# UNIT 1 Complex Numbers

## EXERCISE 1.1

1. Simplify the following:

(i) $i^5$ (ii) $i^{16}$ (iii) $(-i)^{-19}$ (iv) $27i^{-26}$

(v) $i^{11}+i^5$ (vi) $(i^4+i^3+i^2+i)^2$ (vii) $\left(\frac{i^8}{i^5}\right)^{-5}$ (viii) $i^{13}\times i^{29}$

**Solution**

i. $\boldsymbol{i^5} = i.i^4 = i.(i^2)^2 = i.(-1)^2 = i.(1) = i$

ii. $\boldsymbol{i^{16}} = (i^2)^8 = (-1)^8 = 1$

iii. $(-\boldsymbol{i})^{-19} = -i^{-19} = -\frac{1}{i^{19}} = -\frac{1}{i.i^{18}} = -\frac{1}{i.(i^2)^9} = -\frac{1}{i.(-1)^9} = -\frac{1}{i.(-1)} = \frac{1}{i} = \frac{1}{i}\times\frac{i}{i} = \frac{i}{i^2} = -i$

iv. $\boldsymbol{27i^{-26}} = \frac{27}{i^{26}} = \frac{27}{(i^2)^{13}} = \frac{27}{(-1)^{13}} = \frac{27}{(-1)} = -27$

v. $\boldsymbol{i^{11}+i^5} = i.i^{10} + i.i^4 = i.(i^2)^5 + i.(i^2)^2 = i.(-1)^5 + i.(-1)^2$
$= i.(-1) + i.(1) = -i + i = 0$

vi. $(\boldsymbol{i^4+i^3+i^2+i})^2 = [(i^2)^2 + i.i^2 + i^2 + i]^2 = [(-1)^2 + i.(-1) + (-1) + i]^2$
$= [1 - i - 1 + i]^2 = (0)^2 = 0$

vii. $\left(\frac{i^8}{i^5}\right)^{-5} = (i^3)^{-5} = (i.i^2)^{-5} = (-i)^{-5} = -i^{-5} = -\frac{1}{i^5} = -\frac{1}{i.i^4} = -\frac{1}{i.(i^2)^2}$
$= -\frac{1}{i.(-1)^2} = -\frac{1}{i.(1)} = -\frac{1}{i} = -\frac{1}{i}\times\frac{i}{i} = -\frac{i}{i^2} = i$

viii. $\boldsymbol{i^{13}\times i^{29}} = i^{42} = (i^2)^{21} = (-1)^{21} = -1$

2. Write in terms of $i$.

(i) $2+\sqrt{-4}$ (ii) $3-\sqrt{-7}$ (iii) $\frac{2}{5}+\frac{\sqrt{-16}}{5}$ (iv) $\sqrt{2}-\sqrt{-3}$

**Solution**

i. $\mathbf{2+\sqrt{-4}} = 2+\sqrt{4\times -1} = 2+\sqrt{4}\times\sqrt{-1} = 2+2i$

ii. $\mathbf{3-\sqrt{-7}} = 3-\sqrt{-1\times 7} = 3-\sqrt{-1}\times\sqrt{7} = 3-i\sqrt{7}$

iii. $\mathbf{\frac{2}{5}+\frac{\sqrt{-16}}{5}} = \frac{2}{5}+\frac{\sqrt{16\times -1}}{5} = \frac{2}{5}+\frac{\sqrt{16}\times\sqrt{-1}}{5} = \frac{2}{5}+\frac{4i}{5}$

iv. $\mathbf{\sqrt{2}-\sqrt{-3}} = \sqrt{2}-\sqrt{-1\times 3} = \sqrt{2}-\sqrt{-1}\times\sqrt{3} = \sqrt{2}-i\sqrt{3}$

3. Find the values of $x$ and $y$.

(i) $(2x+5)+(y-3)i = 1+2i$ (ii) $(3x+2)-(4-y)i = 5+3i$

(iii) $(2+i)x+(1-2i)y = 3+4i$ (iv) $(1-i)x+(2+i)y = 4-i$

(v) $(3x-1)+(2y-3)i = 8+7i$

**Solution**

| | |
|---|---|
| **i.** $\mathbf{(2x+5)+(y-3)i = 1+2i}$<br>$2x+5=1$ ; $y-3=2$<br>$2x=1-5$ ; $y=2+3$<br>$2x=-4$ ; $y=5$<br>$x=-2$ ; $y=5$ | **ii.** $\mathbf{(3x+2)-(4-y)i = 5+3i}$<br>$3x+2=5$ ; $-(4-y)=3$<br>$3x=5-2$ ; $-4+y=3$<br>$3x=3$ ; $y=3+4$<br>$x=1$ ; $y=7$ |
| **iii.** $\mathbf{(2+i)x+(1-2i)y = 3+4i}$<br>$2x+ix+y-2yi = 3+4i$<br>$2x+y=3$ ………. (i)<br>$x-2y=4$ ………. (ii)<br>$\mathbf{4x+2y=6}$<br>$x-2y=4$<br>$5x=10$<br>$x=2$<br>Put x = 2 in (i)<br>$\Rightarrow 2(2)+y=3 \Rightarrow 4+y=3$<br>$\Rightarrow y=3-4 \Rightarrow y=-1$ | **iv.** $\mathbf{(1-i)x+(2+i)y = 4-i}$<br>$x-ix+2y+iy = 4-i$<br>$x+2y=4$ ………. (i)<br>$-x+y=-1$ ………. (ii)<br>$x+2y=4$<br>$-x+y=-1$<br>$3y=3$<br>$y=1$<br>Put y = 1 in (i)<br>$\Rightarrow x+2(1)=4 \Rightarrow x+2=4$<br>$\Rightarrow x=4-2 \Rightarrow x=2$ |
| **v.** $\mathbf{(3x-1)+(2y-3)i = 8+7i}$<br>$3x-1=8$ ; $2y-3=7$<br>$3x=8+1$ ; $2y=7+3$<br>$3x=9$ ; $2y=10$<br>$x=3$ ; $y=5$ | |

