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Federal Board HSSC-II Examination
Mathematics Model Question Paper

Roll No:

Answer Sheet No: ____________

Signature of Candidate: ____________

Signature of Invigilator: ____________

SECTION – A

Time allowed: 20 minutes Marks: 20

Note: Section-A is compulsory and comprises pages 1-6. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Insert the correct option i.e. A/B/C/D in the empty box provided opposite each part. Each part carries one mark.

i. If f(x) = x and g(x) = $x^2 + 1$ what is the composition of f and g?

A. x
B. $x^2 + 1$
C. $x(x^2 + 1)$
D. $\frac{x}{x^2+1}$

ii. What is the Value of $\underset{x \to 1}{\text{Limit}} \frac{\sqrt{x}-1}{x-1}$?

A. 0
B. $\frac{0}{0}$
C. 1/2
D. -1

DO NOT WRITE ANYTHING HERE

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iii. What is the domain of $2+\sqrt{x-1}$?

A. R

B. $(1, \infty)$

C. $[1, \infty)$

D. $(-\infty, 0]$

iv. What is the derivative of $\cos h^{-1}\left(\frac{x}{2}\right)$?

A. $\dfrac{4}{\sqrt{x^2/4 - 1}}$

B. $\dfrac{4}{\sqrt{1+\frac{x^2}{4}}}$

C. $\dfrac{-4}{\sqrt{\frac{x^2}{4}-1}}$

D. $\dfrac{-4}{1-x^2/4}$

v. What is the maximum value of $(\sqrt{3}Sin\,x + Cos\,x)$?

A. 30°

B. 60°

C. 90°

D. 45°

vi. What is the value of $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$?

A. e^x

B. Sin x

C. $(1+x)^n$

D. Cos x

vii. What is the value of $\int_{-1}^{1} (x^{1/3} + 1)dx$?

A. $\frac{1}{2}$

B. 2

C. $3/4$

D. $4/3$

viii. What is the value of $\int_{a}^{c} (2x+3)dx + \int_{c}^{b} (2x+3)dx$?

A. $\int_{-a}^{b} (2x+3)dx$

B. $\int_{b}^{a} (2x+3)dx$

C. $\int_{a}^{b} (2x+3)dx$

D. $\int_{a}^{b} (2x \pm 3)dx$

ix. What is the value of $\int Sec(Px+q)\tan(Px+q)dx$?

A. tan (Px + q) + C

B. –tan (Px + q) +C

C. –P Sec (Px + q) – C

D. $\frac{1}{P}$ Sec (Px + q) + C

x. Which of the following points is at a distance of 15 units from (0, 0)?

A. $(\sqrt{176}, 7)$

B. (10, -10)

C. (1, 15)

D. (7, 176)

xi. When would the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ be imaginary?

A. When $h^2 = ab$

B. When $h^2 > ab$

C. When $h^2 < ab$

D. When ab = b

xii. What would be the slope of any line perpendicular to 3x – 4y + K = 0

A. –1

B. $3/4$

C. $\frac{4}{3}$

D. $-4/3$

xiii. Which of the following lines passes through (–5, –6) & (3, 0)?

A. 2x – y + 4 = 0

B. 3x – 4y – 9 = 0

C. 2y – x + 7 = 0

D. 2x + y + 4 = 0

xiv. What is the number of order pairs that satisfy the expression $5x + 3y \geq 10$?

A. Finite many
B. Infinite many
C. Two
D. Three

xv. What is the length of the latus rectum of parabola $2x^2 = -32y$?

A. 16
B. – 16
C. – 4
D. 32

xvi. For what value of C would the line y = mx + C be tangent to ellip $\frac{x^2}{9} + \frac{y^2}{4} = 1$?

A. 0
B. $\frac{3}{x}$
C. $\sqrt{9m^2 + 4}$
D. $3\sqrt{1 + m^2}$

xvii. What is the radius of circle $x^2 + y^2 + 2x \cos\theta + 2y \sin\theta = 8$?

A. 1
B. 3
C. $2\sqrt{3}$
D. $\sqrt{10}$

xviii. What is the condition for ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to be a circle?

A. a > b
B. a = b
C. a < b
D. a = 0, b = 0

xix. What does the expression $\underline{a}\times(\underline{b}.\underline{c})$ represent?

A. Vector quantity
B. Scalar quantity
C. Area of parallelogram
D. Nothing

xx. Which of the following vectors is perpendicular to both vectors $\underline{a}\,\&\,\underline{b}$?

