UNIT 4

Functions and Graphs

EXERCISE 4.1

1. If $f(x) = 2x + 5$, $g(x) = 3x - 2$, then find

(i) $f(x) + g(x)$ (ii) $f(x) - g(x)$

Solution

(i) $f(x) + g(x)$

$$\begin{aligned} f(x) + g(x) &= (2x+5) + (3x-2) \\ &= 2x + 3x + 5 - 2 \\ &= 5x + 3. \end{aligned}$$

(ii) $f(x) - g(x)$

$$\begin{aligned} f(x) - g(x) &= (2x+5) - (3x-2) \\ &= 2x + 5 - 3x + 2 \\ &= -x + 7. \end{aligned}$$

2. If $f(x) = x + 2$, $g(x) = 2x + 4$, then find

(i) $f(x)\cdot g(x)$ (ii) $g(x)\cdot f(x)$ (iii) $\dfrac{f(x)}{g(x)}$ (iv) $\dfrac{g(x)}{f(x)}$

3. For the functions f and g, find

Solution

(i) $f(x)\cdot g(x)$

$$\begin{aligned} f(x)\cdot g(x) &= (x+2)(2x+4) \\ &= 2x^2 + 4x + 4x + 8 \\ &= 2x^2 + 8x + 8. \end{aligned}$$

(ii) $g(x)\cdot f(x)$

$$g(x)\cdot f(x) = f(x)\cdot g(x) = 2x^2 + 8x + 8.$$

(iii) $\frac{f(x)}{g(x)}$

$$\frac{f(x)}{g(x)} = \frac{x+2}{2x+4} = \frac{x+2}{2(x+2)} = \frac{1}{2}.$$

(iv) $\frac{g(x)}{f(x)}$

$$\frac{g(x)}{f(x)} = \frac{2x+4}{x+2} = \frac{2(x+2)}{x+2} = 2.$$

3. For the functions f and g, find

(a) $(fog)(x)$ (b) $(gof)(x)$ (c) $(fof)(x)$ (d) $(gog)(x)$

Where,

(i) $f(x) = 2x + 3$; $g(x) = x^3$ (ii) $f(x)=\frac{2}{x}, x \neq 0; g(x) = 2x^2 - 1$

(iii) $f(x) = 2x - 1$; $g(x) = \frac{x+1}{2}$

Solution

(i) $f(x) = 2x + 3$; $g(x) = x^3$	
(a) $(fog)(x)$ $(fog)(x) = f(g(x))$ $= f(x^3)$ $= 2(x^3) + 3$ $= 2x^3 + 3.$	(b) $(gof)(x)$ $(gof)(x) = g(f(x))$ $= g(2x + 3)$ $= (2x + 3)^3.$
(c) $(fof)(x)$ $(fof)(x) = f(f(x))$ $= f(2x + 3)$ $= 2(2x + 3) + 3$ $= 4x + 6 + 3$ $= 4x + 9.$	(d) $(gog)(x)$ $(gog)(x) = g(g(x))$ $= g(x^3)$ $= (x^3)^3$ $= x^9.$
(ii) $f(x) = \frac{2}{x}$, $x \neq 0$; $g(x) = 2x^2 - 1$	
(a) $(fog)(x)$ $(fog)(x) = f(g(x))$ $= f(2x^2 - 1)$ $= \frac{2}{2x^2 - 1}.$	(b) $(gof)(x)$ $(gof)(x) = g(f(x))$ $= g(\frac{2}{x})$ $= 2(\frac{2}{x})^2 - 1$ $= \frac{8}{x^2} - 1.$
(c) $(fof)(x)$ $(fof)(x) = f(f(x))$ $= f(\frac{2}{x})$ $= \frac{2}{\frac{2}{x}}$ $= x.$	(d) $(gog)(x)$ $(gog)(x) = g(g(x))$ $= g(2x^2 - 1)$ $= 2(2x^2 - 1)^2 - 1$ $= 2(4x^4 - 4x^2 + 1) - 1$ $= 8x^4 - 8x^2 + 1.$

