

EXERCISE 7.5

① (i) Given that

$$\underline{u} = 3\underline{i} + 2\underline{k} = 3\underline{i} + 0\underline{j} + 2\underline{k}$$

$$\underline{v} = \underline{i} + 2\underline{j} + \underline{k}$$

$$\underline{w} = -\underline{j} + 4\underline{k} = 0\underline{i} - \underline{j} + 4\underline{k}$$

$$\text{Volume of parallelepiped} = [\underline{u} \ \underline{v} \ \underline{w}]$$

$$= \begin{vmatrix} 3 & 0 & 2 \\ 1 & 2 & 1 \\ 0 & -1 & 4 \end{vmatrix}$$

$$= 3(8+1) - 0 + 2(-1-0)$$

$$= 3(9) + 2(-1) = 27 - 2 = 25$$

(cubic units)

(ii) Given that $\underline{u} = \underline{i} - 4\underline{j} - \underline{k}$

$$\underline{v} = \underline{i} - \underline{j} - 2\underline{k} \text{ and}$$

$$\underline{w} = 2\underline{i} - 3\underline{j} + \underline{k}$$

$$\text{Volume of parallelepiped} = [\underline{u} \ \underline{v} \ \underline{w}]$$

$$= \begin{vmatrix} 1 & -4 & -1 \\ 1 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= 1(-1-6) + 4(1+4) - 1(-3+2)$$

$$= 1(-7) + 4(5) - 1(-1)$$

$$= -7 + 20 + 1 = 14 \text{ (cubic units)}$$

(iii) Given that $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$

$$\underline{v} = 2\underline{i} - \underline{j} - \underline{k}$$

$$\underline{w} = \underline{j} + \underline{k} = 0\underline{i} + \underline{j} + \underline{k}$$

$$\text{Volume of parallelepiped} = [\underline{u} \ \underline{v} \ \underline{w}]$$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1(-1+1) + 2(2-0) + 3(2-0)$$

$$= 1(0) + 2(2) + 3(2) = 0 + 4 + 6$$

$$= 10 \text{ (cubic units)}$$

② Given that $\underline{a} = 3\underline{i} - \underline{j} + 5\underline{k}$

$$\underline{b} = 4\underline{i} + 3\underline{j} - 2\underline{k}$$

$$\underline{c} = 2\underline{i} + 5\underline{j} + \underline{k}$$

$$\underline{a} \cdot (\underline{b} \times \underline{c}) = \begin{vmatrix} 3 & -1 & 5 \\ 4 & 3 & -2 \\ 2 & 5 & 1 \end{vmatrix}$$

$$\begin{aligned} \Rightarrow \underline{a} \cdot (\underline{b} \times \underline{c}) &= 3(3+10) + 1(4+4) + 5(20-6) \\ &= 3(13) + 1(8) + 5(14) \\ &= 39 + 8 + 70 = 117 \end{aligned}$$

$$\Rightarrow \underline{a} \cdot (\underline{b} \times \underline{c}) = 117 \text{ ——— ①}$$

$$\underline{b} \cdot (\underline{c} \times \underline{a}) = \begin{vmatrix} 4 & 3 & -2 \\ 2 & 5 & 1 \\ 3 & -1 & 5 \end{vmatrix}$$

$$= 4(25+1) - 3(10-3) - 2(-2-15)$$

$$= 4(26) - 3(7) - 2(-17)$$

$$= 104 - 21 + 34 = 117$$

$$\Rightarrow \underline{b} \cdot (\underline{c} \times \underline{a}) = 117 \text{ ——— ②}$$

$$\underline{c} \cdot (\underline{a} \times \underline{b}) = \begin{vmatrix} 2 & 5 & 1 \\ 3 & -1 & 5 \\ 4 & 3 & -2 \end{vmatrix}$$

$$= 2(2-15) - 5(-6-20) + 1(9+4)$$

$$= 2(-13) - 5(-26) + 1(13)$$

$$= -26 + 130 + 13 = 117$$

$$\Rightarrow \underline{c} \cdot (\underline{a} \times \underline{b}) = 117 \text{ ——— ③}$$

