)(x+6) = -1$

**Solutions**

(i) $\mathbf{5x^4 - 19x^2 + 12 = 0}$ ⟶ (1)

Let $y = x^2$

$y^2 = (x^2)^2$

$y^2 = x^4$

*puting the value of* $y$ *and* $y^2$ *in equation* 1

$5y^2 - 19y + 12 = 0$

$5y^2 - 15y - 4y + 12 = 0$

$5y(y-3) - 4(y-3) = 0$

$(y-3)(5y-4) = 0$

$y - 3 = 0 \quad , \quad 5y - 4 = 0$

$y = 3 \quad , \quad 5y = 4 \quad y = \frac{4}{5}$

*since* $y = x^2$

$x^2 = 3 \quad , \quad x^2 = \frac{4}{5}$

$x = \pm\sqrt{3} \quad , \quad x = \pm\frac{2}{\sqrt{5}}$

$S.S = \left\{\pm\sqrt{3}, \pm\frac{2}{\sqrt{5}}\right\}$

(ii) $\mathbf{4x^4 - 27x^2 + 18 = 0}$ ⟶ (1)

Let $y = x^2$

$y^2 = (x^2)^2$

$y^2 = x^4$

*puting the value of* $y$ *and* $y^2$ *in equation* 1

$4y^2 - 27y + 18 = 0$

$4y^2 - 24y - 3y + 18 = 0$

$4y(y-6) - 3(y-6) = 0$

$(y-6)(4y-3) = 0$

$y - 6 = 0 \quad , \quad 4y - 3 = 0$

$y = 6 \quad 4y = 3$

$y = \frac{3}{4}$

*since* $y = x^2$

$x^2 = 6 \quad , \quad x^2 = \frac{3}{4}$

$x = \pm\sqrt{6} \quad x = \pm\frac{\sqrt{3}}{2}$

$S.S = \left\{\pm\sqrt{6}, \pm\frac{\sqrt{3}}{2}\right\}$

**(iii)** $5x^4 - 22x^2 + 8 = 0 \longrightarrow (1)$

Let $y = x^2$

$y^2 = (x^2)^2$

$y^2 = x^4$

*now equation* (1) *become*

$5y^2 - 22y + 8 = 0$

$5y^2 - 20y - 2y + 8 = 0$

$5y(y - 40) - 2(y - 4) = 0$

$(y - 4)(5y - 2) = 0$

$y - 4 = 0 \quad , \quad 5y - 2 = 0$

$y = 4 \quad , \quad 5y = 2$

$y = \frac{2}{5}$

*since* $x^2 = y$

$x^2 = 4 \quad , \quad x^2 = \frac{2}{5}$

$x = \pm 2 \qquad x = \pm\sqrt{\frac{2}{5}}$

$S.S = \left\{\pm 2, \pm\sqrt{\frac{2}{5}}\right\}$

**(iv)** $4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x}.2 - 9.2^x + 1 = 0$

$8.2^{2x} - 9.x^2 + 1 = 0 \longrightarrow (1)$

*let* $y = 2^x$

$y^2 = 2^{2x}$

*now equation* (1) *become*

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y - 1) - 1(y - 1) = 0$

$(y - 1)(8y - 1) = 0$

$y - 1 = 0 \quad , \quad 8y - 1 = 0$

$y = 1 \qquad 8y = 1$

$y = \frac{1}{8}$

*since* $2^x = y$

$2^x = 1 \quad , \quad 2^x = \frac{1}{8}$

$2^x = 2^0 \qquad 2^x = 2^{-3}$

$x = 0 \qquad x = -3$

$s.s = \{0, -3\}$

**(v)** $4^{1+x} + 4^{1-x} - 10 = 0$

$4.4^x + 4.4^{-x} - 10 = 0$

$4.4^x + 4.\frac{1}{4^x} - 10 = 0$

$let \quad y = 4^x$

$4y + 4\frac{1}{y} - 10 = 0$

$\frac{4y^2 + 4 - 10y}{y} = 0$

$4y^2 - 10y + 4 = 0$

$4y^2 - 8y - 2y + 4 = 0$

$4y(y-2) - 2(y-2) = 0$

$(y-2)(4y-2) = 0$

| | |
|---|---|
| $y - 2 = 0$ , | $4y - 2 = 0$ |
| $y = 2$ | $4y = 2$ |
| | $y = \frac{2}{4}$ |
| | $y = \frac{1}{2}$ |

$since \ y = 4^x$

| | |
|---|---|
| $4^x = 2$ , | $4^x = \frac{1}{2}$ |
| $2^{2x} = 2^1$ | $2^{2x} = 2^{-1}$ |
| $2x = 1$ | $2x = -1$ |
| $x = \frac{1}{2}$ | $x = \frac{-1}{2}$ |