(iii) $f(x) = 2x - 1 \quad ; \quad g(x) = \frac{x+1}{2}$	
(a) $(fog)(x)$ $(fog)(x) = f(g(x))$ $= f(\frac{x+1}{2})$ $= 2(\frac{x+1}{2}) - 1$ $= x + 1 - 1$ $= x.$	(b) $(gof)(x)$ $(gof)(x) = g(f(x))$ $= g(2x-1)$ $= \frac{(2x-1)+1}{2}$ $= x.$
(c) $(fof)(x)$ $(fof)(x) = f(f(x))$ $= f(2x-1)$ $= 2(2x-1) - 1$ $= 4x - 2 - 1$ $= 4x - 3.$	(d) $(gog)(x)$ $(gog)(x) = g(g(x))$ $= g(\frac{x+1}{2})$ $= \frac{\frac{x+1}{2}+1}{2}$ $= \frac{x+1+2}{4}$ $= \frac{x+3}{4}.$

4. Find the value of k, such that $(fog)(x) = (gof)(x)$, where $f(x) = 3x + 2$, $g(x) = 6x - k$.

Solution

$$(fog)(x) = f(g(x)) = 3(6x - k) + 2 = 18x - 3k$$
$$(gof)(x) = g(f(x)) = 6(3x + 2) - k = 18x + 12$$

Equating,

$$\begin{aligned} 18x - 3k + 2 &= 18x + 12 - k \\ -3k + 2 &= 12 - k \\ -2k &= 10 \\ k &= -5. \end{aligned}$$

5. Given that $f(x) = 3x + 2$ and $g(x) = 2x + 3$. Find
(i) $g(f(4))$ (ii) $f(f(3))$ (iii) $f(g(-2))$

Solution

(i) $g(f(4))$

$$f(4) = 3(4) + 2 = 14,$$
$$g(f(4)) = g(14) = 2(14) + 3 = 31.$$

(ii) $f(f(3))$

$$f(3) = 3(3) + 2 = 11,$$
$$f(f(3)) = f(11) = 3(11) + 2 = 35.$$

(iii) $f(g(-2))$

$$g(-2) = 2(-2) + 3 = -1,$$
$$f(g(-2)) = f(-1) = 3(-1) + 2 = -1.$$

6. Find $f^{-1}(x)$ in each of the following:
(i) $f(x) = 2x - 3$ (ii) $f(x) = 4x^3 - 1$
(iii) $f(x) = \sqrt{x-1}, x \geq 1$ (iv) $f(x) = \frac{x+1}{3x-2}, x \neq \frac{2}{3}$

Solution

(i) $f(x) = 2x - 3$ $y = 2x - 3$ $2x = y + 3$ $x = \frac{y+3}{2}$ $f^{-1}(y) = \frac{y+3}{2}$ $f^{-1}(x) = \frac{x+3}{2}$	(ii) $f(x) = 4x^3 - 1$ $y = 4x^3 - 1$ $4x^3 = y + 1$ $x^3 = \frac{y+1}{4}$ $x = \sqrt[3]{\frac{y+1}{4}}$ $f^{-1}(y) = \sqrt[3]{\frac{y+1}{4}}$ $f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$

(iii) $f(x)=\sqrt{x-1}$, $x\geq 1$	(iv) $f(x)=\frac{x+1}{3x-2}$, $x\neq\frac{2}{3}$
$y=\sqrt{x-1}$ $y^2=x-1$ $x=y^2+1.$ Thus, $f^{-1}(x)=x^2+1,\quad x\geq 0.$	$y=\dfrac{x+1}{3x-2}$ $y(3x-2)=x+1$ $3yx-2y=x+1$ $3yx-x=2y+1$ $x(3y-1)=2y+1$ $x=\dfrac{2y+1}{3y-1}.$ Thus, $f^{-1}(x)=\frac{2x+1}{3x-1},\quad x\neq\frac{1}{3}.$

7. The functions f and g are defined such that $f(x) = 4x + 2$ and $g(x) = 6x - 18$.
(i) Find $f^{-1}(x)$ and $g^{-1}(x)$. (ii) Find x if $f^{-1}(x) = g^{-1}(x)$.