A. $\underline{a}$
B. $\underline{b}$
C. $\underline{a}\times\underline{b}$
D. $\underline{a}.\underline{b}$

For Examiner's use only

Q. No.1: Total Marks: 20

Marks Obtained:

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Federal Board HSSC-II Examination
Mathematics Model Question Paper

Time allowed: 2.40 hours Total Marks: 80

Note: Sections 'B' and 'C' comprise pages 1-3 and questions therein are to be answered on the separately provided answer book. Answer any ten questions from section 'B' and attempt any five questions from section 'C'. Use supplementary answer sheet i.e., sheet B if required. Write your answers neatly and legibly.

SECTION-B
(Marks: 40)

Note: Attempt any **TEN** questions.

Q.2 Evaluate $\underset{x \to 0}{Limit} \quad \dfrac{\sqrt{x+a}-\sqrt{a}}{x}$ (4)

Q.3 Discuss the continuity of function at y = 3 if

$$f(y)=\begin{cases} y-1 & , \quad y<3 \\ 2y+1 & \quad y\geq 3 \end{cases}$$ (4)

Q.4 If y = tan (2tan^{-1} $\dfrac{x}{2}$) then show that $\dfrac{dy}{dx}=\dfrac{4(1+y^2)}{4+x^2}$ (4)

Q.5 Differentiate $\log_a x$ by ab – initio method. (4)

Q.6 Find the coordinates of the turning point of the curve y = 8x + $\dfrac{1}{2x^2}$ and determine whether this point is the maximum or the minimum point. (4)

Q.7 Find the value of K if $\int_2^K 6(1-x)^2\,dx = 52$ (4)

Q.8 Find the area bounded by curve $y = 4 - x^2$ and the x –axis. (4)

Q.9 Using the differential find $\frac{dy}{dx}$ when $\frac{y}{x}$ - ln x = ln c. (4)

Q.10 Determine the value of K for which the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + Ky + 8 = 0$ would meet at a point. (4)

Q.11 Find the equation of the perpendicular bisector of the line segment joining the points (3, 5) and (9, 8). (4)

Q.12 Find the mid point of chord cut off the line $2x + 3y = 13$ by circle $x^2 + y^2 = 26$. (4)

Q.13 Find the equation of the parabola having its focus at the origin and directrix parallel to y – axis. (4)

Q.14 Find the equation of the hyberbola with given data: (4)
foci (±5, 0) vertices (±3, 0)

Q.15 Show that $\text{Cos}^2 \propto + \text{Cos}^2\beta + \text{Cos}^2\gamma = 1$. (4)

SECTION – C
(Marks: 40)

Note: Attempt any **FIVE** questions. Each question carries equal marks.
(Marks 5 × 8=40)

Q.16 Find the graphical solution of the functions x = Sin2x (8)

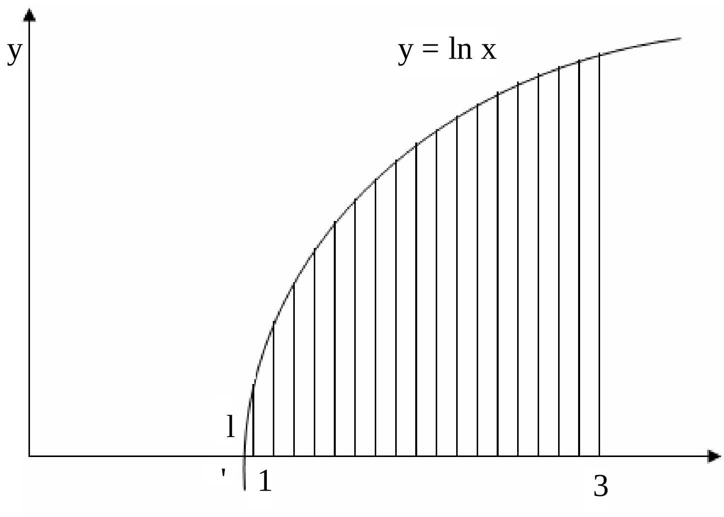
Q.17 If y = a Cos (ln x) + b Sin (ln x); prove that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$ (8)

Q.18 The line y = ax + b is parallel to line y = 2x – 6 and passes through point (-1, 7). Find the value of a & b and fined the Eq of line passing through (7, -1) and perpendicular to line $\frac{x}{3} - y = 7$. (8)

Q.19 If $\underline{a} + \underline{b} + \underline{c} = 0$ then prove that
$\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ (8)

Q.20 Show that $\frac{d}{dx}[x\ln x - x] = \ln x$. The diagram shows part of the curve y = ln x. Find the area of shaded region, correct to two decimal places. (8)



Q.21 Maximize Z = 2x + 3y subject to the constraints $3x + 4y \leq 12$, $2x + y \leq 4$, $2x - y \leq 4$, $x \geq 0$, $y \geq 0$ (8)

Q.22 Find the equation of a tangent to the parabola $y^2 = -6x$ which is parallel to the line 2x + y + 1 = 0. Also find the tangency. (8)