## EXERCISE 1.2

1. Simplify and write in the form $a + bi$:

(i) $(2+5i)+(3-zi)$ (ii) $(16-3i)+(9+2i)$
(iii) $(9-2i)-(7-3i)$ (iv) $(11+9i)-(9-7i)$
(v) $(3+4i)(2-3i)$ (vi) $(5-2i)(3-4i)$
(vii) $(3-5i)\div(2-4i)$ (viii) $(5+2i)\div(6-3i)$

**Solution**

i. $(\mathbf{2}+\mathbf{5}\boldsymbol{i})+(\mathbf{3}-\boldsymbol{zi}) = 2+5i+3-zi = 5+(5-z)i$

ii. $(\mathbf{16}-\mathbf{3}\boldsymbol{i})+(\mathbf{9}+\mathbf{2}\boldsymbol{i}) = 16-3i+9+2i = 25-i$

iii. $(\mathbf{9}-\mathbf{2}\boldsymbol{i})-(\mathbf{7}-\mathbf{3}\boldsymbol{i}) = 9-2i-7+3i = 2+i$

iv. $(\mathbf{11}+\mathbf{9}\boldsymbol{i})-(\mathbf{9}-\mathbf{7}\boldsymbol{i}) = 11+9i-9+7i = 2+16i$

v. $(\mathbf{3}+\mathbf{4}\boldsymbol{i})(\mathbf{2}-\mathbf{3}\boldsymbol{i}) = 6-9i+8i-12i^2 = 6-9i+8i+12 = 18-i$

vi. $(\mathbf{5}-\mathbf{2}\boldsymbol{i})(\mathbf{3}-\mathbf{4}\boldsymbol{i}) = 15-20i-6i+8i^2 = 15-20i-6i-8 = 7-26i$

vii. $(\mathbf{3}-\mathbf{5}\boldsymbol{i})\div(\mathbf{2}-\mathbf{4}\boldsymbol{i}) = \frac{3-5i}{2-4i} = \frac{3-5i}{2-4i}\times\frac{2+4i}{2+4i} = \frac{6+12i-10i-20i^2}{(2)^2-(4i)^2}$

$= \frac{6+12i-10i+20}{4+16} = \frac{26+2i}{20} = \frac{13}{10}+\frac{1}{10}i$

viii. $(\mathbf{5}+\mathbf{2}\boldsymbol{i})\div(\mathbf{6}-\mathbf{3}\boldsymbol{i}) = \frac{5+2i}{6-3i} = \frac{5+2i}{6-3i}\times\frac{6+3i}{6+3i} = \frac{30+15i+12i+6i^2}{(6)^2-(3i)^2}$

$= \frac{30+15i+12i-6}{36+9} = \frac{24+27i}{45} = \frac{8}{15}+\frac{3}{5}i$

2. Write additive inverse for each complex number:

(i) $3+2i$ (ii) $4-3i$ (iii) $5-7i$ (iv) $-\frac{2}{3}+\frac{5}{4}i$

**Solution**

i. $\boldsymbol{z} = \mathbf{3}+\mathbf{2}\boldsymbol{i} \Rightarrow -z = -3-2i$

ii. $\boldsymbol{z} = \mathbf{4}-\mathbf{3}\boldsymbol{i} \Rightarrow -z = -4+3i$

iii. $\boldsymbol{z} = \mathbf{5}-\mathbf{7}\boldsymbol{i} \Rightarrow -z = -5+7i$

iv. $\boldsymbol{z} = -\frac{\mathbf{2}}{\mathbf{3}}+\frac{\mathbf{5}}{\mathbf{4}}\boldsymbol{i} \Rightarrow -z = \frac{2}{3}-\frac{5}{4}i$

3. Find multiplicative inverse for each complex number:

(i) $4+5i$ (ii) $6+2i$ (iii) $7-3i$ (iv) $\sqrt{5}-4i$

**Solution**

i. $z^{-1}=\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=\left(\frac{4}{(4)^2+(5)^2},\frac{-5}{(4)^2+(5)^2}\right)=\left(\frac{4}{16+25},\frac{-5}{16+25}\right)=\left(\frac{4}{41},-\frac{5}{41}\right)=\frac{4}{41}-\frac{5}{41}i$

ii. $z^{-1}=\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=\left(\frac{6}{(6)^2+(2)^2},\frac{-2}{(6)^2+(2)^2}\right)=\left(\frac{6}{36+4},\frac{-2}{36+4}\right)=\left(\frac{6}{40},\frac{-2}{40}\right)=\frac{3}{20}-\frac{1}{20}i$

iii. $z^{-1}=\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=\left(\frac{7}{(7)^2+(-3)^2},\frac{-(-3)}{(7)^2+(-3)^2}\right)=\left(\frac{7}{49+9},\frac{3}{49+9}\right)=\left(\frac{7}{58},\frac{3}{58}\right)=\frac{7}{58}+\frac{3}{58}i$

iv. $z^{-1}=\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=\left(\frac{\sqrt{5}}{\left(\sqrt{5}\right)^2+(-4)^2},\frac{-(-4)}{\left(\sqrt{5}\right)^2+(-4)^2}\right)=\left(\frac{\sqrt{5}}{5+16},\frac{4}{5+16}\right)=\left(\frac{\sqrt{5}}{21},\frac{4}{21}\right)=\frac{\sqrt{5}}{21}+\frac{4}{21}i$

4. If $z_1=2+5i$, $z_2=1-3i$ and $z_3=2+i$, then verify that

(i) $z_1+z_2=z_2+z_1$ (ii) $z_1z_2=z_2z_1$

(iii) $(z_1+z_2)+z_3=z_1+(z_2+z_3)$ (iv) $(z_1z_2)z_3=z_1(z_2z_3)$

(v) $z_1+(-z_1)=(-z_1)+z_1=0$

**Solution**

**i.** $L.H.S=z_1+z_2=(2+5i)+(1-3i)=2+5i+1-3i=3+2i$
$R.H.S=z_2+z_1=(1-3i)+(2+5i)=1-3i+2+5i=3+2i$
Hence $\boldsymbol{z_1+z_2=z_2+z_1}$