Solution

(i) Find $f^{-1}(x)$ and $g^{-1}(x)$.	(ii) Find x if $f^{-1}(x)=g^{-1}(x)$.
$f(x)=4x+2$ $y=4x+2$ $4x=y-2$ $x=\dfrac{y-2}{4}.$ $f^{-1}(x)=\frac{x-2}{4}.$ $g(x)=6x-18$ $y=6x-18$ $6x=y+18$ $x=\dfrac{y+18}{6}.$ $g^{-1}(x)=\frac{x+18}{6}.$	$\dfrac{x-2}{4}=\dfrac{x+18}{6}$ $6(x-2)=4(x+18)$ $6x-12=4x+72$ $6x-4x=72+12$ $2x=84$ $x=42.$

8. Verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

(i) $f(x) = x - 6$ (ii) $f(x) = 7x - 4$

(iii) $f(x) = \dfrac{x-3}{4}$ (iv) $f(x) = \dfrac{x-4}{x+2}, \ x \neq -2$

Solution

(i) $f(x) = x - 6$ $f^{-1}(x) = x + 6.$ $f(f^{-1}(x)) = f(x+6) = (x+6) - 6 = x,$ $f^{-1}(f(x)) = f^{-1}(x-6) = (x-6) + 6 = x.$	(ii) $f(x) = 7x - 4$ $f^{-1}(x) = \dfrac{x+4}{7}.$ $f(f^{-1}(x)) = f(\dfrac{x+4}{7}) = 7(\dfrac{x+4}{7}) - 4 = x,$ $f^{-1}(f(x)) = f^{-1}(7x-4) = \dfrac{7x-4+4}{7} = x.$
(iii) $f(x) = \frac{x-3}{4}$ $f^{-1}(x) = 4x + 3.$ $f(f^{-1}(x)) = f(4x+3) = \dfrac{4x+3-3}{4} = x,$ $f^{-1}(f(x)) = f^{-1}(\dfrac{x-3}{4}) = 4(\dfrac{x-3}{4}) + 3 = x.$	(iv) $f(x) = \frac{x-4}{x+2}, \ x \neq -2$ $f^{-1}(x) = \dfrac{2x+4}{1-x}, \quad x \neq 1.$ $f(f^{-1}(x)) = f(\dfrac{2x+4}{1-x}) = \dfrac{\frac{2x+4}{1-x} - 4}{\frac{2x+4}{1-x} + 2} = x,$ $f^{-1}(f(x)) = f^{-1}(\dfrac{x-4}{x+2}) = x.$

9. Without finding $f^{-1}(x)$, find domain and range of $f^{-1}(x)$:

(i) $f(x) = 12x - 3$ (ii) $f(x) = \dfrac{1}{2}x + 8$

(iii) $f(x) = \dfrac{x}{1+x}, \ x \neq -1$ (iv) $f(x) = \sqrt{x-2}, \ x \geq 2$

Solution

(i) Domain = $(-\infty, \infty)$; Range = $(-\infty, \infty)$ **(ii)** Domain = $(-\infty, \infty)$; Range = $(-\infty, \infty)$

(iii) Domain = $(-\infty, 1) \cup (1, \infty)$; Range = $(-\infty, -1) \cup (-1, \infty)$

(iv) Domain = $[0, \infty)$; Range = $[2, \infty)$

10. Given that $f(x) = x^2 + 9$ and $g(x) = x + 21$. Find the values of a such that: $f(a) = g(a)$

Solution

$$f(a) = g(a)$$
$$a^2 + 9 = a + 21$$
$$a^2 - a - 12 = 0$$
$$(a-4)(a+3) = 0.$$

Thus,

$$a = 4 \quad \text{or} \quad a = -3.$$

EXERCISE 4.2

1. Plot graph for the following absolute valued functions:

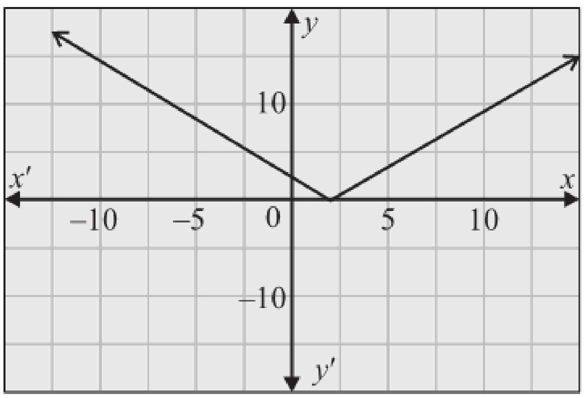
(i) $f(x) = |x - 2|$ (ii) $f(x) = 3|x + 3| - 4$