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Federal Board HSSC – II Examination
Mathematics – Mark Scheme

SECTION A

Q.1

i.	B	ii.	C	iii.	C
iv.	B	v.	B	vi.	A
vii.	C	viii.	C	ix.	D
x.	A	xi.	C	xii.	D
xiii.	B	xiv.	B	xv.	A
xvi.	C	xvii.	B	xviii.	B
xix.	D	xx.	C		

(20 × 1 = 20)

SECTION B

Q.2 **(4)**

$$\underset{x\to 0}{Limit}\ \frac{\sqrt{x+a}-\sqrt{a}}{x}\quad \left(\frac{0}{0}\right)$$ (1 mark)

$$\underset{x\to 0}{Limit}\ \frac{\sqrt{x+a}-\sqrt{a}}{x}\times\frac{\sqrt{x+a}+\sqrt{a}}{\sqrt{x+a}+\sqrt{a}}$$ (1 mark)

$$\underset{x\to 0}{Limit}\ \frac{x+\cancel{a}-\cancel{a}}{x\sqrt{x+a}+\sqrt{a}}$$

$$\underset{x\to 0}{Limit}\ \frac{\cancel{x}}{\cancel{x}(\sqrt{x+a}+a)}$$ (1 mark)

$$=\frac{1}{\sqrt{0+a}+\sqrt{a}}=\frac{1}{2\sqrt{a}}$$ (1 mark)

Q.3 **(4)**

L.H. Limit

$$\underset{y\to 3^-}{Limit} f(y)=\underset{y\to 3^-}{Limit} y-1$$

$= 3 – 1 = 2$ ⟶ **(I)** (1 mark)

R.H. Limit

$$\underset{y \to 3^+}{Limit}\ fy = \underset{y \to 3^+}{Limit}\ 2y + 1$$

$= 6 + 1 = 7 \longrightarrow$ **(II)** (1 mark)

Direct Limit

Put y = 3

f(y) = 2y + 1

f(3) = 2(3) + 1 = 7 $\longrightarrow$ **(III)** (1 mark)

From I, II & III L.H. Limit ≠ R.H. Limit = Direct Limit

So f(y) is not continuous at y = 3 (1 mark)

Q.4 **(4)**

$$y = \tan(2\tan^{-1}\frac{x}{2})$$

$$\tan^{-1} y = 2\tan^{-1} x/2$$ (1 mark)

Diff w.r.t. x

$$\frac{1}{1+y^2}\frac{dy}{dx} = 2.\frac{2}{1+\frac{x^2}{4}}.\frac{1}{2}$$ (2 marks)

$$\frac{dy}{dx} = \frac{4(1+y^2)}{4+x^2}$$ (1 mark)

Q.5 **(4)**

Let y = $\log_a x$

y + Δy = $\log_a(x + \Delta x)$

Δy = $\log_a(x + \Delta x) - \log_a x$ (1 mark)

$$\Delta y = \log_a\left(\frac{x+\Delta x}{x}\right) = \log\left(1+\frac{\Delta x}{x}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{x}\frac{x}{\Delta x}\log\left(1+\frac{\Delta x}{x}\right)$$ (1 mark)

$$\frac{\Delta y}{\Delta x} = \frac{1}{x}\log_a\left(1+\frac{\Delta x}{x}\right)^{x/\Delta x}$$ (1 mark)

$$\underset{\Delta x \to 0}{Limit}\frac{\Delta y}{\Delta x} = \frac{1}{x}\log_a\left[\underset{\Delta x \to 0}{Limit}\left(1+\frac{\Delta x}{x}\right)^{x/\Delta x}\right]$$

$$\frac{dy}{dx} = \frac{1}{x}\log_a e$$ (1 mark)

Q.6 **(4)**

$$y = 8x + \frac{1}{2x^2}$$

$$y = 8x + \frac{1}{2}x^{-2}$$

Difference with reference to x

$$\frac{dy}{dx} = 8 + \frac{1}{2}(-2x^{-3})$$

$$\frac{dy}{dx} = 8 - \frac{1}{x^3}, \quad Put\,\frac{dy}{dx} = 0$$ (1 mark)

$$8 - \frac{1}{x^3} = 0 \Rightarrow 8 = \frac{1}{x^3}$$

$$x^3 = \frac{1}{8} \Rightarrow x = \frac{1}{2}$$ (1 mark)

$$\frac{dy}{dx} = 8 - x^{-3}$$

Difference again

$$\frac{d^{2y}}{dx^2} = 3x^{-4} = \frac{3}{x^4}$$

$$\left.\frac{d^{2y}}{dx^2}\right| = \frac{3}{\left(\frac{1}{2}\right)^4} = 3\times 16 = 48 > 0$$ (1 mark)

$$x = \frac{1}{2}$$

So y will be minimum at $x = \frac{1}{2}$

Put $x = \frac{1}{2}$ = y

$$y = 8\left(\frac{1}{2}\right) + \frac{1}{2\left(\frac{1}{4}\right)} = 4 + 2 = 6$$ (1 mark)