$s.s = \left\{\frac{-1}{2}, \frac{1}{2}\right\}$

**(vi)** $3^x + 3^{3-x} - 12 = 0$

$3^x + 3^3.3^{-x} - 12 = 0$

$3^x + 27.\frac{1}{3^x} - 12 = 0$

$let \ y = 3^x$

$y + 27.\frac{1}{y} - 12 = 0$

$\frac{y^2 + 27 - 12y}{y} = 0$

$y^2 - 12y + 27 = 0$

$y^2 - 9y - 3y + 27 = 0$

$y(y-9) - 3(y-9) = 0$

$(y-9)(y-3) = 0$

| | |
|---|---|
| $y - 9 = 0$ , | $y - 3 = 0$ |
| $y = 9$ | $y = 3$ |

$since \ y = 3^x$

| | |
|---|---|
| $3^x = 9$ | $3^x = 3$ |
| $3^x = 3^2$ | $3^x = 3^1$ |
| $x = 2$ | $x = 1$ |

$S.S = \{1,2\}$

**(vii)** $5^{1+3x} + 5^{2-3x} - 126 = 0$

$5.5^{3x} + 5^2.5^{-3x} - 126 = 0$

$5.5^{3x} + 25.\frac{1}{5^{3x}} - 126 = 0$

*let* $y = 5^{3x}$

$5y + 25\frac{1}{y} - 126 = 0$

$\frac{5y^2 + 25 - 126y}{y} = 0$

$5y^2 - 126y + 25 = 0$

$5y^2 - 125y - y + 25 = 0$

$5y(y - 25) - 1(y - 25) = 0$

$(y - 25)(5y - 1) = 0$

$y - 25 = 0 \quad , \quad 5y - 1 = 0$

$y = 25 \qquad 5y = 1$

$y = \frac{1}{5}$

*since* $y = 5^{3x}$

$5^{3x} = 25 \quad , \quad 5^{3x} = \frac{1}{5}$

$5^{3x} = 5^2 \qquad 5^{3x} = 5^{-1}$

$3x = 2 \qquad 3x = -1$

$x = \frac{2}{3} \qquad x = \frac{-1}{3}$

$S.S = \left\{\frac{-1}{3}, \frac{2}{3}\right\}$

**(viii)** $\left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) - 33 = 0$

*let* $y = x + \frac{1}{x}$

$(y)^2 = \left(x + \frac{1}{x}\right)^2$

$y^2 = x^2 + \frac{1}{x^2} + 2$

$y^2 - 2 = x^2 + \frac{1}{x^2}$

$(y^2 - 2) + 2y - 33 = 0$

$y^2 + 2y - 35 = 0$

$y^2 + 7y - 5y - 35 = 0$

$y(y + 7) - 5(y + 7) = 0$

$(y + 7)(y - 5) = 0$

$y + 7 = 0 \quad , \quad y - 5 = 0$

$y = -7 \qquad y = 5$

*since* $y = x + \frac{1}{x}$

$x + \frac{1}{x} = -7 \quad , \quad x + \frac{1}{x} = 5$

$\frac{x^2 + 1}{x} = -7$

$x^2 + 1 = -7x$

$x^2 + 7x + 1 = 0$

$a = 1, \ b = 7, \ c = 1$

$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{49 - 4}}{2}$

$x = \frac{-7 \pm \sqrt{45}}{2}$

$x = \frac{-7 \pm 3\sqrt{5}}{2}$

*Now*

$x + \frac{1}{x} = 5$

$x^2 - 5x + 1 = 0$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{5 \pm \sqrt{25 - 4}}{2}$