(iii) $f(x) = 5|x|$ (iv) $f(x) = |x + 2| + 3$

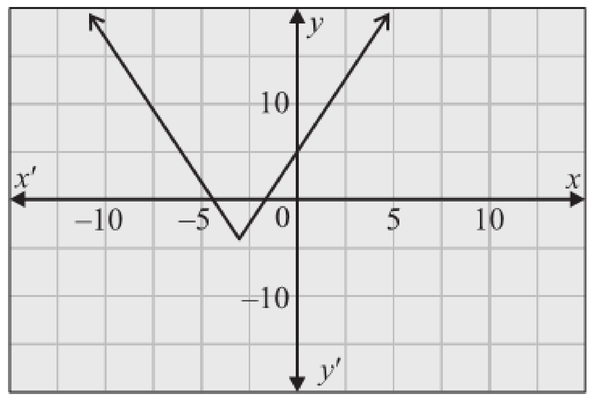
(v) $f(x) = 2|x + 4| - 3$ (vi) $f(x) = 2|x + 1| - 6$

Solution

(i)



(ii)



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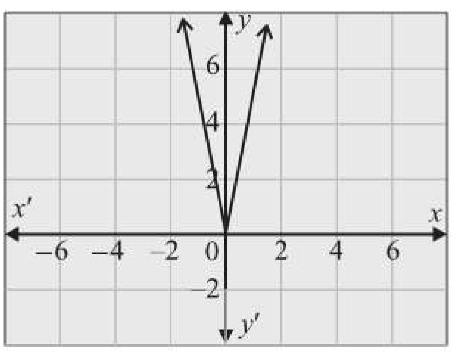

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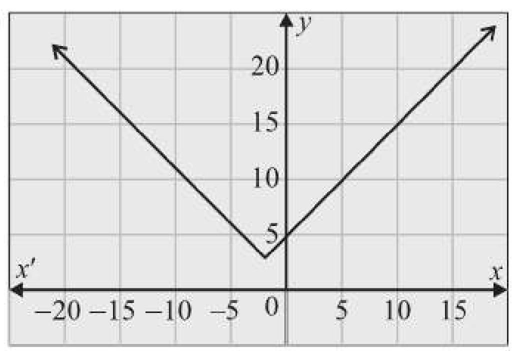

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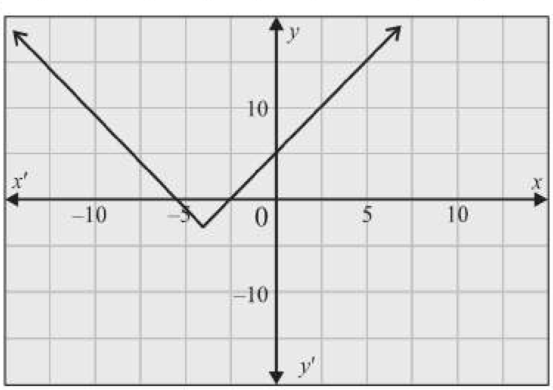
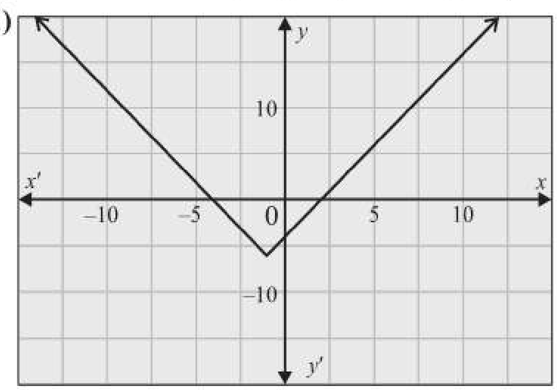