**ii.** $L.H.S=z_1z_2=(2+5i)(1-3i)=2-6i+5i-15i^2=2-6i+5i+15=17-i$
$R.H.S=z_2z_1=(1-3i)(2+5i)=2+5i-6i-15i^2=2+5i-6i+15=17-i$
Hence $\boldsymbol{z_1z_2=z_2z_1}$

**iii.** $L.H.S=(z_1+z_2)+z_3=[(2+5i)+(1-3i)]+(2+i)$
$=[2+5i+1-3i]+(2+i)=3+2i+2+i=5+3i$
$R.H.S=z_1+(z_2+z_3)=(2+5i)+[(1-3i)+(2+i)]$
$=(2+5i)+1-3i+2+i=2+5i+3-2i=5+3i$
Hence $\boldsymbol{(z_1+z_2)+z_3=z_1+(z_2+z_3)}$

**iv.** $L.H.S=(z_1z_2)z_3=[(2+5i)(1-3i)](2+i)=(2-6i+5i-15i^2)(2+i)$
$=(2-6i+5i+15)(2+i)=(17-i)(2+i)=34+17i-2i-i^2$
$=34+17i-2i+1=35+15i$
$R.H.S=z_1(z_2z_3)=(2+5i)[(1-3i)(2+i)]=(2+5i)[2+i-6i-3i^2]$
$=(2+5i)[2+i-6i+3]=(2+5i)(5-5i)=10-10i+25i-25i^2$
$=10-10i+25i+25=35+15i$
Hence $\boldsymbol{(z_1z_2)z_3=z_1(z_2z_3)}$

**v.** $L.H.S=z_1+(-z_1)=(2+5i)+(-2-5i)=2+5i-2-5i=0$
$R.H.S=(-z_1)+z_1=(-2-5i)+(2+5i)=-2-5i+2+5i=0$
Hence $\boldsymbol{z_1+(-z_1)=(-z_1)+z_1}$

5. If $\dfrac{(1+i)^2}{2-i} = x+iy$, then find the values of $x$ and $y$.

**Solution**

$x+iy = \frac{(1+i)^2}{2-i}$

$x+iy = \frac{1+i^2+2i}{2-i} = \frac{1-1+2i}{2-i} = \frac{2i}{2-i} = \frac{2i}{2-i} \times \frac{2+i}{2+i} = \frac{4i+2i^2}{(2)^2-i^2} = \frac{4i-2}{4+1} = \frac{4i-2}{5} = \frac{4}{5}i - \frac{2}{5} = -\frac{2}{5} + \frac{4}{5}i$

$x = -\frac{2}{5}$ ; $y = \frac{4}{5}$ comparing real and imaginary parts

6. If $(2x+iy)(1-i) = 4+2i$, then find the values of $x$ and $y$.

**Solution**

$(2x+iy)(1-i) = 4+2i$

$2x - 2xi + iy - i^2y = 4+2i$

$2x - 2xi + iy + y = 4+2i$

$2x + y = 4$ ………. (i) and $-2x + y = 2$ ………. (ii)

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| $2x+y=4$<br>$-2x+y=2$ |
| $2y = 6$<br>$y = 3$ |

Put y = 3 in (i)

$\Rightarrow 2x + (3) = 4 \Rightarrow 2x = 4-3 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

Hence $x = \frac{1}{2}$ ; $y = 3$

7. Find the values of $a$ and $b$, if $(a+bi)(1+3i) = -8+11i$.

**Solution**

$(a+bi)(1+3i) = -8+11i$

$a + 3ai + bi + 3bi^2 = -8+11i$

$a + 3ai + bi - 3b = -8+11i$

$a - 3b = -8$ ………. (i) and $3a + b = 11$ ………. (ii)

| |
|---|
| $\mathbf{3}a - \mathbf{9}b = \mathbf{-24}$<br>$\pm 3a \pm b = \pm 11$ |
| $-10b = -35$<br>$b = \dfrac{7}{2}$ |

Put b = $\frac{7}{2}$ in (i)

$\Rightarrow a - 3\left(\frac{7}{2}\right) = -8 \Rightarrow a - \frac{21}{2} = -8 \Rightarrow a = -8 + \frac{21}{2} \Rightarrow a = \frac{-16+21}{2} \Rightarrow a = \frac{5}{2}$

Hence $a = \frac{5}{2}$ ; $b = \frac{7}{2}$

# EXERCISE 1.3

1. Find the modulus of the following complex numbers:

(i) $4+3i$ (ii) $-5-4i$ (iii) $\frac{3}{5}-\frac{4}{5}i$ (iv) $-\sqrt{2}-\sqrt{3}i$

**Solution**

**i.** $|4+3i| = \sqrt{(4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

**ii.** $|-5-4i| = \sqrt{(-5)^2+(-4)^2} = \sqrt{25+16} = \sqrt{41}$

**iii.** $\left|\frac{3}{5}-\frac{4}{5}i\right| = \sqrt{\left(\frac{3}{5}\right)^2+\left(-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{9+16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$

**iv.** $|-\sqrt{2}-\sqrt{3}i| = \sqrt{\left(-\sqrt{2}\right)^2+\left(-\sqrt{3}\right)^2} = \sqrt{2+3} = \sqrt{5}$

2. If $z_1 = 2+7i$ and $z_2 = 4-3i$, then verify that

(i) $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$ (ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ (iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

**Solution**

**i.** $L.H.S = \overline{z_1+z_2} = \overline{(2+7\imath)+(4-3\imath)} = \overline{2+7\imath+4-3\imath} = \overline{6+4\imath} = 6-4i$

$R.H.S = \overline{z_1}+\overline{z_2} = \overline{(2+7\imath)}+\overline{(4-3\imath)} = 2-7i+4+3i = 6-4i$

Hence $\overline{\boldsymbol{z_1}+\boldsymbol{z_2}} = \overline{\boldsymbol{z_1}}+\overline{\boldsymbol{z_2}}$

**ii.** $L.H.S = \overline{z_1 z_2} = \overline{(2+7\imath)(4-3\imath)} = \overline{8-6\imath+28\imath-21\imath^2}$

$= \overline{8-6\imath+28\imath+21} = \overline{29+22\imath} = 29-22i$

$R.H.S = \overline{z_1}\,\overline{z_2} = \overline{(2+7\imath)}.\overline{(4-3\imath)} = (2-7i)(4+3i)$