Q.7 **(4)**

$$\int_2^K 6(1-x)^2\,dx = 52$$

$$\left|\frac{6(1-x)^3}{-3}\right|_2^K = 52$$ (1 mark)

$$-2(1-x^3)\Big|_2^K = 52$$

$$(1-x^3)\Big|_2^K = -26$$ (1 mark)

$(1–K)^3 – (1–2)^3 = –26$

$(1–K)^3 + 1 = –26$

$(1–K)^3 = –27$ (1 mark)

$1–K = –3 \Rightarrow K = +4$ (1 mark)

Q.8 **(4)**

$y = 4 - x^2$

Put $y = 0 \Rightarrow 4 - x^2 = 0$ $x = \pm 2$ (1 mark)

$A = \int_{-2}^{2} y\,dx$

$A = \int_{-2}^{2} (4 - x^2)dx = 4\int_{-2}^{2} dx - \int_{-2}^{2} x^2 dx$

$A = 4[x]_{-2}^{2} - \left[\frac{x^3}{3}\right]_{-2}^{2}$ (1 mark)

$A = 4[2+2] - \frac{1}{3}[8+8]$

$A = 16 - \frac{16}{3}$

$A = \frac{48-16}{3}$ (1 mark)

$A = \frac{32}{3}$ Square Units (1 mark)

Q.9 **(4)**

$\frac{y}{x} - \ln x = \ln c$

$y - x\ln x = x\ln c$ (1 mark)

Taking Differential

$dy - dx\ln x - x \frac{1}{x} dx = dx\ln c$ (1 mark)

$dy = dx\ln c + dx\ln x + dx$

$dy = dx[\ln c + \ln x + 1]$ **(I)** (1 mark)

Dividing by dx by **(I)**

$\frac{dy}{dx} = \ln c + \ln x + 1$ (1 mark)

Q.10 **(4)**

$2x - 3y - 1 = 0$

$3x - y - 5 = 0$

$3x + Ky + 8 = 0$

Since lines are concurrent so the det. of their coeff. will be zero (1 mark)

$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & K & 8 \end{vmatrix} = 0$ (1 mark)

Exp. by R_1

$2(-8+5K)+3(24+15)-1(3K+3) = 0$

$-16+10K+117-3K-3 = 0$

$7K+98 = 0$

K = -14 (2 marks)

Q.11 **(4)**

Line is perpendicular to AB & A(3, 5) B(9, 8)

mid point of AB = $(6, \frac{13}{2})$ (1 mark)

slope of AB = $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{8-5}{9-3}$

$m_1 = 3/6 = 1/2$ (1 mark)

Line perpendicular to AB has slope, $m_2 = -2$

Required line passes through $(6, \frac{13}{2})$ So its equation is:

$y - y_1 = m(x - x_1)$ (1 mark)

$y - \frac{13}{2} = -2(x-6)$

$2y - 13 = -4(x-6)$

$2y - 13 = -4x + 24$

$4x + 2y - 37 = 0$ (1 mark)

Q.12 **(4)**

$x^2+y^2 = 26$ **(I)**

$2x+3y = 13$ **(II)**

From **(II)** $x = \frac{13-3y}{2}$ putting in **(I)**

$\left(\frac{13-3y}{2}\right)^2 + y^2 = 26$

$169+9y^2-78y+y^2 = 104$

$13y^2-78y+65 = 0$ (1 mark)

$\div by 13, y^2 - 6y + 5 = 0$

$y^2-5y-y+5 = 0$

$y(y-5)-1(y-5) = 0$

$(y-1)(y-5) = 0$

$y = 5, \quad y = 1$ (1 mark)

when y = 5, then x = -1 So A(-1, 5)

when y = 1, then x = 5 So B(5, 1) (1 mark)

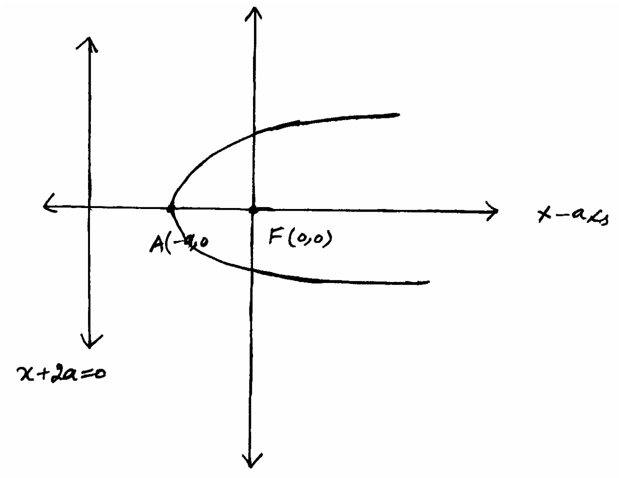
AB is chord its mid point is

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$=\left(\frac{-1+5}{2},\frac{5+1}{2}\right)$$