$x = \frac{5 \pm \sqrt{21}}{2}$

$S.S = \left\{\frac{-7 \pm 3\sqrt{5}}{2}, \frac{5 \pm \sqrt{21}}{2}\right\}$

**(ix)** $2\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)-17=0$

Let $y = x-\frac{1}{x}$

$$y^2=\left(x-\frac{1}{x}\right)^2$$

$$y^2=x^2+\frac{1}{x^2}-2$$

$$y^2+2=x^2+\frac{1}{x^2}$$

$$2(y^2+2)-25y-17=0$$

$$2y^2+4-25y-17=0$$

$$2y^2-25y-13=0$$

$$2y^2-26y+y-13=0$$

$$2y(y-13)+1(y-13)=0$$

$$(y-13)(2y+1)=0$$

$$y-13=0 \quad , \quad 2y+1=0$$

$$y=13 \qquad y=\frac{-1}{2}$$

$since\ y = x-\frac{1}{x}$

$$x-\frac{1}{x}=13$$

$$x^2-13x-1=0$$

$$a=1 \quad b=-13 \quad c=-1$$

$$x=\frac{-(-13)\pm\sqrt{(-13)^2-4(1)(-1)}}{2(1)}$$

$$x=\frac{13\pm\sqrt{169+4}}{2}$$

$$x=\frac{13\pm\sqrt{173}}{2}$$

Now

$$y=\frac{-1}{2}$$

$$x-\frac{1}{x}=\frac{-1}{2}$$

$$\frac{x^2-1}{x}=\frac{-1}{2}$$

$$2(x^2-1)=-x$$

$$2x^2-2=-x$$

$$2x^2+x-2=0$$

$$a=2 \quad b=1 \quad c=-2$$

$$x=\frac{-(1)\pm\sqrt{(1)^2-4(2)(-2)}}{2(2)}$$

$$x=\frac{-1\pm\sqrt{1+16}}{4}$$

$$x=\frac{-1\pm\sqrt{17}}{4}$$

$$S.S=\left\{\frac{13\pm\sqrt{173}}{2},\frac{-1\pm\sqrt{17}}{4}\right\}$$

**(x)** $2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$

*Dividied By* $x^2$

$$2\frac{x^4}{x^2} - 5\frac{x^3}{x^2} - 14\frac{x^2}{x^2} - 5\frac{x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 - 5x - 14 - 5\frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} - 5x - 5\frac{1}{x} - 14 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 14 = 0$$

*Let* $y = x + \frac{1}{x}$

$$(y)^2 = \left(x + \tfrac{1}{x}\right)^2$$

$$y^2 = x^2 + \tfrac{1}{x^2} + 2$$

$$y^2 - 2 = x^2 + \tfrac{1}{x^2}$$

$$2(y^2 - 2) - 5y - 14 = 0$$

$$2y^2 - 4 - 5y - 14 = 0$$

$$2y^2 - 5y - 18 = 0$$

$$2y^2 - 9y + 4y - 18 = 0$$

$$y(2y - 9) + 2(2y - 9) = 0$$

$$(2y - 9)(y + 2) = 0$$

$2y - 9 = 0$ , $y + 2 = 0$

$2y = 9$ $y = -2$

$y = \frac{9}{2}$

*since* $y = x + \frac{1}{x}$

$x + \frac{1}{x} = \frac{9}{2}$ $x + \frac{1}{x} = -2$

$\frac{x^2+1}{x} = \frac{9}{2}$ $x^2 + 1 + 2x = 0$

$2x^2 + 2 = 9x$ $x^2 + 2x + 1 = 0$

$2x^2 - 9x + 2 = 0$ $(x + 1)^2 = 0$

$a = 2 \quad b = -9 \quad c = 2$ $x + 1 = 0$

$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(2)}}{2(2)}$ $x = -1$

$$x = \frac{9 \pm \sqrt{81 - 16}}{4}$$

$$x = \frac{9 \pm \sqrt{65}}{4}$$

$$S.S = \left\{-1, \frac{9 \pm \sqrt{65}}{4}\right\}$$

**(xi)** $(x+1)(x+2)(x-4)(x-5)+8=0$

$[(x+1)(x-4)][(x+2)(x-5)]+8=0$

$[x^2-4x+x-4][x^2-5x+2x-10]+8=0$

$(x^2-3x-4)(x^2-3x-10)+8=0$

$Let\ y=x^2-3x$

$(y-4)(y-10)+8=0$

$y^2-10y-4y+40+8=0$

$y^2-14y+48=0$

$y^2-8y-6y+48=0$

$y(y-8)-6(y-8)=0$

$(y-8)(y-6)=0$

$y-8=0 \quad , \quad y-6=0$

$y=8 \quad , \quad y=6$

$since\ y=x^2-3x$

$x^2-3x=8$

$x^2-3x-8=0$

$a=1 \quad b=-3 \quad c=-8$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-8)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9+32}}{2}$$

$$x=\frac{3\pm\sqrt{41}}{2}$$

$Now$

$x^2-3x=6$

$x^2-3x-6=0$

$a=1 \quad b=-3 \quad c=-6$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-6)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9+24}}{2}$$

$$x=\frac{3\pm\sqrt{33}}{2}$$

$$S.S=\left\{\frac{3\pm\sqrt{41}}{2},\frac{3\pm\sqrt{33}}{2}\right\}$$

**(xii)** $(x+3)(x+4)(x+5)(x+6) = -1$

$[(x+3)(x+6)][(x+4)(x+5)] = -1$

$[x^2+6x+3x+18][x^2+5x+4x+20] = -1$

$(x^2+9x+18)(x^2+9x+20) = -1$

$Let\ y = x^2+9x$

$(y+18)(y+20) = -1$

$y^2+20y+18y+360 = -1$

$y^2+38y+360+1 = 0$

$y^2+38y+361 = 0$

$y^2+2(y)(19)+(19)^2 = 0$

$(y+19)^2 = 0$

$y+19 = 0$

$y = -19$

$Since\ y = x^2+9x$

$x^2+9x = -19$

$x^2+9x+19 = 0$

$a = 1 \quad b = 9 \quad c = 19$

$$x = \frac{-(9) \pm \sqrt{(9)^2-4(1)(19)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81-76}}{2}$$

