MathCity.org *Merging man and maths*	**Definitions: Mathematics HSSC-II** **Calculus and Analytic Geometry, MATHEMATICS 12** Collected by: **Muhammad Waqas Sulaiman**

Unit # 01 (Functions and Limits)

Function: A function is a rule or correspondence, relating to two sets in such a way that each element in the first set corresponds to one and only one element in the second set. Or
A function from X to Y is a rule that assigns to each element x in X a unique element y in Y.
e.g. $A = x^2$, that is, A is a function of x.

Domain: In a function $f : X \to Y$ the set X is called the domain of function.

Range: In a function $f : X \to Y$ the set of corresponding elements y in Y is called range of function.

Independent and Dependent Variables: If a variable y depends on a variable x in such a way that each value x determines exactly one value of y, then we say that y is function of x. The variable x is called independent variable and y is called dependent variable.
e.g. in $y = ax + b$, the x variable is called independent and y is called dependent variable of function.

Algebraic Function: The function which are defined by variable expressions. e.g. $f(x) = 2x - 6$.

Polynomial Function: A function of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$, where $a_0, a_1, a_2, \ldots, a_n$ are real number and exponent are non-negative integers is called polynomial function.

Linear Function: If degree of polynomial function is one it is called linear function. e.g $f(x) = ax + b$.

Identity Function: For any set X, a function $I : X \to X$ of form $I(x) = x$ is called Identity function.

Constant Function: A function $C : X \to Y$ defined by $C(x) = a$, where a some real numbers, is called constant function.

Rational Function: A function R defined by $R(x) = P(x)/Q(x)$, where both $P(x)$, $Q(x)$ are polynomial function and $Q(x) \neq 0$, is called rational function .

Logarithmic Function: A function in which the variable appears as an argument of logarithmic is called logarithmic function. Example $f(x) = \log a,\ a > 0$.

Exponential Function: A function in which the variable appears as exponent is called exponential function. Example $f(x) = a^x,\ f(x) = 2^x$.

Discontinuous Function: A function $f(x)$ is said to be discontinuous at *a* point if either f is not defined at a or limit of f at a does not exist or $\lim\limits_{x \to a} f(x) \neq f(a)$.

Explicit Function: A function in which independent variable is easily expressed in terms of the independent variable is called explicit function. Example $y = ax + b$.

Implicit Function: If the independent and the dependent variable are mixed up in such a way that the dependent and independent variable cannot be expressed is called implicit function.
Example: $x^2+y^2+2gx+2fy+c=0$

Parametric Function: A function in which x and y are expressed as functions of a third variable is called parametric function. Example $x=at^2,\ y=2at.$

Even Function: A function f is said to be an even function if $f(-x) = f(x)$ for every x in the domain of function. Example $f(x)= x^4,\ g(x)=\cos x$.

Odd Function: A function f is said to be an odd function if $f(-x) = -f(x)$ for every x in the domain of function. Example $f(x) = x^3,\ g(x) = \sin x$.

Continuous Function: A function f is said to be continuous at a number "c" if and only if the following three conditions are satisfied:

(i) $f(c)$ is defined. (ii) $\lim\limits_{x\to c} f(c)$ exists. (iii) $\lim\limits_{x\to c} f(x)=f(c)$

Unit # 02 (Differentiation)

Differentiation: Instantaneous rate of change of one variable with respect to other variable is called derivative or differentiation. Its denoted by dy/dx.

Increasing function: A *function* f *is* defined on an interval $[a,b]$ is said to be increasing function on $[a,b]$ if $f(x_1)<f(x_2)$ whenever $x_1<x_2$, where x_1 and x_2 are any numbers in the interval $[a,b]$.

Decreasing function: A function f is defined on an interval $[a,b]$ is said to be decreasing function on $[a,b]$ if $f(x_1)>f(x_2)$, whenever $x_1<x_2$, where x_1 and x_2 are any numbers in the interval $[a,b]$.

Critical point: If $c\in D_f$ and $f'(c)=0$ or $f'(c)$ does not exists then c is called critical value or point.

Stationary point: Those critical points on the graph of f at which $f'=0$ are called stationary point of f.