$= 8+6i-28i-21i^2 = 8+6i-28i+21 = 29-22i$

Hence $\overline{\boldsymbol{z_1}\boldsymbol{z_2}} = \overline{\boldsymbol{z_1}}\,\overline{\boldsymbol{z_2}}$

**iii.** $L.H.S = \overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{2+7\imath}{4-3\imath}\right)} = \overline{\left(\frac{2+7\imath}{4-3\imath}\times\frac{4+3\imath}{4+3\imath}\right)} = \overline{\left(\frac{8+6\imath+28\imath+21\imath^2}{(4)^2-(3\imath)^2}\right)} = \overline{\left(\frac{8+6\imath+28\imath-21}{16+9}\right)}$

$= \overline{\left(\frac{-13+34\imath}{25}\right)} = \overline{-\frac{13}{25}+\frac{34}{25}\imath} = -\frac{13}{25}-\frac{34}{25}i$

$R.H.S = \frac{\overline{z_1}}{\overline{z_2}} = \frac{\overline{(2+7\imath)}}{\overline{(4-3\imath)}} = \frac{2-7i}{4+3i} = \frac{2-7i}{4+3i}\times\frac{4-3i}{4-3i} = \frac{8-6i-28i+21i^2}{(4)^2-(3i)^2} = \frac{8-6i-28i-21}{16+9}$

$= \frac{-13-34i}{25} = -\frac{13}{25}-\frac{34}{25}i$

Hence $\overline{\left(\frac{\boldsymbol{z_1}}{\boldsymbol{z_2}}\right)} = \frac{\overline{\boldsymbol{z_1}}}{\overline{\boldsymbol{z_2}}}$

3. If $z = 5 - 2i$, then verify that

(i) $\bar{\bar{z}} = z$ (ii) $|z| = |\bar{z}|$ (iii) $|z| = |-z|$

(iv) $z\bar{z} = |z|^2$ (v) $|z| = |-\bar{z}|$

**Solution**

i. $z = 5 - 2i \Rightarrow \bar{z} = 5 + 2i \Rightarrow \bar{\bar{z}} = 5 - 2i \Rightarrow \bar{\bar{\boldsymbol{z}}} = \boldsymbol{z}$

ii. $L.H.S = |z| = |5 - 2i| = \sqrt{(5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$R.H.S = |\bar{z}| = |5 + 2i| = \sqrt{(5)^2 + (2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$\Rightarrow |\boldsymbol{z}| = |\bar{\boldsymbol{z}}|$

iii. $L.H.S = |z| = |5 - 2i| = \sqrt{(5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$R.H.S = |-z| = |-(5 - 2i)| = |-5 + 2i| = \sqrt{(-5)^2 + (2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$\Rightarrow |\boldsymbol{z}| = |-\boldsymbol{z}|$

iv. $L.H.S = z\bar{z} = (5 - 2i)(5 + 2i) = (5)^2 - (2i)^2 = 25 + 4 = 29$

$R.H.S = |z|^2 = |5 - 2i|^2 = \left(\sqrt{(5)^2 + (-2)^2}\right)^2 = 25 + 4 = 29$

$\Rightarrow \boldsymbol{z}\bar{\boldsymbol{z}} = |\boldsymbol{z}|^2$

v. $L.H.S = |z| = |5 - 2i| = \sqrt{(5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$R.H.S = |-\bar{z}| = |-(5 + 2i)| = |-5 - 2i| = \sqrt{(-5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$\Rightarrow |\boldsymbol{z}| = |-\bar{\boldsymbol{z}}|$

4. If $z = 4 - 3i$, then verify that $|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}|$.

**Solution**

$|z| = |4 - 3i| = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$|-z| = |-(4 - 3i)| = |-4 + 3i| = \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$\because \bar{\bar{z}} = z \therefore |\bar{\bar{z}}| = |z| = |4 - 3i| = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$|-\bar{z}| = |-(\overline{4 - 3i})| = |-(4 + 3i)| = |-4 - 3i| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Clearly $|\boldsymbol{z}| = |-\boldsymbol{z}| = |\bar{\bar{\boldsymbol{z}}}| = |-\bar{\boldsymbol{z}}|$

5. If $z_1 = 2 + 3i$, $z_2 = -1 + i$, then evaluate:

(i) $Re(z_1 z_2)$ (ii) $Im(z_1 z_2)$

**Solution**

$z_1 z_2 = (2 + 3i)(-1 + i) = -2 + 2i - 3i + 3i^2 = -2 + 2i - 3i - 3$

$z_1 z_2 = -5 - i$

i. $\mathrm{Re}(z_1 z_2) = -5$

ii. $\mathrm{Im}(z_1 z_2) = -1$

# EXERCISE 1.4

1. Find the real and imaginary parts of the following complex numbers:

(i) $(8-3i)^2$ (ii) $(5+3i)^{-1}$ (iii) $(4-5i)^{-1}$

(iv) $(4-3i)^{-2}$ (v) $\left(\dfrac{3+2i}{4+3i}\right)^{-1}$ (vi) $\left(\dfrac{2-i}{2+i}\right)^{-2}$

(vii) $\left(\dfrac{1-2i}{1+i}\right)^{2}$

**Solution**

**i.** $\mathbf{(8-3i)^2} = (8)^2+(3i)^2-2(8)(3i) = 64+9i^2-48i = 64-9-48i$
$= 55-48i \Rightarrow \text{Re}(z)=55, \text{Im}(z)=-48$

**ii.** $\mathbf{(5+3i)^{-1}} = \frac{1}{5+3i} = \frac{1}{5+3i}\times\frac{5-3i}{5-3i} = \frac{5-3i}{(5)^2-(3i)^2} = \frac{5-3i}{25+9} = \frac{5-3i}{34} = \frac{5}{34}-\frac{3}{34}i \Rightarrow \text{Re}(z)=\frac{5}{34}, \text{Im}(z)=-\frac{3}{34}$