= (2, 3) (1 mark)

Q.13 **(4)**



(1 mark)

According to problem F(0, 0) and directrix is x+2a = 0

Let P(x, y) be any point on parabola so (1 mark)

$|PF| = |PM|$

$$\sqrt{(x-0)^2+(y-0)^2} = \frac{|x+2a|}{\sqrt{(1)^2+(0)^2}}$$ (1 mark)

Seq on both sides

$x^2+y^2 = (x+2a)^2$

$\cancel{x}^2 + y^2 = \cancel{x}^2 + 4ax + 4a^2$

$y^2 = 4a(x+a)$ (1 mark)

Q.14 **(4)**

F(5, 0) $\quad F'$(-5, 0)	A(3, 0) $\quad A'$(-3, 0)
$2c = \|FF\|$	$2a = \|AA\|$
$2c = \sqrt{(10)^2+(0)^2}$	$2a = 6$
$2c = 10$	$a = 3$
$c = 5$	

(1 mark)

We have

$b^2 = c^2 - a^2$

$b^2 = 25 - 9$

$b^2 = 16$

$b = 4$ (1 mark)

Hyberbola is along x – axis and its centre is at (0, 0) so its equation is (1 mark)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$ (1 mark)

Q.15 **(4)**

Let $\underline{r} = x\underline{i} + y\underline{i} + z\underline{k}$,

$|\underline{r}| = \sqrt{x^2 + y^2 + z^2}$ (1 mark)

Then $\frac{r}{|r|} = \left[\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right]$ is unit vector in the direction of $\underline{\gamma} = \overline{OP}$ (1 mark)

It is supposed that OAP is right angled length

So $Cos \propto = \frac{OA}{OP} \Rightarrow Cos \propto = \frac{x}{r}$

$Cos\beta = \frac{y}{r}, \quad Cos\gamma = \frac{z}{r}$ (1 mark)

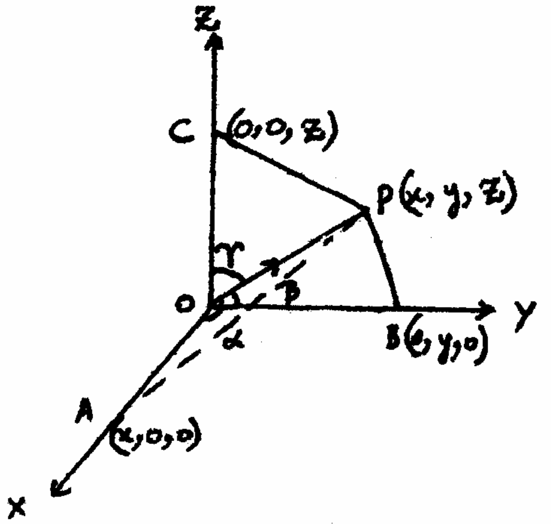
Since Cos$\propto$, Cosβ, Cosγ are called direction cosins of OP. So

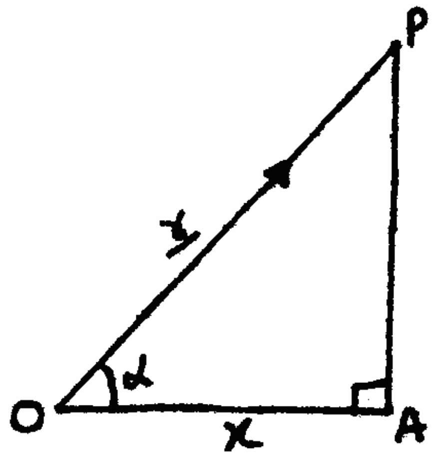
$Cos^2\propto + Cos^2\beta + Cos^2\gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$

$= \frac{x^2 + y^2 + z^2}{r^2}$

$= \frac{r^2}{r^2}$

$= 1$ (1 mark)