$\therefore$ from ①, ② and ③, we get

$$\underline{a} \cdot (\underline{b} \times \underline{c}) = \underline{b} \cdot (\underline{c} \times \underline{a}) = \underline{c} \cdot (\underline{a} \times \underline{b})$$

(proved)

③ Let $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$

$$\underline{v} = -2\underline{i} + 3\underline{j} - 4\underline{k}$$

$$\underline{w} = \underline{i} - 3\underline{j} + 5\underline{k}$$

$$[\underline{u} \ \underline{v} \ \underline{w}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(15-12) + 2(-10+4) + 3(6-3)$$

$$= 1(3) + 2(-6) + 3(3)$$

$$= 3 - 12 + 9 = 0$$

$\therefore \underline{u}, \underline{v}$ and $\underline{w}$ are coplanar.

④ (i) Let $\underline{u} = \underline{i} - \underline{j} + \underline{k}$

$$\underline{v} = \underline{i} - 2\underline{j} - 3\underline{k}$$

$$\underline{w} = 3\underline{i} - \alpha\underline{j} + 5\underline{k}$$

$\therefore \underline{u}, \underline{v}$ and $\underline{w}$ are coplanar

$$\therefore [\underline{u} \ \underline{v} \ \underline{w}] = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -3 \\ 3 & -\alpha & 5 \end{vmatrix} = 0$$

$$\Rightarrow 1(-10-3\alpha) + 1(5+9) + 1(-\alpha+6) = 0$$

$$\Rightarrow -10 - 3\alpha + 14 - \alpha + 6 = 0$$

$$\Rightarrow -4\alpha + 10 = 0$$

$$\Rightarrow -4\alpha = -10$$

$$\Rightarrow \alpha = \frac{-10}{-4} \Rightarrow \alpha = \frac{5}{2} \text{ Ans.}$$

(ii) Let $\underline{u} = \underline{i} - 2\underline{j} - \underline{k}$

$\underline{v} = \underline{i} - \underline{j} + 2\underline{k}$ and

$\underline{w} = \alpha \underline{i} - \underline{j} + \underline{k}$

$\therefore \underline{u}, \underline{v}$ and $\underline{w}$ are coplanar

$\therefore [\underline{u} \ \underline{v} \ \underline{w}] = 0$

$\Rightarrow \begin{vmatrix} 1 & -2\alpha & -1 \\ 1 & -1 & 2 \\ \alpha & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(-1+2) + 2\alpha(1-2\alpha) - 1(-1+\alpha) = 0$

$\Rightarrow 1 + 2\alpha - 4\alpha^2 + 1 - \alpha = 0$

$\Rightarrow -4\alpha^2 + \alpha + 2 = 0$

$\Rightarrow -1(4\alpha^2 - \alpha - 2) = 0$

$\Rightarrow 4\alpha^2 - \alpha - 2 = 0$

$\Rightarrow \alpha = \frac{1 \pm \sqrt{1+32}}{8} = \frac{1 \pm \sqrt{33}}{8}$ Ans.

(5)(a)(i) $2\underline{i} \times 2\underline{j} \cdot \underline{k} = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
 $= 2(2-0) - 0 + 0 = 2(2) = 4$ Ans.

(ii) $3\underline{j} \cdot \underline{k} \times \underline{i} = \begin{vmatrix} 0 & 3 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0 - 3(0-1) + 0$
 $= 3$ Ans.

(iii) $[\underline{k} \ \underline{i} \ \underline{j}] = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0 - 0 + 1(1-0) = 1$ Ans.

(iv) $[\underline{i} \ \underline{i} \ \underline{k}] = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(0-0) - 0 + 0 = 0$ Ans.