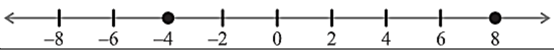
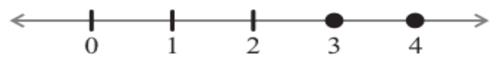
(vi)



2. Solve and express the solution on number line:

(i) $|x-2|=6$ (ii) $|2x+1|=7$ (iii) $|4x-9|=3$ (iv) $|7-2x|=1$

Solution

(i) $\lvert x-2\rvert=6$ $x-2=\pm 6 \implies x=8$ or $x=-4$. 	**(ii)** $\lvert 2x+1\rvert=7$ $2x+1=\pm 7 \implies x=3$ or $x=-4$. 
(iii) $\lvert 4x-9\rvert=3$ $4x-9=\pm 3 \implies x=3$ or $x=\frac{3}{2}$. 	**(iv)** $\lvert 7-2x\rvert=1$ $7-2x=\pm 1 \implies x=3$ or $x=4$. 

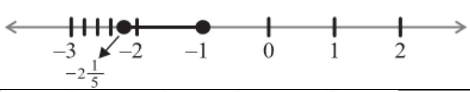
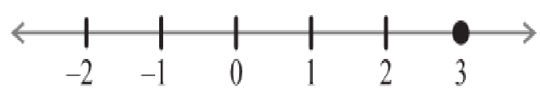
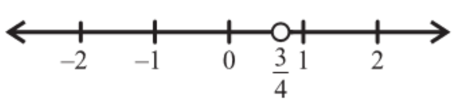
3. Solve and express the solution on number line:

(i) $|5x+8|\le 3$ (ii) $|4x-12|\le 0$

(iii) $|3-4x|>0$ (iv) $1-2\left|\frac{3}{2}x-5\right|>-3$

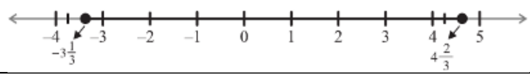
(v) $|3x-2|\le 12$ (vi) $|1-2x|>5$

Solution

(i) $\lvert 5x+8\rvert\le 3$ $-3\le 5x+8\le 3$ $-3-8\le 5x\le 3-8$ $-11\le 5x\le -5$ $\frac{-11}{5}\le x\le -1.$ $\left[-2\frac{1}{5},-1\right]$ 	2. **(ii)** $\lvert 4x-12\rvert\le 0$ $4x-12=0$ (absolute value can only be zero) $4x=12$ $x=3.$ 
(iii) $\lvert 3-4x\rvert>0$ $3-4x\ne 0$ $x\ne\frac{3}{4}.$ $\left(-\infty,\frac{3}{4}\right)\cup\left(\frac{3}{4},\infty\right)$ 	**(iv)** $1-2\lvert\frac{3}{2}x-5\rvert>-3$ $1-2\lvert\frac{3}{2}x-5\rvert>-3$ $-2\lvert 3x-10\rvert>-4$ $\lvert 3x-10\rvert<2$ $-2<3x-10<2$ $8<3x<12$ $\frac{8}{3}<x<4.$

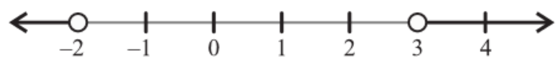
(v) $|3x-2| \le 12$

$$-12 \le 3x-2 \le 12$$
$$-12+2 \le 3x \le 12+2$$
$$-10 \le 3x \le 14$$
$$\frac{-10}{3} \le x \le \frac{14}{3}.$$
$$\left[-3\frac{1}{3}, 4\frac{2}{3}\right]$$



(vi) $|1-2x| > 5$

$$1-2x < -5 \quad \text{or} \quad 1-2x > 5$$
$$-2x < -6 \quad \text{or} \quad -2x > 4$$
$$x > 3 \quad \text{or} \quad x < -2.$$
$$(-\infty, -2) \cup (3, \infty)$$



EXERCISE 4.3

1. A function $B(t) = 5,000 + 200t$ represents the total balance (in rupees) after t months. What will be the balance after 6 months?