$$x = \frac{-9 \pm \sqrt{5}}{2}$$

$$S.S = \left\{\frac{-9\pm\sqrt{5}}{2}\right\}$$

# EXERCISE 5.5

1. A train travels a distance of 240 km at a uniform rate, if it had finished 4 km an hour slower, it would have taken 2 hours more over the journey. Find its rate of travelling.

**Solution**

Distance (D) = 240km

Initial speed = x

Time = $\frac{distance}{speed}$

$T_1 = \frac{240}{x}$

Final speed = x-4

$T_2 = \frac{240}{x-4}$

$T_2 - T_1 = 2$

$\frac{240}{x-4} - \frac{240}{x} = 2$

Divided by 2

$\frac{120}{x-4} - \frac{120}{x} = 1$

$\frac{120x-120(x-4)}{x(x-4)} = 1$

$\frac{120x-120x+480}{x^2-4x} = 1$

$$120x - 120x + 480 = x^2 - 4x$$

$$x^2 - 4x - 480 = 0$$

$$x^2 - 24x + 20x - 480 = 0$$

$$x(x-24) + 20(x-24) = 0$$

$$(x-24)(x+20) = 0$$

$$x - 24 = 0 \ , x + 20 = 0$$

$$x = 24 \quad , x = -20$$

Speed cannot be negative, so the answer is 24 km/h

2. Arshia and Ibraheem complete a job together in 4 hours. If Arshia takes 6 hours, then find how much time will Ibraheem take?

**Solution**

$$\text{Combined rate} = \frac{1}{4}\left(\frac{job}{hours}\right)$$

$$\text{Arshia rate} = \frac{1}{6}$$

$$\text{Let Ibraheem rate} = \frac{1}{x}$$

$$Arshia\ rate + Ibraheem\ rate = Combined\ rate$$

$$\frac{1}{6} + \frac{1}{x} = \frac{1}{4}$$

$$\frac{1}{x} = \frac{1}{4} - \frac{1}{6}$$

$$\frac{1}{x} = \frac{6-4}{24}$$

$$\frac{1}{x} = \frac{2}{24}$$

$$\frac{1}{x} = \frac{1}{12}$$

$$x = 12 \text{ hours}$$

3. One pipe fills water in 5 hours, another in 8 hours. Second pipe closed after 2 hours. Find the total time.

**Solution**

$1^{st}$ pipe rate = $\frac{1}{5}$ $\left(\frac{tank}{hour}\right)$ ; $2^{nd}$ pipe rate = $\frac{1}{8}$

Assuming they start together, and $2^{nd}$ pipe closed ofter 2 hours

$$\left(\frac{1}{5}+\frac{1}{8}\right)\times 2 = \left(\frac{8+5}{40}\right)\times 2$$

$$= \frac{13}{20} \quad \text{(Filling rate)}$$

**Remaining work:**

$$= 1 - \frac{13}{20}$$

$$= \frac{20-13}{20}$$

$$= \frac{7}{20}$$

**Time for $1^{st}$ pipe to finish rest**

Time = $\frac{work}{rate}$

$$= \frac{\frac{7}{20}}{\frac{1}{5}}$$

$$= \frac{7}{20}\times 5$$

$$= \frac{7}{4}$$

$$= 1.75\ hours$$

Total time = time for both pipe + time for $1^{st}$ pipe

= 2 hours + 1.75 hours

= 3.75 hours

= 3 hours 45 minutes

4. Huria can complete a project in 12 hours by working alone. If Abdul Hadi join her and they finish it together in 5 hours, how long would it take Abdul Hadi to do the project alone?

**Solution**

Huria works rate = $\frac{1}{12}$ $\left(\frac{job}{hour}\right)$

Combined work rate = $\frac{1}{5}$

let Abdul Hadi time rate x, so

Abdul hadi rate = $\frac{1}{x}$

Huria works rate + Abdul hadi works rate = combined works rate

$\frac{1}{12}+\frac{1}{x}=\frac{1}{5}$

$\frac{1}{x}=\frac{1}{5}-\frac{1}{12}$

$\frac{1}{x}=\frac{12-5}{60}$

$\frac{1}{x}=\frac{7}{60}$

$x=\frac{60}{7}$

$x = 8.57\ hours$

$x = 8\ hours\ 34\ minutes$

5. Two cars start from opposite towns and head towards each other. The distance between them is 240 km. One travels at $x$ km/h and the other at $(x + 10)$ km/h. They meet after 2 hours. Find the speed of each car.