**iii.** $\mathbf{(4-5i)^{-1}} = \frac{1}{4-5i} = \frac{1}{4-5i}\times\frac{4+5i}{4+5i} = \frac{4+5i}{(4)^2-(5i)^2} = \frac{4+5i}{16+25} = \frac{4+5i}{41} = \frac{4}{41}+\frac{5}{41}i \Rightarrow \text{Re}(z)=\frac{4}{41}, \text{Im}(z)=\frac{5}{41}$

**iv.** $\mathbf{(4-3i)^{-2}} = \frac{1}{(4-3i)^2} = \frac{1}{16+9i^2-24i} = \frac{1}{16-9-24i} = \frac{1}{7-24i} = \frac{1}{7-24i}\times\frac{7+24i}{7+24i}$
$= \frac{7+24i}{(7)^2-(24i)^2} = \frac{7+24i}{49+576} = \frac{7+24i}{625} = \frac{7}{625}+\frac{24}{625}i \Rightarrow \text{Re}(z)=\frac{7}{625}, \text{Im}(z)=\frac{24}{625}$

**v.** $\left(\mathbf{\frac{3+2i}{4+3i}}\right)^{-1} = \frac{4+3i}{3+2i} = \frac{4+3i}{3+2i}\times\frac{3-2i}{3-2i} = \frac{12-8i+9i-6i^2}{(3)^2-(2i)^2} = \frac{12-8i+9i+6}{9+4} = \frac{18+i}{13}$
$= \frac{18}{13}+\frac{1}{13}i \Rightarrow \text{Re}(z)=\frac{18}{13}, \text{Im}(z)=\frac{1}{13}$

**vi.** $\left(\mathbf{\frac{2-i}{2+i}}\right)^{-2} = \left(\frac{2+i}{2-i}\right)^2 = \left(\frac{2+i}{2-i}\times\frac{2+i}{2+i}\right)^2 = \left(\frac{4+2i+2i+i^2}{(2)^2-(i)^2}\right)^2 = \left(\frac{4+2i+2i-1}{4+1}\right)^2 = \left(\frac{3+4i}{5}\right)^2$
$= \frac{(3+4i)^2}{25} = \frac{9+16i^2+24i}{25} = \frac{9-16+24i}{25} = -\frac{7}{25}+\frac{24}{25}i \Rightarrow \text{Re}(z)=-\frac{7}{25}, \text{Im}(z)=\frac{24}{25}$

**vii.** $\left(\mathbf{\frac{1-2i}{1+i}}\right)^{2} = \left(\frac{1-2i}{1+i}\times\frac{1-i}{1-i}\right)^2 = \left(\frac{1-i-2i+2i^2}{(1)^2-(i)^2}\right)^2 = \left(\frac{1-i-2i-2}{1+1}\right)^2 = \left(\frac{-1-3i}{2}\right)^2 = \frac{(-1-3i)^2}{4}$
$= \frac{(-1)^2+(-3i)^2+2(-1)(-3i)}{4} = \frac{1-9+6i}{4} = -\frac{8}{4}+\frac{6}{4}i \Rightarrow \text{Re}(z)=-2, \text{Im}(z)=\frac{3}{2}$

2. Solve the following simultaneous linear equations with complex coefficients for $w$ and $z$:

(i) $3z+(2+i)w=11-i$
$(2-i)z-w=-1+i$

(ii) $2z+(3+i)w=9-i$
$-iz-i\,w=-1+i$

(iii) $z-4w=3i$
$2z+3w=11-5i$

(iv) $z+w=3i$
$2z+3w=2$

(v) $2z+(3+i)w=1$
$-z-(1-i)w=2$

**Solution**

**i.** $\boldsymbol{3z+(2+i)w=11-i}$ ; $\boldsymbol{(2-i)z-w=-1+i}$

**Solution**

$3z+(2+i)w=11-i$ ……….(i)

$(2-i)z-w=-1+i$ ……….(ii)

Multiplying eq (i) by 1 and eq (ii) by $(2+i)$ and then adding them

$3z+(2+i)w=11-i$

$(2-i)(2+i)z-(2+i)w=(2+i)(-1+i)$

---

$3z+(2-i)(2+i)z=11-i+(2+i)(-1+i)$

$3z+(4-i^2)z=11-i-2+2i-i+i^2$

$3z+(4+1)z=11-i-2+2i-i-1$

$3z+5z=8\Rightarrow 8z=8\Rightarrow \boldsymbol{z}=1$

Put z = 1 in eq (i)

$3(1)+(2+i)w=11-i$

$3+(2+i)w=11-i$

$(2+i)w=8-i$

$w=\frac{8-i}{2+i}$

$w=\frac{8-i}{2+i}\times\frac{2-i}{2-i}=\frac{16-8i-2i+i^2}{(2)^2-(i)^2}=\frac{16-8i-2i-1}{4+1}=\frac{15-10i}{5}=\frac{15}{5}-\frac{10}{5}i$

$\boldsymbol{w=3-2i}$

**ii.** $\boldsymbol{2z+(3+i)w=9-i}$ ; $\boldsymbol{-iz-iw=-1+i}$

**Solution**

$2z+(3+i)w=9-i$ ……….(i)

$-iz-iw=-1+i$ ……….(ii)

Multiplying eq (i) by $i$ and eq (ii) by 2 and then adding them

$2iz+(3+i)iw=9i-i^2$

$-2iz-2iw=-2+2i$

---

$(3+i)iw-2iw=9i-i^2-2+2i$

$3iw+i^2w-2iw=9i+1-2+2i$

$iw-w=11i-1$

$(i-1)w = 11i - 1$

$w = \frac{11i-1}{i-1}$

$w = \frac{11i-1}{i-1} \times \frac{i+1}{i+1} = \frac{11i^2+11i-i-1}{(i)^2-(1)^2} = \frac{-11+11i-i-1}{-1-1} = \frac{-12+10i}{-2} = \frac{-12}{-2} + \frac{10}{-2}i$

$\boldsymbol{w = 6 - 5i}$

Put $w = 6 - 5i$ in eq (i)