SECTION C

Q.16 **(8)**

x = Sin2x

Let y = x = Sin2x

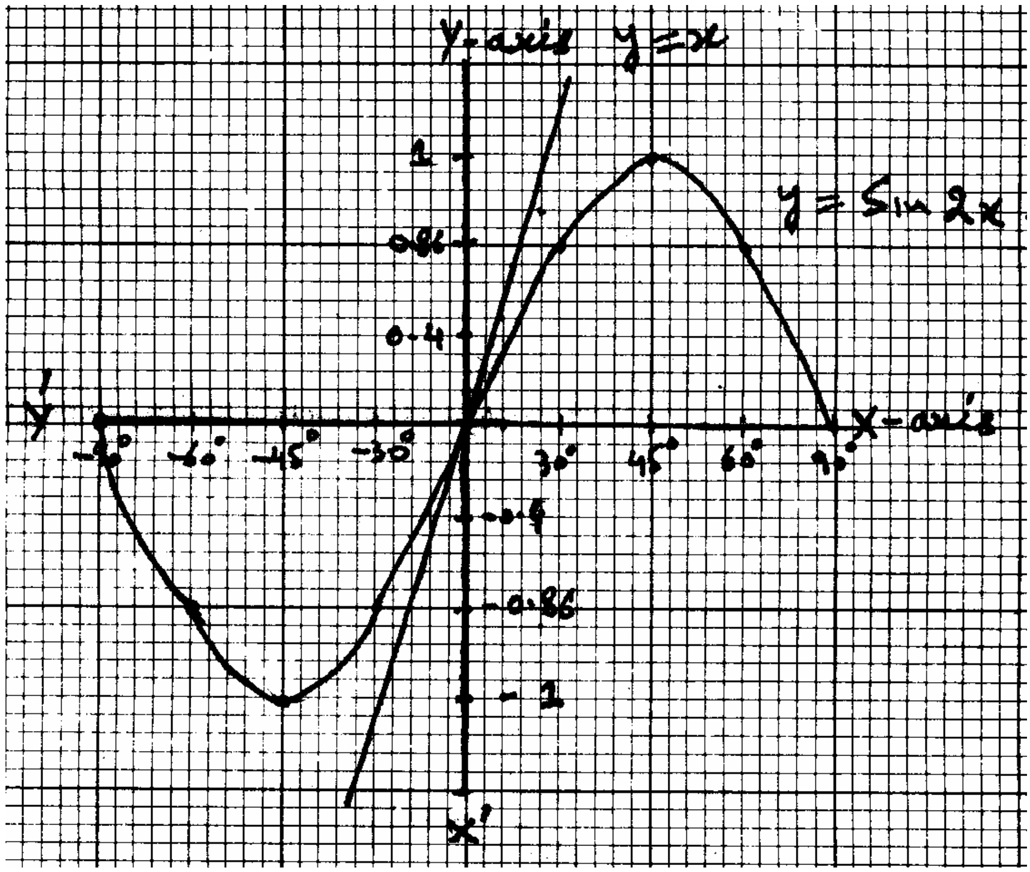
y = x, y = Sin2x

y = x is a straight line passing through origin and bisecting Ist & IIIrd quad:

(1 mark)

x	0°	15°	30°	45°	60°	75°	90°	-90°	-75°	-60°	-45°	-30°	-15°
y	0	0.5	0.86	1	0.86	0.5	0	0	-0.5	-0.8	-1	-0.86	-0.5

(4 marks)



(1 mark)

These two graphs intersect at origin

so x = 0, y = 0 is only solution set (1 mark)

Q.17 **(8)**

y = a Cos(ln x) + b Sin(ln x)

Diff w.r.t.

$$\frac{dy}{dx} = a(-Sin\ln x)\frac{1}{x} + bCos(\ln x)\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x}(b\,Cos\ln x - a\,Sin\ln x)$$ (3 marks)

Diff w.r.t. x again:

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}[b\,Cos(\ln x) - a\,Sin(\ln x)] + \frac{1}{x}[-b\,Sin(\ln x) - a\,Cos(\ln x)\frac{1}{x}]$$

$$= \frac{-1}{x^2}[b\,Cos(\ln x) - a\,Sin(\ln x) + b\,Sin(\ln x) + a\,Cos(\ln x]$$ (3 marks)

Now

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$-b\,Cos(\ln x) + a\,Sin(\ln x) - b\,Sin(\ln x) - a\,Cos(\ln x) + b\,Cos\ln x - a\,Sin\ln x$$

$$+ a\,Cos(\ln x) + b\,Sin\ln x = 0$$

Proved: 0 = 0 (2 marks)

Q.18 **(8)**

y = ax + b (I)

y = 2x–6 (II)

Comparing I&II, a = 2

y = 2x + b, line passing through (-1, 7)

7 = -2 + b $\Rightarrow$ b = 9

Hence req. line is y = 2x + 9 (4 marks)

Given that $\frac{x}{3} - y = 7$

x-3y = 7

Slop of line m = $\frac{-1}{-3}$

m = $\frac{1}{3}$

Slop of line perpendicular is '-3', line passing through (7, -1) then eq of line is