Solution

$B(t) = 5,000 + 200t$

$B(6) = 5\,000 + 200(6) = 6\,200$ rupees

2. A function $f(k) = 150 + 20k$ represents the total fare (in rupees) for k kilometres. How much will the fare be for a 12 kilometres ride?

Solution

$f(k) = 150 + 20k$

$f(12) = 150 + 20(12) = 390$ rupees

3. The cost of manufacturing fancy sofa set would be fixed charges Rs. 5500 which is modeled as $f(n) = 5500\,n$, where n is the number of sofa sets. Find the cost of 50 sofa sets.

Solution

$f(n) = 5500\,n$

$f(50) = 5500(50) = 275000$ rupees

4. A function $T(d) = \dfrac{d}{60}$ represents the time T in hours to travel a distance d kilometres. How long will it take to travel 180 km?

Solution

$T(d) = \dfrac{d}{60}$

$T(180) = \frac{180}{60} = 3$ hours

5. A company charges Rs. 100 for an encoding work. In addition, the company charges Rs. 5 per page of printed output. The model of function $f(x) = 100 + 5x$, where x represents the number of pages printed out. How much will company charge for 55 page encoding and printing work?

Solution

$$f(x) = 100 + 5x$$

$f(55) = 100 + 5(55) = 375$ rupees

6. A chemical reaction is stable at $37°C$. The process must be stopped if the temperature deviates by more than $2.5°C$. The condition is modeled as: $|T - 37°| > 2.5°$, T be the temperature. For what temperature values must the process be stopped?

Solution

$$|T - 37°| > 2.5°,$$

$$T - 37 < -2.5 \quad \text{or} \quad T - 37 > 2.5$$

$$\Rightarrow \quad T < 34.5 \quad \text{or} \quad T > 39.5.$$

7. A factory produces metal rods that must be 2.5 metres long, with a tolerance of ± 0.04 metres. An absolute value inequality models this: $|x - 2.5| \leq 0.04$. What is the range of acceptable lengths?

Solution

$$|x - 2.5| \leq 0.04$$

$$2.5 - 0.04 \leq x \leq 2.5 + 0.04$$

$$\Rightarrow \quad 2.46 \leq x \leq 2.54$$

8. A machine part must be aligned so that its centre is exactly at 0. If it shifts more than 0.1 mm, the part is rejected. The model is given by $|x| > 0.1$. What positions of the centre cause rejection?

Solution

$$|x| > 0.1$$

$$x < -0.1 \quad \text{or} \quad x > 0.1.$$

Rejection for shifts beyond ± 0.1 mm.

REVIEW EXERCISE 4

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) If $f(x) = \frac{5x-6}{3}$, then what is the value of $f(3)$?

(a) -1 ✓(b) 3 (c) 9 (d) 15

(ii) A function f from X to Y is represented by:

(a) $f: XY$ (b) $f: Y \to X$ ✓(c) $f: X \to Y$ (d) $f: \frac{X}{Y}$

(iii) $(fog)(x) =$

(a) $(f+g)(x)$ (b) $(f-g)(x)$ ✓(c) $f(g(x))$ (d) $f(x) \div g(x)$

(iv) If $f(x) = 2x + 3$, $g(x) = x + 1$, then $f(x) + g(x) =$

(a) $3x$ ✓(b) $3x + 4$ (c) 4 (d) $2x^2 + 3$

(v) If $f(x) = 5x + 2$, $h(x) = 2x - 2$, then $f(x) - h(x) =$

(a) $3x$ (b) $5x^2 - 4$ ✓(c) $3x + 4$ (d) $-3x - 4$

(vi) If $f(x) = 3x + 1$, $g(x) = 2x$, then $g(x) \times f(x) =$

(a) $6x + 2x$ (b) $5x^2 + 1$ (c) $x + 1$ ✓(d) $6x^2 + 2x$

(vii) If $f(x) = x^2 - 4$, $g(x) = x + 2, x \neq -2$, then $\frac{f(x)}{g(x)} =$

(a) $\frac{1}{x-2}$ (b) $\frac{1}{x+2}$ (c) $x + 2$ ✓(d) $x - 2$

(viii) What is the shape of the graph of an absolute value function?