$2z + (3+i)(6-5i) = 9 - i$

$2z + 18 - 15i + 6i - 5i^2 = 9 - i$

$2z + 18 - 15i + 6i + 5 = 9 - i$

$2z + 23 - 9i = 9 - i$

$2z = 9 - i - 23 + 9i$

$2z = -14 + 8i$

$z = -\frac{14}{2} + \frac{8}{2}i$

$\boldsymbol{z = -7 + 4i}$

**iii.** $\boldsymbol{z - 4w = 3i \quad ; \quad 2z + 3w = 11 - 5i}$

**Solution**

$z - 4w = 3i$ ……….(i)

$2z + 3w = 11 - 5i$ ……….(ii)

Multiplying eq (i) by 2 and subtracting both

$2z - 8w = 6i$

$\underline{\pm 2z \pm 3w = \pm 11 \mp 5i}$

$-11w = -11 + 11i$

$w = -\frac{11}{-11} + \frac{11}{-11}i$

$\boldsymbol{w = 1 - i}$

Put $w = 1 - i$ in eq (i)

$z - 4(1-i) = 3i$

$z - 4 + 4i = 3i$

$z = 3i + 4 - 4i$

$\boldsymbol{z = 4 - i}$

**iv.** $\boldsymbol{z + w = 3i \quad ; \quad 2z + 3w = 2}$

**Solution**

$z + w = 3i$ ……….(i)

$2z + 3w = 2$ ……….(ii)

Multiplying eq (i) by 2 and subtracting both

$2z + 2w = 6i$
$\underline{\pm 2z \pm 3w = \pm 2}$
$-w = -2 + 6i$
$\boldsymbol{w = 2 - 6i}$
Put $w = 2 - 6i$ in eq (i)
$z + (2 - 6i) = 3i$
$z = 3i - 2 + 6i$
$\boldsymbol{z = -2 + 9i}$

**v.** $\boldsymbol{2z + (3 + i)w = 1}$ **;** $\boldsymbol{-z - (1 - i)w = 2}$

**Solution**
$2z + (3 + i)w = 1$ ……….(i)
$-z - (1 - i)w = 2$ ……….(ii)
Multiplying eq (ii) by 2 and adding both
$2z + (3 + i)w = 1$
$\underline{-2z - 2(1 - i)w = 4}$
$(3 + i)w - 2(1 - i)w = 5$
$3w + iw - 2w + 2iw = 5$
$w + 3iw = 5$
$(1 + 3i)w = 5$
$w = \frac{5}{1+3i}$
$w = \frac{5}{1+3i} \times \frac{1-3i}{1-3i} = \frac{5-15i}{(1)^2-(3i)^2} = \frac{5-15i}{1+9} = \frac{5-15i}{10} = \frac{5}{10} - \frac{15}{10}i$
$\boldsymbol{w = \frac{1}{2} - \frac{3}{2}i}$
Put $w = \frac{1}{2} - \frac{3}{2}i$ in eq (i)
$2z + (3 + i)\left(\frac{1}{2} - \frac{3}{2}i\right) = 1$
$2z + \frac{3}{2} - \frac{9}{2}i + \frac{i}{2} - \frac{3}{2}i^2 = 1$
$2z + \frac{3}{2} - \frac{9}{2}i + \frac{i}{2} + \frac{3}{2} = 1$
$2z = 1 - \frac{3}{2} + \frac{9}{2}i - \frac{i}{2} - \frac{3}{2}$
$2z = \frac{2-3+9i-i-3}{2}$
$2z = \frac{-4+8i}{2}$
$z = \frac{-4+8i}{4} = -\frac{4}{4} + \frac{8}{4}i$
$\boldsymbol{z = -1 + 2i}$

# REVIEW EXERCISE 1

1. Four possible answers are given for the following questions. Choose the correct answer:

(i) $i^2 + i^4 =$

(a) $-1$ (b) 0 (c) 1 (d) 2

(ii) Real part of $(2-3i)(2+3i)$ is:

(a) $-3$ (b) 1 (c) 4 (d) 13

(iii) Imaginary part of $(2-i)(2+i)$ is:

(a) 0 (b) 1 (c) 7 (d) 9

(iv) $x + iy$ will be pure imaginary number, when:

(a) $y = 0$ (b) $x = 0$ (c) $i = 0$ (d) $x = 0, y = 0$

(v) What is additive inverse of $5 - 2i$?

(a) $5 + 2i$ (b) $-5 - 2i$ (c) $5 - 2i$ (d) $-5 + 2i$

(vi) What is multiplicative inverse of $z = 1 + i$?

(a) $1 - i$ (b) $i$ (c) $\frac{1}{2} - \frac{1}{2}i$ (d) $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

(vii) If $z = 4 - 3i$, then $z\bar{z} =$

(a) 3 (b) 9 (c) 16 (d) 25

(viii) Conjugate of $9 - 4i$ is:

(a) $-9 - 4i$ (b) $9 + 4i$ (c) $9 + 9i$ (d) $4 - 9i$

(ix) If $z = 4 + 4i$, then $z + \bar{z} =$

(a) 8 (b) $8 + 8i$ (c) $8i$ (d) 0

(x) If $z = 5 + 4i$, then $|z| =$

(a) 9 (b) 25 (c) 41 (d) $\sqrt{41}$

2. (i) Is "0" a complex number? Explain.
(ii) What is the result of multiplying a complex number by its conjugate?
(iii) State the condition for two complex numbers to be equal.

