```
////////// Procedures to  Compute  SH-bases of subalgebra/////////

LIB "algebra.lib";

LIB"elim.lib";

////////////////////////////////////////////////

//// For a polynomial f, following procedure compute the maximal part of f.

 proc interm (poly g)

"USAGE: interm(f); f polynomial.

 RETURN: a polynomial h. "

{  int n=deg(g);

  poly f=g;

  poly h;

  while (deg(f)==n)

   {
     h=h+f[1];
     f=f-f[1];
   }

return(h);


}

//// SINGULAR Example

// ring r= 0,(x,y),dp;
// poly f= x3y+xy3+x2y+xy;
// interm(f);

// x3y+xy3

///////////////////////////////////////

////// For the given set of generators, following procedure compute the maximal
part of each generator.

proc intermI (ideal I)

"USAGE: intermI(I); I ideal.

 RETURN: an ideal j. "

{ ideal i=I;
  ideal j;
   int n,z;
   n=size(i);
  for (z=1;z<=n;z++)
   { j[z]=interm(i[z]);


    }
return(j);

}
```

```
//// SINGULAR Example

// ring r= 0,(x,y),dp;
// ideal I= x3y+xy3+x2y+xy,x+y+1,xy+x2-y;
// intermI(I);

//_[1]=x3y+xy3
//_[2]=x+y
//_[3]=x2+xy


/////////////////////////// Procedure to compute iterative d-reduction (Algorithm
1)/////////////////

//////////// For a polynomial f and a finite set of polynomials G, following
procedure
//////////// perform iterated d-reductions of f with respect to G.


proc shred(poly f,ideal I)

"USAGE: shred(f,I ); f polynomial, I ideal.

 RETURN: a polynomial h. "


{
  ideal G=I;
  poly h=f;
  poly h1,j;
  list L;
  map psi;
  while(h!=0 && h1!=h)
  {
     L= algebra_containment(interm(h),intermI(G),1);

     h1=h;  //change (to terminate)
     if (L[1]==1)
     {
        def s= L[2];

        psi= s,maxideal(1),G;
        j= psi(check);

        h=h-j;
     kill s;
     }
   }

   return (h);
}

//// SINGULAR Example

// ring r= 0,(x,y),dp;
// poly f= x3y+xy3+x2y+xy;
// ideal I=x2,y2,xy+y;
// shred(f,I);

// -y3+xy

/////////////////////////////////////

/// For a finite set of polynomials G, following procedure  Compute the <
P_1(G),....P_m(G)> such that P_1,...P_m
/// are the generators of the ideal of algberiac relations betweem the maximal
part of the polynomial in G.
```

```
proc shSpoly(ideal id)

"USAGE: shSpoly(I);  I ideal.

 RETURN: an ideal P ."

{
def bsr= basering ;

ideal vars = maxideal(1) ;

int n=nvars(bsr) ;

int m=ncols(id) ;

int z;

ideal p;

if(id==0)
{ return(p); }

else
{
execute("ring R1=("+charstr(bsr)+"),("+varstr(bsr)+", @y(1..m)),(Dp(n),Dp(m)) ;")
;

ideal id =imap(bsr,id) ;

ideal A ;

for (z=1;z<=m;z++)
{
  A[z]= @y(z)-interm(id[z]) ;
}

 A=std(A) ;

ideal kern=nselect(A,1..n);



setring bsr ;

map phi= R1,vars,id;


p=simplify(phi(kern),2) ;

return (p);
}

}

//// SINGULAR Example


// ring r= 0,(x,y),dp;
// ideal I=x2,y2,xy+y;
// shSpoly(I);

// _[1]=-2xy2-y2

/////////////////// SH-Basis Construction Algorithm (Algorithm
2)///////////////////////
```

```
/// For a given set of generators, following procedure compute the SH-basis of
the subalgebra generated by G.

proc sh(ideal id)

"USAGE: sh(I);  I ideal.

 RETURN: an ideal S."

{
 ideal S,oldS,Red ;
 list L ;
 int z;
 int n;
 S=id ;
 while( size(S)!=size(oldS))
 {

   L=shSpoly(S) ;
   n=size(L);
   for (z=1;z<=n;z++)
   {
     Red=L[1][z];

     Red=shred(Red[1],S);  //change2  (Red is not a poly)
     oldS=S ;
     S=S+Red ;

   }

 }

return(S);

}


//// SINGULAR Example


// ring r= 0,(x,y),dp;
// ideal I=x2,y2,xy+y;
// sh(I);

// _[1]=x2
// _[2]=y2
// _[3]=xy+y
// _[4]=-2xy2-y2

///////////////////////// Partial SH-bases Construction//////////////////


/// For a given set of generators and integer c, following procedure compute the
SH-basis of the subalgebra generated by G upto step c.


proc shpart(ideal id, int c)

"USAGE: sh(I,c);  I ideal, integer c.

 RETURN: an ideal S."

{
 ideal S,oldS,Red ;
 int counter;
 list L ;
 int z;
 int n;
```

```
 S=id ;
 while( size(S)!=size(oldS) && (counter<=c))
 {

   L=shSpoly(S) ;
   n=size(L);
   for (z=1;z<=n;z++)
   {
     Red=L[1][z];

     Red=shred(Red[1],S);  //change2  (Red is not a poly)
     oldS=S ;
     S=S+Red ;
     counter=counter+1;


   }

 }

return(S);

}


//// SINGULAR Example


// ring r= 0,(x,y,z),dp;
// ideal I=xz+y,xyz,xy2z;
// shpart(I,3);

//_[1]=xz+y
//_[2]=xyz
//_[3]=xy2z
//_[4]=xy3z
//_[5]=xy4z
//_[6]=xy5z
//_[7]=xy6z
```