(b) Prove that

$\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v})$
 $= 3 \underline{u} \cdot (\underline{v} \times \underline{w})$

Sol: - L.H.S. = $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v})$
 $= [\underline{u} \ \underline{v} \ \underline{w}] + [\underline{v} \ \underline{w} \ \underline{u}] + [\underline{w} \ \underline{u} \ \underline{v}]$
 $= [\underline{u} \ \underline{v} \ \underline{w}] + [\underline{u} \ \underline{v} \ \underline{w}] + [\underline{u} \ \underline{v} \ \underline{w}]$
 $\therefore [\underline{v} \ \underline{w} \ \underline{u}] = [\underline{u} \ \underline{v} \ \underline{w}]$
 $\& [\underline{w} \ \underline{u} \ \underline{v}] = [\underline{u} \ \underline{v} \ \underline{w}]$
 $\therefore \text{L.H.S} = 3 [\underline{u} \ \underline{v} \ \underline{w}] = 3 \underline{u} \cdot (\underline{v} \times \underline{w}) = \text{R.H.S.}$

(6) Let $A(0,1,2), B(3,2,1), C(6,2,1)$
 and $D(5,5,6)$

Now $\overrightarrow{AB} = (3-0)\underline{i} + (2-1)\underline{j} + (1-2)\underline{k} = 3\underline{i} + \underline{j} - \underline{k}$

$\overrightarrow{AC} = (1-0)\underline{i} + (2-1)\underline{j} + (1-2)\underline{k} = \underline{i} + \underline{j} - \underline{k}$

$\overrightarrow{AD} = (5-0)\underline{i} + (5-1)\underline{j} + (6-2)\underline{k} = 5\underline{i} + 4\underline{j} + 4\underline{k}$

Volume of tetrahedron = $\frac{1}{6} [\overrightarrow{AB} \ \overrightarrow{AC} \ \overrightarrow{AD}]$

$= \frac{1}{6} \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & -1 \\ 5 & 4 & 4 \end{vmatrix}$

$= \frac{1}{6} [3(4+4) - 1(4+5) - 1(4-5)]$

$= \frac{1}{6} [24 - 9 + 1] = \frac{1}{6} (16) = \frac{16}{6} = \frac{8}{3}$
 (Cubic units) Ans.

(ii) Let $A(2,1,8), B(3,2,9), C(2,1,4)$
 and $D(3,3,10)$.

Now $\overrightarrow{AB} = (3-2)\underline{i} + (2-1)\underline{j} + (9-8)\underline{k} = \underline{i} + \underline{j} + \underline{k}$

$\overrightarrow{AC} = (2-2)\underline{i} + (1-1)\underline{j} + (4-8)\underline{k} = 0\underline{i} + 0\underline{j} - 4\underline{k}$

$\overrightarrow{AD} = (3-2)\underline{i} + (3-1)\underline{j} + (10-8)\underline{k} = \underline{i} + 2\underline{j} + 2\underline{k}$

Volume of tetrahedron = $\frac{1}{6} [\overrightarrow{AB} \ \overrightarrow{AC} \ \overrightarrow{AD}]$

$= \frac{1}{6} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & -4 \\ 1 & 2 & 2 \end{vmatrix}$

$= \frac{1}{6} [1(0+8) - 1(0+4) + 1(0-0)]$

$= \frac{1}{6} [8 - 4 + 0] = \frac{1}{6} (4) = \frac{4}{6} = \frac{2}{3}$ Ans.
 (Cubic units)

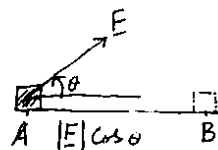
Work Done

If a constant force $\underline{F}$ acts on a body at any angle θ to the direction of motion, displaces it from A to B. Then

Work done = $|\underline{F}| \cos \theta |\overrightarrow{AB}|$

$= \underline{F} \cdot \overrightarrow{AB}$

$= \underline{F} \cdot \underline{d}$



(7) Given that $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$

$P_1(3,1,-2)$ and $P_2(2,4,6)$

$\underline{d} = \overrightarrow{P_1 P_2} = (2-3)\underline{i} + (4-1)\underline{j} + (6+2)\underline{k}$