**Solution**

Total distance = 240 km

Relative speed = x+(x+10)

= 2x+10

Time = 2 hours

Distance = Speed × Time

$240 = (2x + 10) \times 2$

divided by 2

$120 = 2x + 10$

$120 - 10 = 2x$

$2x = 110$

$x = 55$

speed of car A = 55 km/h

speed of car B = 55 + 10

= 65 km/h

6. Rashid can paint a house in 6 days, but if he gets a helper he can do it in 4 days. How long would it take the helper to paint the house alone?

**Solution**

Rashid rate = $\frac{1}{6}$

combined rate = $\frac{1}{4}$

let the helper time x, so

helper time rate = $\frac{1}{x}$

Rashid rate + helper rate = combined rate

$$\frac{1}{6}+\frac{1}{x}=\frac{1}{4}$$

$$\frac{1}{x}=\frac{1}{4}-\frac{1}{6}$$

$$\frac{1}{x}=\frac{6-4}{24}$$

$$\frac{1}{x}=\frac{2}{24}$$

$$\frac{1}{x}=\frac{1}{12}$$

$$x = 12\ days$$

7. Ishmal runs 10 km in $\frac{2x}{x+4}$ hours. If her average speed is 8 km/h, find the value of $x$.

**Solution**

distance (D) = 10km

time (T) = $\frac{2x}{x+4}$

speed = 8 km/h

distance = speed × time

$10 = 8 \times \frac{2x}{x+4}$

$10(x+4) = 16x$

$10x + 40 = 16x$

$40 = 16x - 10x$

$40 = 6x$

$\frac{40}{6} = x$

$x = 6.67$

8. Fahad runs 600 m at a certain pace, and then doubling his pace, does another 600 m. If he took $2\frac{1}{2}$ to cover the distance 1200 m, find the pace he started at, in metres per seconds.

**Solution**

**first part :-**

distance = 600m

speed = x

time ($t_1$) = $\frac{600}{x}$

**second part:-**

distance = 600m

speed = 2x

time($t_2$) = $\frac{600}{2x} = \frac{300}{x}$

total time = 2.5 minutes= 150 second

$t_1+t_2$=total time

$\frac{600}{x} + \frac{300}{x} = 150$

$\frac{600+300}{x} = 150$

$900 = 150x$

$x = 6m/s$

9. A cyclist travels 30 km at a certain speed. If the speed had been 5 km/h faster, the journey would have taken 1 hour less. Find the speed.

**Solution**

distance = 30km

original speed = x

time = $\frac{30}{x}$

new speed = x+5

time = $\frac{30}{x+5}$

$$\frac{30}{x} - \frac{30}{x+5} = 1$$

$$\frac{30(x+5)-30x}{x(x+5)} = 1$$

$$30x + 150 - 30x = x^2 + 5x$$

$$x^2 + 5x - 150 = 0$$

$$x^2 + 15x - 10x - 150 = 0$$

$$x(x + 15) - 10(x + 15) = 0$$

$$(x + 15)(x - 10) = 0$$

$$x + 15 = 0 \quad , \quad x - 10 = 0$$

$$x = -15 \text{ (reject)} \qquad x = 10km/h$$

# REVIEW EXERCISE 5

1. Four possible answers are given for the following questions. Choose the correct answer:

(i) The expression $\frac{2x-1}{x^2+4}$ is:

(a) a polynomial (b) an algebraic fraction
(c) a numerical fraction (d) an equation

(ii) Lowest form of $\frac{18a^2bc^2}{12ac}$ is:

(a) $\frac{9abc}{6}$ (b) $\frac{3abc}{2}$ (c) $\frac{18b}{12}$ (d) $\frac{3}{2}$

(iii) Lowest form of $\frac{15xyz}{10}$ is:

(a) $\frac{3xyz}{2}$ (b) $\frac{3}{2}$ (c) $xyz$ (d) $\frac{xyz}{2}$

(iv) Simplified form of $\frac{3x+1}{5}+\frac{2x+3}{5}$ is:

(a) $\frac{5x+4}{10}$ (b) $\frac{5x+4}{5}$ (c) $\frac{5x+4}{25}$ (d) $5x+4$

(v) Simplified form of $\left(\frac{x^2-y^2}{x+y}\right)(x-y)$ is:

(a) $(x+y)^2$ (b) $x^2+y^2$ (c) $x-y$ (d) $(x-y)^2$

(vi) $(x^3-y^3)\div(x-y)$ in simplified form is:

(a) $x^2-xy+y^2$ (b) $x^2+xy+y^2$ (c) $x^2-y^2$ (d) $x^2+y^2$

(vii) An equation of the form $\frac{5}{x}+\frac{1-x}{3}=\frac{1}{6x}$ is called:

(a) radical equation (b) reciprocal equation
(c) fractional equation (d) exponential equation

(viii) An equation of the form $5^x+64\cdot5^{-x}-20=0$ is called:

(a) radical equation (b) exponential equation
(c) reciprocal equation (d) fractional equation

(ix) Roots of $y^2-24y+128=0$ are:

(a) 8, –16 (b) 8, 16 (c) –8, 16 (d) –8, –16

(x) Linear factors of $3x^2+10x+3=0$ are:

(a) $(x+3),(3x+1)$ (b) $(x+3),(3x-1)$
(c) $(x-3),(3x+1)$ (d) $(x-3),(3x-1)$

2. Reduce the following to the lowest form:

(i) $\dfrac{x^2-7x+12}{x^2-6x+9}$ (ii) $\dfrac{x^2+3x-18}{7x-21}$

**Solution**

**(i)** $\dfrac{x^2-7x+12}{x^2-6x+9}$

$= \dfrac{x^2-4x-3x+12}{x^2-3x-3x+9}$

$= \dfrac{x(x-4)-3(x-4)}{x(x-3)-3(x-3)}$

$= \dfrac{(x-4)(x-3)}{(x-3)(x-3)}$

$= \dfrac{x-4}{x-3}$

**(ii)** $\dfrac{x^2+3x-18}{7x-21}$

$= \dfrac{x^2+6x-3x-18}{7(x-3)}$

$= \dfrac{x(x+6)-3(x+6)}{7(x-3)}$

$= \dfrac{(x+6)(x-3)}{7(x-3)}$

$= \dfrac{x+6}{7}$

3. Simplify:

(i) $\dfrac{5}{x^2+x-6} - \dfrac{1}{2x^2-7x+6}$ (ii) $\dfrac{x^3-27}{x^2-9} \div \dfrac{x^2+3x+9}{x^2+6x+9}$

(iii) $\dfrac{a^2-(b-c)^2}{(a+b)^2-c^2} \times \dfrac{a^2-(b+c)^2}{(a-b)^2-c^2}$

**Solution**

**(i)** $\dfrac{5}{x^2+x-6} - \dfrac{1}{2x^2-7x+6}$

$= \dfrac{5}{x^2+3x-2x-6} - \dfrac{1}{2x^2-4x-3x+6}$

$= \dfrac{5}{x(x+3)-2(x+3)} - \dfrac{1}{2x(x-2)-3(x-2)}$

$$= \frac{5}{(x+3)(x-2)} - \frac{1}{(x-2)(2x-3)}$$

$$= \frac{5(2x-3)-(x+3)}{(x+3)(x-2)(2x-3)}$$

$$= \frac{10x-15-x-3}{(x+3)(x-2)(2x-3)}$$

$$= \frac{9x-18}{(x+3)(x-2)(2x-3)}$$

$$= \frac{9(x-2)}{(x+3)(x-2)(2x-3)}$$

$$= \frac{9}{(x+3)(2x-3)}$$

**(ii)** $\frac{x^3-27}{x^2-9} \div \frac{x^2+3x+9}{x^2+6x+9}$

$$= \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)} \div \frac{x^2+3x+9}{x^2+3x+3x+9}$$

$$= \frac{(x^2+3x+9)}{(x+3)} \div \frac{x^2+3x+9}{x(x+3)+3(x+3)}$$

$$= \frac{(x^2+3x+9)}{(x+3)} \div \frac{x^2+3x+9}{(x+3)(x+3)}$$

$$= \frac{x^2+3x+9}{(x+3)} \times \frac{(x+3)(x+3)}{x^2+3x+9}$$

$= x + 3$

**(iii)** $\frac{a^2-(b-c)^2}{(a+b)^2-c^2} \times \frac{a^2-(b+c)^2}{(a-b)^2-c^2}$

$$= \frac{[a-(b-c)][a+b-c]}{(a+b-c)(a+b+c)} \times \frac{[a-(b+c)][a+b+c]}{(a-b-c)(a-b+c)}$$

$$= \frac{(a-b+c)(a+b-c)}{(a+b-c)(a+b+c)} \times \frac{(a-b-c)(a+b+c)}{(a-b-c)(a-b+c)}$$

$= 1$

4. Solve the following equations:

(i) $x^4-16x^2+63=0$ (ii) $4x^4-16x^2+15=0$

(iii) $3^{2x}-12\cdot 3^x+27=0$ (iv) $\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-2=0$

(v) $\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=4$ (vi) $(x+9)(x-3)(x-7)(x+5)=385$