(a) U-shaped ✓(b) V-shaped
(c) L-shaped (d) M-shaped

(ix) If a graph represents a function, then every vertical line must intersect it at:

(a) 4 points (b) 3 points (c) 2 points ✓(d) 1 point

(x) If $f(x) = x^3$, then $f(-2) =$

✓(a) -8 (v) 8 (c) 4 (d) -6

2. If $f(x) = 25 - x^2$ and $g(x) = 5 + x$, then find

(i) $f(x) + g(x)$ (ii) $f(x) - g(x)$ (iii) $f(x) \cdot g(x)$

(iv) $\dfrac{f(x)}{g(x)}$ (v) $f(7)$ (vi) $g(-8)$

Solution

(i) $f(x) + g(x) = (25 - x^2) + (5 + x) = 30 + x - x^2$.	(ii) $f(x) - g(x) = (25 - x^2) - (5 + x) = 20 - x - x^2$.
(iii) $f(x) \cdot g(x) = (25 - x^2)(5 + x) = 125 + 25x - 5x^2$	(iv) $\frac{f(x)}{g(x)} = \frac{25 - x^2}{5 + x} = \frac{(5 - x)(5 + x)}{5 + x} = 5 - x$
(v) $f(7) = 25 - 7^2 = 25 - 49 = -24$	(vi) $g(-8) = 5 + (-8) = -3$.

3. If $f(x) = x^3$ and $g(x) = 14 + 2x$, then find

(i) $(fog)(x)$ (ii) $(gof)(x)$

(iii) $(fof)(x)$ (iv) $(gog)(x)$

Solution

(i) $(fog)(x) = f(g(x)) = (14 + 2x)^3$.

(ii) $(gof)(x) = g(f(x)) = 14 + 2(x^3) = 14 + 2x^3$.

(iii) $(fof)(x) = f(f(x)) = (x^3)^3 = x^9$.

(iv)

$(gog)(x) = g(g(x)) = 14 + 2(14 + 2x) = 42 + 4x$.

4. Find $f^{-1}(x)$, if

(i) $f(x) = 9x - 1$ (ii) $f(x) = \dfrac{5}{x-1},\ x \neq 1$

(iii) $f(x) = \sqrt{x-5},\ x \geq 5$ (iv) $f(x) = \dfrac{3-x}{2}$

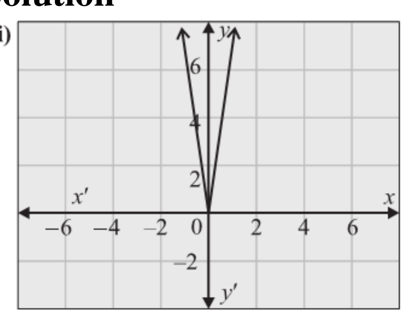
Solution

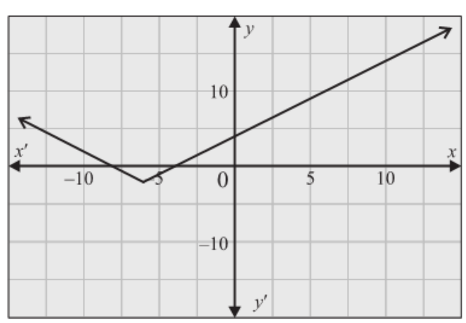
(i) $f(x) = 9x - 1$ $y = 9x - 1$ $9x = y + 1$ $x = \frac{y+1}{9}$ $f^{-1}(y) = \frac{y+1}{9}$ $f^{-1}(x) = \frac{x+1}{9}$	(ii) $f(x) = \frac{5}{x-1}$ $y = \frac{5}{x-1}$ $yx - y = 5$ $yx = 5 + y$ $x = \frac{5+y}{y}$ $f^{-1}(y) = \frac{5+y}{y}$ $f^{-1}(x) = \frac{5+x}{x}$
(iii) $f(x) = \sqrt{x-5}$ $y = \sqrt{x-5}$ $y^2 = x - 5$ $x = y^2 + 5$ $f^{-1}(y) = y^2 + 5$ $f^{-1}(x) = x^2 + 5$	(iv) $f(x) = \frac{3-x}{2}$ $y = \frac{3-x}{2}$ $2y = 3 - x$ $x = 3 - 2y$ $f^{-1}(y) = 3 - 2y$ $f^{-1}(x) = 3 - 2x$