Relative maxima: f has relative maxima at c if $f''(c)<0$.

Relative minima: f has relative manima at c if $f''(c)>0$.

Turning point: A stationary point is called turning point if it is either a maximum point or a minimum point.

Point of inflection: A stationary point is called point of inflection if a function have neither local maxima nor local minima at that point.

Unit # 03 (Integration)

Integration or Anti-derivative: Inverse process of differentiation is called integration.

Differential equation: An equation containing at least one derivation of a dependent variable w.r.t an independent variable. e.g. $y\dfrac{dy}{dx}+x=0$.

order of differential equation: The order of a differential is the order of the highest derivative in the equation.

Unit # 04 (Analytical geometry)

Coordinate Axes: In a mutually perpendicular number lines $X'OX$ and $Y'OY$, one horizontal and the other vertical , are called the coordinate axes.

Coordinate of point: Coordinate of a point represents an ordered set of numbers that defines the position of a point in a plane or space.

Analytic geometry: Analytic geometry (coordinate geometry) is the branch of mathematics. Analytical geometry is a fusion of algebraic equation and geometry curves.

Centroid of a triangle: The centroid of a triangle is the point of intersection of its medians.

Ortho-center: The ortho-center of a triangle is the point of intersection of the lines through the vertices and perpendicular to the opposite side (altitudes) of the triangle.

Circum-center: The point of intersection of the perpendicular bisectors of the sides of a triangle is called cirum-center.

In-center: The point of intersection of the internal bisectors of the angles of a triangle is called its in-center. It is the center of the circle, which touches the sides of the triangle.

Inclination of a line: The angle $\alpha\left(0<\alpha<180^\circ\right)$ measured counter clockwise from +ve axis to a non-horizontal straight line is called the inclination of line.

Slope of line: The slop (m) of the non-vertical straight line with α as inclination is define by $m=\tan\alpha$.

Homogeneous equation: An equation in two variable x and y is said to be homogeneous equation if sum of exponents of x and y in each term remains same. The equation in this form $ax^2+2hxy+y^2=0$ (where a, b and h simultaneously not zero) is called general form of Homogeneous equation.

Unit # 05 (Linear Inequalities and Linear Programming)

Inequality: An inequality involving any one of the symbols $<$ (less then) , $>$ (greater then) , $\leq$ (less then equal to), $\geq$ (greater then equal to) is called inequality. If $ax+b<c$ is called linear inequality.

Problem Constrain: In a certain problem from everyday life each linear inequality concerning the problem is called the problem constrain.

Feasible region: The solution region of an inequality restricted to the first quadrant is called feasible region.

Vertex or corner point: A point of a solution region where two of its boundary lines intersects is called vertex.

Decision variable: The variables used in the system of linear inequalities relating to the problems of everyday life are nonnegative constrain or decision variables.

Feasible Solution: Each point of the feasible region is called the feasible solution of linear inequalities.

Feasible Solution set: A set consisting of all the feasible solutions of the system of linear inequalities is called a feasible solution set.
Convex Region: If the line segment joining any two points of a certain region lies entirely within the region, then such a region is called convex region.
Linear Programming: The mathematical technique which deals with the problems to get optimization (maximization or minimization) of a linear function of variable is called linear programming.
Objective Function: A function which is to be a maximized or minimized is called an objective function.
Optimal solution: The feasible solution which maximizes or minimizes the objective function is called optimal solution.

Unit # 07 (Vectors)

Vector: A physical quantity defined by its magnitude and direction is also called vector. e.g. force, length.
Scalar: A physical quantity which is defined only by its magnitude is called scalar. e.g. mass, time.
Magnitude or length: Absolute value of vector is called magnitude or length its denoted by $|\overrightarrow{AB}|$.

Unit Vector: A vector whose magnitude is unity or 1 , $\hat{v}=\dfrac{v}{|v|}$.

Equal Vector: Two vectors $|\overrightarrow{AB}|$ and $|\overrightarrow{PQ}|$ are equal if they have same magnitude and direction $|\overrightarrow{AB}|=|\overrightarrow{PQ}|$

Parallel Vector: Two vectors are parallel if and only if they are non-zero scalar multiple of each $a=\alpha b$
Position vector: The vector of whose initial point is the origin O terminal point is P .