(vii) $(x-1)(x-2)(x+5)(x-8)=-360$

**Solution**

**(i)** $\mathbf{x^4-16x^2+63=0}$

Let $y=x^2$

$y^2=x^4$

$y^2-16y+63=0$

$y^2-9y-7y+63=0$

$y(y-9)-7(y-9)=0$

$(y-9)(y-7)=0$

Y-9=0 , y-7=0

Y=9 y=7

Since $y=x^2$

$x^2=9$ , $x^2=7$

$x=\pm 3$ $x=\pm\sqrt{7}$

$S.S=\{\pm 3,\pm\sqrt{7}\}$

**(ii)** $\mathbf{4x^4-16x^2+15=0}$

let $y=x^2, y^2=x^4$

$4y^2-16y+15=0$

$4y^2-10y-6y+15=0$

$2y(2y-5)-3(2y-5)=0$

$(2y-5)(2y-3)=0$

$2y-5=0\,,\,2y-3=0$

$y=\frac{5}{2}$ , $y=\frac{3}{2}$

$x^2=\frac{5}{2}$ , $x^2=\frac{3}{2}$

$x=\pm\sqrt{\frac{5}{2}}$ , $x=\pm\sqrt{\frac{3}{2}}$

$x=\pm\frac{\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ , $x=\pm\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$x=\pm\frac{\sqrt{10}}{2}$ , $x=\pm\frac{\sqrt{3}}{2}$

$S.S=\left\{\pm\frac{\sqrt{10}}{2},\pm\frac{\sqrt{3}}{2}\right\}$

**(iii)** $3^{2x} - 12.3^x + 27 = 0$

Let $y = 3^x$ , $y^2 = 3^{2x}$

$y^2 - 12y + 27 = 0$

$y^2 - 9y - 3y + 27 = 0$

$y(y-9) - 3(y-9) = 0$

$(y-9)(y-3) = 0$

$y - 9 = 0$ , $y - 3 = 0$

$y = 9$ $\quad$ $y = 3$

$3^x = 9$ $\quad$ $3^x = 3$

$x = 2$ $\quad$ $x = 1$

**(iv)** $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 2 = 0$

$let\ y = x + \frac{1}{x}$

$y^2 - 2 = x^2 + \frac{1}{x^2}$

$(y^2 - 2) - 3y - 2 = 0$

$y^2 - 3y - 4 = 0$

$y^2 - 4y + y - 4 = 0$

$y(y-4) + 1(y-4) = 0$

$(y-4)(y+1) = 0$

$y = 4$ , $y = -1$

$y = 4$

$x + \frac{1}{x} = 4$

$x^2 - 4x + 1 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16-4}}{2}$

$x = \frac{4 \pm \sqrt{12}}{2}$

$x = \frac{2(2 \pm \sqrt{3})}{2}$

$x = 2 \pm \sqrt{3}$

$y = -1$

$x + \frac{1}{x} = -1$

$x^2 + x + 1 = 0$

$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

$S.S = \left\{2 \pm \sqrt{3}, \frac{-1 \pm \sqrt{-3}}{2}\right\}$

**(v)** $\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=4$

Let $y=x+\frac{1}{x}$

$$y^2-2=x^2+\frac{1}{x^2}$$

$$y^2-2+y-4=0$$

$$y^2+y-6=0$$

$$y^2+3y-2y-6=0$$

$$y(y+3)-2(y+3)=0$$

$$(y+3)(y-2)=0$$

$$y+3=0\ ,\ y-2=0$$

$$y=-3 \qquad y=2$$

$y=-3$

$$x+\frac{1}{x}=-3$$

$$x^2+3x+1=0$$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{-3\pm\sqrt{9-4}}{2}$$

$$x=\frac{-3\pm\sqrt{5}}{2}$$

$y=2$

$$x+\frac{1}{x}=2$$

$$x^2-2x+1=0$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4-4}}{2}=\frac{2\pm\sqrt{0}}{2}=\frac{2}{2}$$