$\Rightarrow \underline{d} = -\underline{i} + 3\underline{j} + 8\underline{k}$

Work done = $\underline{F} \cdot \underline{d}$

$= (4\underline{i} + 3\underline{j} + 5\underline{k}) \cdot (-\underline{i} + 3\underline{j} + 8\underline{k})$

$= (4)(-1) + (3)(3) + (5)(8)$

$= -4 + 9 + 40 = 45 \text{ units.}$

(8) Given that $\underline{F}_1 = 4\underline{i} + \underline{j} - 3\underline{k}$

$\underline{F}_2 = 3\underline{i} - \underline{j} - \underline{k}$

$\therefore \text{net force} = \underline{F} = \underline{F}_1 + \underline{F}_2$

$$\Rightarrow \underline{F} = 4\underline{i} + \underline{j} - 3\underline{k} + 3\underline{i} - \underline{j} - \underline{k}$$

$$\Rightarrow \underline{F} = 7\underline{i} + 0\underline{j} - 4\underline{k}$$

$$A(1, 2, 3), B(5, 4, 1)$$

$$\therefore \underline{d} = \underline{AB} = (5-1)\underline{i} + (4-2)\underline{j} + (1-3)\underline{k}$$

$$\Rightarrow \underline{d} = 4\underline{i} + 2\underline{j} - 2\underline{k}$$

$$\therefore \text{Work done} = \underline{F} \cdot \underline{d}$$

$$= (7\underline{i} + 0\underline{j} - 4\underline{k}) \cdot (4\underline{i} + 2\underline{j} - 2\underline{k})$$

$$= (7)(4) + (0)(2) + (-4)(-2)$$

$$= 28 + 0 + 8 = 36 \text{ units} \quad \underline{\text{Ans}}$$

$$\textcircled{9} \text{ Given that } A(5, -5, -7) \text{ and } B(6, 2, -2)$$

$$\underline{d} = \underline{AB} = (6-5)\underline{i} + (2+5)\underline{j} + (-2+7)\underline{k}$$

$$\Rightarrow \underline{d} = \underline{i} + 7\underline{j} + 5\underline{k}$$

$$\underline{F}_1 = 10\underline{i} - \underline{j} + 11\underline{k}$$

$$\underline{F}_2 = 4\underline{i} + 5\underline{j} + 9\underline{k}$$

$$\underline{F}_3 = -2\underline{i} + \underline{j} - 9\underline{k}$$

$$\therefore \text{net force} = \underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$\Rightarrow \underline{F} = 12\underline{i} + 5\underline{j} + 11\underline{k}$$

$$\therefore \text{Work done} = \underline{F} \cdot \underline{d}$$

$$= (12\underline{i} + 5\underline{j} + 11\underline{k}) \cdot (\underline{i} + 7\underline{j} + 5\underline{k})$$

$$= (12)(1) + (5)(7) + (11)(5)$$

$$= 12 + 35 + 55 = 102 \text{ units.} \quad \underline{\text{Ans.}}$$

$$\textcircled{10} \text{ Given that } |\underline{F}| = 6 \text{ units}$$

$$\therefore \underline{F} = |\underline{F}| \cdot \hat{\underline{E}} \Rightarrow \underline{F} = 6 \hat{\underline{E}} \quad \text{--- (1)}$$

$$\therefore \underline{F} \parallel 2\underline{i} - 2\underline{j} + \underline{k}$$

$$\therefore \hat{\underline{F}} = \frac{2\underline{i} - 2\underline{j} + \underline{k}}{\sqrt{4+4+1}} = \frac{2\underline{i} - 2\underline{j} + \underline{k}}{3}$$

$$\therefore \textcircled{1} \text{ becomes}$$

$$\underline{F} = 6 \left(\frac{2\underline{i} - 2\underline{j} + \underline{k}}{3} \right)$$

$$\Rightarrow \underline{F} = 4\underline{i} - 4\underline{j} + 2\underline{k}$$

$$\text{Let } A(1, 2, 3) \text{ and } B(5, 3, 7). \text{ Then}$$

$$\underline{d} = \underline{AB} = (5-1)\underline{i} + (3-2)\underline{j} + (7-3)\underline{k}$$