**Solution**

**i.** The real numbers are a subset of the complex numbers, so **zero is by definition a complex number**. i.e. $0 \in R \subseteq C$

**ii.** When a complex number is multiplied by its conjugate - the result is **real number**. i.e. $z\bar{z} = |z|^2 \in R$

**iii.** Two complex numbers are equal **if their real parts are the same and their imaginary parts are the same**. i.e. $(a, b) = (c, d) \Leftrightarrow a = c, b = d$

3. Simplify:
(i) $i^{37}$ (ii) $i^{13} \times i^{11}$ (iii) $(-i)^{-9}$
(iv) $(3-4i)(5-6i)$ (v) $(3+4i) \div (5-7i)$

**Solution**

i. $\boldsymbol{i^{37}} = i.i^{36} = i.(i^2)^{18} = i.(-1)^{18} = i.(1) = i$

ii. $\boldsymbol{i^{13}} \times \boldsymbol{i^{11}} = i^{24} = (i^2)^{12} = (-1)^{12} = 1$

iii. $(-\boldsymbol{i})^{-9} = -i^{-9} = -\frac{1}{i^9} = -\frac{1}{i.i^8} = -\frac{1}{i.(i^2)^4} = -\frac{1}{i.(-1)^4} = -\frac{1}{i.(1)} = -\frac{1}{i} = -\frac{1}{i} \times \frac{i}{i} = -\frac{i}{i^2} = i$

iv. $(\boldsymbol{3} - \boldsymbol{4i})(\boldsymbol{5} - \boldsymbol{6i}) = 15 - 18i - 20i + 24i^2 = 15 - 18i - 20i - 24 = -9 - 38i$

v. $(\boldsymbol{3} + \boldsymbol{4i}) \div (\boldsymbol{5} - \boldsymbol{7i}) = \frac{3+4i}{5-7i} = \frac{3+4i}{5-7i} \times \frac{5+7i}{5+7i} = \frac{15+21i+20i+28i^2}{(5)^2-(7i)^2}$

$= \frac{15+21i+20i-28}{25+49} = \frac{-13+41i}{74} = -\frac{13}{74} + \frac{41}{74}i$

4. Find additive and multiplicative inverse of $z = 8 + 9i$.

**Solution**

Let $z = 8 + 9i$

Additive inverse of z $= -z = -8 - 9i$

Multiplicative inverse of z $= z^{-1} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right) = \left(\frac{8}{(8)^2+(9)^2}, \frac{-9}{(8)^2+(9)^2}\right)$

$= \left(\frac{8}{64+81}, \frac{-9}{64+81}\right) = \left(\frac{8}{145}, -\frac{9}{145}\right) = \frac{8}{145} - \frac{9}{145}i$

5. If $z_1 = 3+4i$ and $z_2 = 2+3i$, then verify that

(i) $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$ (ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ (iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

(iv) $|z_1| = |-\overline{z_1}|$ (v) $\overline{\overline{z_2}} = z_2$ (vi) $z_1\overline{z_1} = |z_1|^2$

**Solution**

**i.** $L.H.S = \overline{z_1+z_2} = \overline{(3+4\imath)+(2+3\imath)} = \overline{3+4\imath+2+3\imath} = \overline{5+7\imath} = 5-7i$

$R.H.S = \overline{z_1}+\overline{z_2} = \overline{(3+4\imath)}+\overline{(2+3\imath)} = 3-4i+2-3i = 5-7i$

Hence $\overline{\boldsymbol{z_1}+\boldsymbol{z_2}} = \overline{\boldsymbol{z_1}}+\overline{\boldsymbol{z_2}}$

**ii.** $L.H.S = \overline{z_1 z_2} = \overline{(3+4\imath)(2+3\imath)} = \overline{6+9\imath+8\imath+12\imath^2}$

$= \overline{6+9\imath+8\imath-12} = \overline{-6+17\imath} = -6-17i$

$R.H.S = \overline{z_1}\overline{z_2} = \overline{(3+4\imath)}.\overline{(2+3\imath)} = (3-4i)(2-3i)$

$= 6-9i-8i+12i^2 = 6-9i-8i-12 = -6-17i$

Hence $\overline{\boldsymbol{z_1}\boldsymbol{z_2}} = \overline{\boldsymbol{z_1}}\,\overline{\boldsymbol{z_2}}$

**iii.** $L.H.S = \overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{3+4\imath}{2+3\imath}\right)} = \overline{\left(\frac{3+4\imath}{2+3\imath}\times\frac{2-3\imath}{2-3\imath}\right)} = \overline{\left(\frac{6-9\imath+8\imath-12\imath^2}{(2)^2-(3\imath)^2}\right)} = \overline{\left(\frac{6-9\imath+8\imath+12}{4+9}\right)}$

$= \overline{\left(\frac{18-\imath}{13}\right)} = \overline{\frac{18}{13}-\frac{1}{13}\imath} = \frac{18}{13}+\frac{1}{13}i$

$R.H.S = \frac{\overline{z_1}}{\overline{z_2}} = \frac{\overline{(3+4\imath)}}{\overline{(2+3\imath)}} = \frac{3-4i}{2-3i} = \frac{3-4i}{2-3i}\times\frac{2+3i}{2+3i} = \frac{6+9i-8i-12i^2}{(2)^2-(3i)^2} = \frac{6+9i-8i+12}{4+9}$

$= \frac{18+i}{13} = \frac{18}{13}+\frac{1}{13}i$

Hence $\overline{\left(\frac{\boldsymbol{z_1}}{\boldsymbol{z_2}}\right)} = \frac{\overline{\boldsymbol{z_1}}}{\overline{\boldsymbol{z_2}}}$

**iv.** $L.H.S = |z_1| = |3+4i| = \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$R.H.S = |-\overline{z_1}| = \left|-\overline{(3+4\imath)}\right| = |-(3-4i)| = |-3+4i| = \sqrt{(-3)^2+(4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$\Rightarrow |\boldsymbol{z_1}| = |-\overline{\boldsymbol{z_1}}|$

**v.** $z_2 = 2+3i \Rightarrow \overline{z_2} = 2-3i \Rightarrow \overline{\overline{z_2}} = 2+3i \Rightarrow \overline{\overline{\boldsymbol{z_2}}} = \boldsymbol{z_2}$

**vi.** $L.H.S = z_1\overline{z_1} = (3+4i)(3-4i) = (3)^2-(4i)^2 = 9+16 = 25$

$R.H.S = |z_1|^2 = |3+4i|^2 = \left(\sqrt{(3)^2+(4)^2}\right)^2 = 9+16 = 25$

$\Rightarrow \boldsymbol{z_1}\overline{\boldsymbol{z_1}} = |\boldsymbol{z_1}|^2$

6. If $z_1 = 5 + 4i$, $z_2 = 3 + 2i$, then find

(i) $z_1 z_2$ (ii) $\frac{z_1}{z_2}$ (iii) $\bar{z}_1 \bar{z}_2$ (iv) $|z_1 z_2|$