$$x=1$$

$$S.S=\left\{1,\frac{-3\pm\sqrt{5}}{2}\right\}$$

**(vi)** $(x+9)(x-3)(x-7)(x+5)=385$

$[(x+9)(x-7)][(x-3)(x+5)]=385$

$[x^2-7x+9x-63][x^2+5x-3x-15]=385$

$(x^2+2x-63)(x^2+2x-15)=385$

$let\ y=x^2+2x$

$(y-63)(y-15)=385$

$y^2-15y-63y+945=385$

$y^2-78y+945-385=0$

$y^2-78y+560=0$

$y^2-70y-8y+560=0$

$y(y-70)-8(y-70)=0$

$(y-70)(y-8)=0$

$y-70=0\ ,\quad y-8=0$

$y=70 \qquad y=8$

$y=70$

$x^2+2x=70$

$x^2+2x-70=0$

$x=\frac{-(2)\pm\sqrt{(2)^2-4(1)(-70)}}{2(1)}$

$x=\frac{-2\pm\sqrt{4+280}}{2}$

$x=\frac{-2\pm\sqrt{284}}{2}$

$x=\frac{-2\pm\sqrt{4\times71}}{2}$

$x=\frac{-2\pm2\sqrt{71}}{2}$

$x=-1\pm\sqrt{71}$

$y=8$

$x^2+2x=8$

$x^2+2x-8=0$

$x^2+4x-2x-8=0$

$x(x+4)-2(x+4)=0$

$(x+4)(x-2)=0$

$x+4=0\ ,\ x-2=0$

$x=-4\ ,\quad x=2$

$S.S=\{-4,2,-1\pm\sqrt{71}\}$

**(vii)** $(x-1)(x-2)(x+5)(x-8) = -360$

$[(x-1)(x-2)][(x+5)(x-8)] = -360$

$(x^2-2x-x+2)(x^2-8x+5x-40) = -360$

$(x^2-3x+2)(x^2-3x-40) = -360$

$let\ y = x^2-3x$

$(y+2)(y-40) = -360$

$y^2-40y+2y-80 = -360$

$y^2-38y-80+360 = 0$

$y^2-38y+280 = 0$

$y^2-28y-10y+280 = 0$

$y(y-28)-10(y-28) = 0$

$(y-28)(y-10) = 0$

$y-28=0 \quad , \quad y-10=0$

$y = 28 \qquad y = 10$

$x^2-3x = 28$

$x^2-3x-28 = 0$

$x^2-7x+4x-28 = 0$

$x(x-7)+4(x-7) = 0$

$(x-7)(x+4) = 0$

$x-7=0 \quad , \quad x+4=0$

$x = 7 \qquad x = -4$

$x^2-3x = 10$

$x^2-3x-10 = 0$

$x^2-5x+2x-10 = 0$

$x(x-5)+2(x-5) = 0$

$(x-5)(x+2) = 0$

$x-5=0 \quad , \quad x+2=0$

$x = 5 \qquad x = -2$

$S.S = \{-2,-4,5,7\}$

5. Ahmad takes 2 hours to paint 50 glasses. Faiza takes 2 hours to paint 45 glasses. Working together, how long should it take them to paint 150 glasses?

**Solution**

Ahamd works 2 hours = 50 glasses

Faiza works in 2 hours = 45 glasses

Total glasses to be paint = 150

Time taken working together =?

Combined rate = ahamd rate + faiza rate

$$= \frac{work}{time} + \frac{work}{time}$$

$$= \frac{50}{2} + \frac{45}{2}$$

$$= 25 + 22.5$$

Combined rate = 47.5 glasses

$$\text{Time taken together working} = \frac{total\ work}{combined\ rate}$$

$$= \frac{150}{47.5}$$

$$= 3.16 \text{ hours}$$

6. A tap can fill a tank in 6 hours. Another tap can fill it in 9 hours. If both taps are opened together, how long will it take to fill the empty tank?

**Solution**

Time for tap A to fill the tank = 6 hours

Time for tap B to fill the tank = 9 hours

Total time whrn both are open = ?

Work done by Tap A in 1h = $\frac{1}{6}$

Work done by Tap B in 1h = $\frac{1}{9}$

Total work per hour = $\frac{1}{6} + \frac{1}{9}$

$$\frac{1}{T} = \frac{3}{18} + \frac{2}{18}$$

$$\frac{1}{T} = \frac{5}{18}$$

$$T = \frac{18}{5} = 3.6 \text{ hours}$$

7. Maham bought a certain number of toys for Rs. 300. If each toy had cost Rs. 5 less, she could have bought two more for the same amount. How many toys did she buy?

**Solution**

Let x be the number of toys maham originally bought

The orignal price per toy= $\frac{300}{x}$

If the price were Rs.5 less,

The new price = $\frac{300}{x} - 5$

At this lower price, she could buy x+2 toys for same Rs.300

$$(x+2)\left(\frac{300}{x} - 5\right) = 300$$

$$\frac{300x}{x} - 5x + \frac{600}{x} - 10 = 300$$

$$300 - 5x + \frac{600}{x} - 10 - 300 = 0$$

$$-5x^2 + 600 - 10x = 0$$

$$-5(x^2 - 120 + 2x) = 0$$

$$x^2 + 2x - 120 = 0$$

$$x^2 + 12x - 10x - 120 = 0$$

$$x(x+12) - 10(x+12) = 0$$

$$(x+12)(x-10) = 0$$

$$x + 12 = 0 \quad , \quad x - 10 = 0$$

$$x = -12 \qquad x = 10$$

$$(x = -12\, reject)$$