5. Plot graph for the following absolute valued functions:

(i) $f(x) = 7|x|$ (ii) $f(x) = |x+6| - 2$

Solution

(i)


(ii)


6. Solve and express the solution on number line:

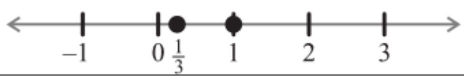
(i) $|3x-2|=1$ (ii) $|6x+1|=9$

Solution

(i) $|3x-2|=1$.

$$3x-2=1 \quad \text{or} \quad 3x-2=-1$$
$$x=1 \quad \text{or} \quad x=\frac{1}{3}.$$

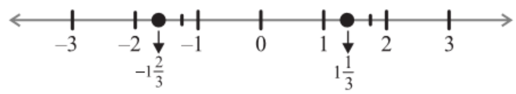
$\left\{\frac{1}{3}, 1\right\}$



(ii) $|6x+1|=9$.

$$6x+1=9 \quad \text{or} \quad 6x+1=-9$$
$$x=\frac{4}{3} \quad \text{or} \quad x=-\frac{5}{3}.$$

$\left\{-1\frac{2}{3}, 1\frac{1}{3}\right\}$



7. Solve and express the solution on number line:

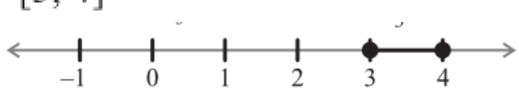
(i) $|7-2x|\le 1$ (ii) $|6x+18|\le 24$

Solution

(i) $|7-2x|\le 1$.

$$-1\le 7-2x\le 1$$
$$-8\le -2x\le -6$$
$$3\le x\le 4.$$

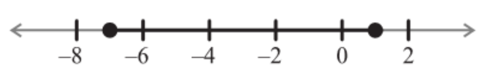
[3, 4]



(ii) $|6x+18|\le 24$.

$$-24\le 6x+18\le 24$$
$$-42\le 6x\le 6$$
$$-7\le x\le 1.$$

[–7, 1]



8. Given that $f(x)=3x+7$, find

(i) $f^{-1}(x)$ (ii) the value of x for which $f(x)=f^{-1}(x)$

Solution

(i) $f(x)=3x+7$

$y=3x+7 \Rightarrow 3x=y-7$

$x=\frac{y-7}{3}$

$f^{-1}(y)=\frac{y-7}{3}$

$f^{-1}(x)=\frac{x-7}{3}$

(i) $f(x)=f^{-1}(x)$

$3x+7=\frac{x-7}{3}$

$9x+21=x-7$

$9x-x=-7-21$

$8x=-28$

$x=-\frac{7}{2}$

9. A company earns a profit of Rs. 100 for each item sold after a fixed monthly expense of Rs. 10,000. A function $P(x) = 100x - 10{,}000$ represents the profit after selling x items. How many items must be sold to break even (profit = 0)?

Solution

$P(x) = 100x - 10\,000$.

$$P(x) = 0$$
$$100x - 10\,000 = 0$$
$$x = \frac{10\,000}{100} = 100.$$

100 items must be sold to break even.

10. A store offers a 15% discount on a product. A function $D(p) = 0.85p$ gives the selling price after the discount on an original price p. What is the selling price of an item that originally costs Rs. 2,000?

Solution

$D(p) = 0.85p$.

For an original price $p = 2\,000$,

$$D(p) = 0.85 \times 2\,000 = 1\,700.$$

The selling price after discount is **Rs. 1,700**.

11. A GPS system is considered accurate if the actual position differs from the reported position by no more than 6 metres. If the actual location is at point $x = 100$, the allowed range is defined as: $|x - r| \le 6$, where r is the reported location. What is the range of acceptable reported locations?

Solution

GPS accuracy range $|r - A| \le 6$ with

actual location $A = 100$.

$$|r - 100| \le 6$$
$$-6 \le r - 100 \le 6$$
$$94 \le r \le 106.$$

The acceptable reported locations are in the interval **[94, 106]**. 📍📊