$y - y_1 = m(x - x_1)$

y + 1 = -3(x – 7)

y + 1 = -3x + 21

3x + y – 20 = 0 (4 marks)

Q.19 **(8)**

Given that

$$\underline{a} + \underline{b} + \underline{c} = 0$$

Taking cross mult with $\underline{a}$

$$\underline{a} \times (\underline{a} + \underline{b} + \underline{c}) = 0$$

$$\underline{a} \times \underline{a} + \underline{a} \times \underline{b} + \underline{a} \times \underline{c} = 0$$

$$\underline{a} \times b = -(\underline{a} \times \underline{c})$$

$$\underline{a} \times \underline{b} = \underline{c} \times \underline{a}$$ (I) (4 marks)

Let

$$\underline{a}+\underline{b}+\underline{c}=0$$

Taking $\underline{b}$ cross mult

$$\underline{b}\times(\underline{a}+\underline{b}+\underline{c})=0$$

$$\underline{b}\times\underline{a}+\underline{b}\times\underline{b}+\underline{b}\times\underline{c}=0$$

$$\underline{b}\times\underline{c}=-(\underline{b}\times\underline{a})$$

$$\underline{b}\times\underline{c}=\underline{a}\times\underline{b} \qquad \text{(II)}$$

From (I) & (II) $\underline{a}\times\underline{b}=\underline{b}\times\underline{c}=\underline{c}\times\underline{a}$ (4 marks)

Q.20 **(8)**

$$\frac{d}{dx}[x\ln x-x]=\ln x$$

L.H.S

$$\frac{d}{dx}[x\ln x-x]$$

$$\ln x+x.\,{}^{1}\!/\!_{x}-1=\ln x=RHS$$ (2 marks)

Now

$$\ln x=\frac{d}{dx}(x\ln x-x)$$

$$\int \ln x\,dx=\int\frac{d}{dx}(x\ln x-x)dx$$

$$\int \ln x\,dx=x\ln x-x+c$$ (2 marks)

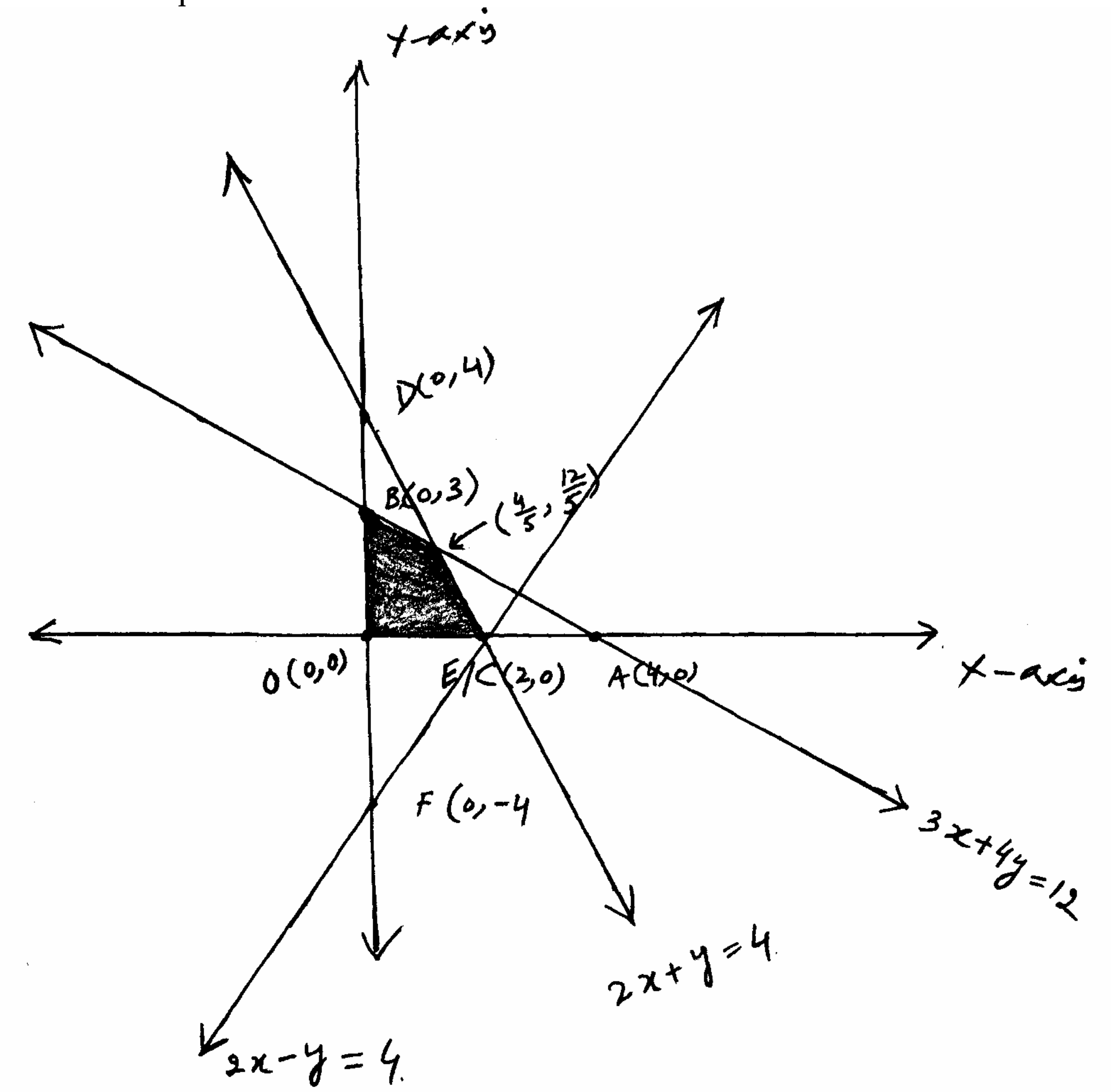
$$\int_1^3 \ln x\,dx=[x\ln x-x]_1^3$$ (1 mark)