Unit # 06 (Conic Section)

Conic sections: Conic sections or conic are the curves obtained by cutting a cone by a plane.
Circle: If the cone is cut by a plane perpendicular to the axis of the cone then the section is a circle. We can also define circle as: The of all points in a plane which are equidistance from some fixed point in the plane is called a circle.

Elements of a circle:

Radius: A locus of a point which remains at a fixed distance from a certain point. The point is called center of circle and fixed distance is called radius of a circle.
Center: The fixed point from which all the points of a circle are equidistant is called the center of a circle
Diameter: A line segment passing through the center of a circle with its endpoints lying on the circle is called the diameter of a circle.
Write Circle Equation with centre C(h,k) and radius : Let $P(x,y)$ be any point of circle , then by the definition of circle the distance P from C must be equal to the radius r i.e. $|CP|=r$

$$\sqrt{(x-h)^2(y-k)^2}=r$$

Chord of a circle: A line segment whose endpoints lie on a circle is called the chord of the circle.

Elements of a Parabola:

Parabola: If the intersecting plane is parallel to a generator of the cone but cuts one nape only is called parabola.
We can also define Parabola as: A parabola is the set of all points in a plane which are equidistant from a fixed point and a fixed line. The fixed point is called the focus of parabola and fixed line is called the directrix of parabola. A standard equation of parabola is $y^2 = 4ax$.
Vertex: The mid-point between the focus and the directrix on the parabola is called the vertex of the parabola.
Chord of a parabola: A line joining two distinct points on a parabola is called the chord of the parabola.
Axis: Line through the vertex and focus is called the axis of a parabola.
Focal chord: Any chord is passing through the focus of the parabola is called focal chord of the parabola
Latusrectum: The focal chord perpendicular to the axis of the parabola is called the latusrectum of the parabola. There is only one latusrectum in parabola.

Elements of a Ellipse:

Center of Ellipse: The midpoint of foci or midpoint of vertices is called center of Ellipse.
Major axis: Longest diameter of ellipse is called major axis. OR The line segment joining two points of ellipse while passing through foci is called major axis. It is axis of symmetry for ellipse.
Equation of Major axis for horizontal line y=c: if $c(0,0)$ then $y = 0$. if $c(h,k)$ then $y = k$.
Equation of Major axis for vertical line x=c: if $c(0,0)$ then $x = 0$. if $c(h,k)$ then $x = h$.
Minor axis: Shortest diameter of ellipse is called major axis. OR The line segment joining two points of ellipse while passing through center and perpendicular to major axis is called minor axis. It is axis of symmetry for ellipse.
Equation of Minor axis for horizontal line y=c: if $c(0,0)$ then $y = 0$. if $c(h,k)$ then $y = k$.
Equation of Minor axis for vertical line x=c: if $c(0,0)$ then $x = 0$. if $c(h,k)$ then $x = h$.
Vertices: The ends points of major axis or longest diameter are called vertices of ellipse.
Covertices: The ends points of minor axis or shortest diameter is called covertices.
Chord: A line segment joining any two distinct points of ellipse is called chord of ellipse.
There can be an infinite number of chords in an ellipse
Focal chord: A chord passing through any focus is called focal chord. There can be an infinite number of focal chords in an ellipse.
Latusrectum: A focal chord perpendicular to major axis is called lastusretum. There can be only two latusrectum for an ellipse.

Elements of a Hyperbola:

Hyperbola: If the cone is cut by a plane and the cutting plane is parallel to the axis of cone and intersecting both its napes, then curve of intersection is hyperbola.
We can also define Parabola as: A hyperbola is the set of all points in the plane, the difference of whose distances from two fixed distinct points is a given positive constant that is less the distance between the fixed points.
Center of hyperbola: The midpoint of the line segment joining the foci is called center of hyperbola.
Focal axis or transverse axis: The line through the foci of the hyperbola is called focal axis.

Conjugate axis: The line through the center and perpendicular to the focal axis is called conjugate axis of hyperbola.
Vertices: The points where the hyperbola meets its focal axis are called its vertices.
Branches: The two separates parts of a hyperbola are called its branches.

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