**Solution**

i. $\boldsymbol{z_1 z_2} = (5+4i)(3+2i) = 15 + 10i + 12i + 8i^2 = 15 + 10i + 12i - 8 = 7 + 22i$

ii. $\frac{z_1}{z_2} = \frac{5+4i}{3+2i} = \frac{5+4i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{15-10i+12i-8i^2}{(3)^2-(2i)^2} = \frac{15-10i+12i+8}{9+4} = \frac{23+2i}{13} = \frac{23}{13} + \frac{2}{13} i$

iii. $\bar{z}_1 \bar{z}_2 = \overline{(5+4i)}.\overline{(3+2i)} = (5-4i)(3-2i) = 15 - 10i - 12i + 8i^2$
$= 15 - 10i - 12i - 8 = 7 - 22i$

iv. $|\boldsymbol{z_1 z_2}| = ???$

**Solution**

$\boldsymbol{z_1 z_2} = (5+4i)(3+2i) = 15 + 10i + 12i + 8i^2 = 15 + 10i + 12i - 8 = 7 + 22i$

$|\boldsymbol{z_1 z_2}| = |7 + 22i| = \sqrt{(7)^2 + (22)^2} = \sqrt{49 + 484} = \sqrt{533}$

7. Find real and imaginary parts of $z = (2 + 7i)^{-1}$.

**Solution**

$(\boldsymbol{2 + 7i})^{-1} = \frac{1}{2+7i} = \frac{1}{2+7i} \times \frac{2-7i}{2-7i} = \frac{2-7i}{(2)^2-(7i)^2} = \frac{2-7i}{4+49} = \frac{2-7i}{53} = \frac{2}{53} - \frac{7}{53} i$

$\Rightarrow \text{Re(z)} = \frac{2}{53}, \text{Im(z)} = -\frac{7}{53}$

8. Solve the given simultaneous linear equations with complex coefficients for $z$ and $w$:

$$iz + (2-i)w = 4 + i$$
$$iz + (3+i)w = 3 + 3i$$

**Solution**

$iz + (2-i)w = 4 + i$ ……….(i)

$iz + (3+i)w = 3 + 3i$ ……….(ii)

Subtracting both

$iz + (2-i)w = 4 + i$

$\underline{\pm iz \pm (3+i)w = \pm 3 \pm 3i}$

$(2-i)w - (3+i)w = 1 - 2i$

$2w - iw - 3w - iw = 1 - 2i$

$-w - 2iw = 1 - 2i$

$(-1-2i)w = 1 - 2i$

$w = \frac{1-2i}{-1-2i}$

$w = \frac{1-2i}{-1-2i} \times \frac{-1+2i}{-1+2i} = \frac{-1+2i+2i-4i^2}{(-1)^2-(2i)^2} = \frac{-1+2i+2i+4}{1+4} = \frac{3+4i}{5}$

$\boldsymbol{w = \frac{3}{5} + \frac{4}{5} i}$

Put $w = \frac{3}{5} + \frac{4}{5}i$ in eq (i)

$iz + (2 - i)w = 4 + i$

$iz + (2 - i)\left(\frac{3}{5} + \frac{4}{5}i\right) = 4 + i$

$iz + \frac{6}{5} + \frac{8}{5}i - \frac{3}{5}i - \frac{4}{5}i^2 = 4 + i$

$iz + \frac{6}{5} + \frac{8}{5}i - \frac{3}{5}i + \frac{4}{5} = 4 + i$

$iz = 4 + i - \frac{6}{5} - \frac{8}{5}i + \frac{3}{5}i - \frac{4}{5}$

$iz = 4 - \frac{6}{5} - \frac{4}{5} + i - \frac{8}{5}i + \frac{3}{5}i$

$iz = \frac{20-6-4}{5} + \frac{5-8+3}{5}i$

$iz = \frac{10}{5} + \frac{0}{5}i$

$iz = 2 + 0i$

$iz = 2$

$z = \frac{2}{i} = \frac{2}{i} \times \frac{i}{i} = \frac{2i}{i^2}$

$\boldsymbol{z = -2i}$

9. Solve $(3 - 4i)(a + bi) = 1 + 0i$ and find the values of $a$ and $b$.

**Solution**

$(3 - 4i)(a + bi) = 1 + 0i$

$3a + 3bi - 4ai - 4bi^2 = 1 + 0i$

$3a + 3bi - 4ai + 4b = 1 + 0i$

$3a + 4b = 1$ ……….(i) and $-4a + 3b = 0$ ……….(ii)

Multiplying eq (i) by 4, eq (ii) by 3 and adding both

| |
|---|
| $\mathbf{12}a + \mathbf{16}b = \mathbf{4}$ <br> $-\mathbf{12}a + \mathbf{9}b = 0$ |
| $25b = 4$ <br> $b = \frac{4}{25}$ |

Put b $= \frac{4}{25}$ in (i)

$\Rightarrow 3a + 4\left(\frac{4}{25}\right) = 1 \Rightarrow 3a + \frac{16}{25} = 1 \Rightarrow 3a = 1 - \frac{16}{25} \Rightarrow 3a = \frac{9}{25} \Rightarrow a = \frac{3}{25}$

Hence $a = \frac{3}{25}$ ; $b = \frac{4}{25}$

10. Solve the equation for $x$ and $y$:

$$(3-2i)(x+yi)=2(x-2yi)+2i-1$$

**Solution**

$(3-2i)(x+yi)=2(x-2yi)+2i-1$

$3x+3yi-2xi-2yi^2=2x-4yi+2i-1$

$3x+3yi-2xi+2y=2x-4yi+2i-1$

$3x+3yi-2xi+2y-2x+4yi=2i-1$

$x+7yi-2xi+2y=2i-1$

$x+2y=-1$ ……….(i) and $-2x+7y=2$ ……….(ii)

Multiplying eq (i) by 2 and adding both

| |
|---|
| $\mathbf{2}x+\mathbf{4}y=-\mathbf{2}$<br>$-2x+7y=2$ |
| $11y=0$<br>$y=0$ |

Put y = 0 in (i)

$\Rightarrow x+2(0)=-1 \Rightarrow x=-1$

Hence $x=-1$ ; $y=0$