= (3ln3-3) – (1ln1-1)
= xln3 – 3 + 1
= 3ln3 – 2 (3 marks)

Q.21 **(8)**

Z = 2x + 3y subject to:
3x + 4y ≤ 12, 2x + y ≤ 4, 2x – y ≤ 4, x ≥ 0, y ≥ 0
Associated eq. are
3x + 4y = 12 (I)
2x + y = 4 (II)
2x – y = 4 (III)
x = 0, y = 0 (1 mark)
X Intercept of I put y = 0 ⇒ x = 4 A(4, 0)
Y Intercept of I put x = 0 ⇒ y = 3 B(0, 3)
Put (0, 0) in given inequality 0 < 12 so solution region lies towards origin.
X Intercept of II put y = 0 ⇒ x = 2 C(2, 0)

Y   Intercept of II put x = 0 $\Rightarrow$ y = 4   D(0, 4)
Put (0, 0) so 0 < 12 soln region towards origin.
X   Intercept of III put y = 0 $\Rightarrow$ x = 2   E(2, 0)
Y   Intercept of I put x = 0 $\Rightarrow$ y = -4   F(0, -4)   (3 marks)
Put (0, 0) solution region towards origin as 0 < 12 $x \geq 0$ & $y \geq 0$ lies on 1[st] quad.



(2 marks)

So corner points are (0, 3) (0, 0) (2, 0) and $\left(\frac{4}{5}, \frac{12}{5}\right)$

$$\begin{array}{rl} 3x + 4y = & 12 \\ \underline{-8x \pm 4y} = & \underline{-16} \\ -5x \quad = & -4 \end{array}$$

x = $\frac{4}{5}$ Put in (II)

$\frac{8}{5} + y = 4$

y = $4 - \frac{8}{5} \Rightarrow \frac{12}{5}$ (1 mark)

f(x, y)	2x + 3y
(0, 0)	0 + 0 = 0
(0, 3)	0 + 9 = 9
(2, 0)	4 + 0 = 4
$\left(\frac{4}{5},\frac{12}{5}\right)$	$\frac{8}{5}+\frac{36}{5}=\frac{44}{5}=8.8$

So Z = 2x + 3y is maximum at (0, 3) (1 mark)

Q.22 **(8)**

$y^2 = -6x$ & $2x + y + 1 = 0$

Slope of required line = -2 = m (1 mark)

In parabola

$y^2 = -6x$

$a = -6/4 = -3/2$ (1 mark)

Eq of Tangent to parabola

$y = mx + \frac{a}{m}$ (1 mark)

$y = -2x + \frac{-3/2}{-2} \Rightarrow y = \frac{-8x+3}{4}$

8x + 4y – 3 = 0 Req. Eq (1 mark)

Put $y = \frac{-8x+3}{4}$ in $y^2 = -6x$

$$\left(\frac{-8x+3}{4}\right)^2 = -6x$$

$$\frac{64x^2 - 48x + 9}{16} = -6x \Rightarrow 64x^2 + 48x + 9 = 0$$

$(8x+3)^2 = 0 \Rightarrow x = -3/8$ (3 marks)

Putting the value of x in $y = \frac{-8x+3}{4}$

$$y = \frac{-8\left(-3/8\right)+3}{4}$$

$y = \frac{3}{2}$ (1 mark)

Hence point of Tangency is $\left(-3/8, 3/2\right)$ (1 mark)