$$\Rightarrow \underline{d} = 4\underline{i} + \underline{j} + 4\underline{k}$$

$$\therefore \text{Work done} = \underline{F} \cdot \underline{d}$$

$$= (4\underline{i} - 4\underline{j} + 2\underline{k}) \cdot (4\underline{i} + \underline{j} + 4\underline{k})$$

$$\underline{\underline{28}}$$

$$= (4)(4) + (-4)(1) + (2)(4)$$

$$= 16 - 4 + 8 = 20 \text{ units}$$

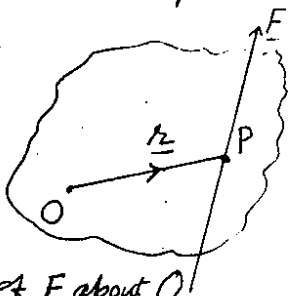
Moment of Force

The turning effect of a force about a point is called moment of the force about that point.

If a force $\underline{F}$ acts on a body at point P and rotates the body about point O .

Then moment of $\underline{F}$ about O

$$= \underline{M} = \underline{r} \times \underline{F}$$



$$\textcircled{11} \text{ Given that } \underline{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$$

Let $A(1, -1, 2)$ (point of application)

and $B(2, -1, 3)$

$$\underline{r} = \underline{BA}$$

$$\underline{r} = (1-2)\underline{i} + (-1+1)\underline{j} + (2-3)\underline{k}$$

$$\underline{r} = -\underline{i} + 0\underline{j} - \underline{k}$$

$$\therefore \underline{M} = \underline{r} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$\Rightarrow \underline{M} = \underline{i}(0+2) - \underline{j}(-4+3) + \underline{k}(-2-0)$$

$$\Rightarrow \underline{M} = 2\underline{i} - 7\underline{j} - 2\underline{k} \quad \underline{\text{Ans.}}$$

$$\textcircled{12} \text{ Given that } \underline{F} = 4\underline{i} - 3\underline{k} = 4\underline{i} + 0\underline{j} - 3\underline{k}$$

$A(2, -2, 5)$ (point of application) and about

$B(1, -3, 1)$

$$\underline{r} = \underline{BA} = (2-1)\underline{i} + (-2+3)\underline{j} + (5-1)\underline{k}$$

$$\Rightarrow \underline{r} = \underline{i} + \underline{j} + 4\underline{k}$$

$$\therefore \underline{M} = \underline{r} \times \underline{F}$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 4 \\ 4 & 0 & -3 \end{vmatrix}$$

$$= \underline{i}(-3-0) - \underline{j}(-3-16) + \underline{k}(0-4)$$

$$= -3\underline{i} + 19\underline{j} - 4\underline{k} \quad \underline{\text{Ans.}}$$

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(13) Given that

$$\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$$

$$A(1, -2, 1) \text{ and } B(2, 0, -2)$$

$$\underline{r} = \overrightarrow{BA} = (1-2)\underline{i} + (-2-0)\underline{j} + (1+2)\underline{k}$$

$$\underline{r} = -\underline{i} - 2\underline{j} + 3\underline{k}$$

$$\therefore \underline{M} = \underline{r} \times \underline{F}$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & -2 & 3 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \underline{i}(6-3) - \underline{j}(3-6) + \underline{k}(-1+4)$$

$$= 3\underline{i} - 3\underline{j} + 3\underline{k} \quad \underline{\text{Ans.}}$$

(14) Given that $A(1, 1, 1)$ and $P(2, 0, 1)$

$$\underline{r} = \overrightarrow{AP} = (2-1)\underline{i} + (0-1)\underline{j} + (1-1)\underline{k}$$

$$\underline{r} = \underline{i} - \underline{j} + 0\underline{k}$$

$$\underline{F}_1 = \underline{i} - 2\underline{j}$$

$$\underline{F}_2 = 3\underline{i} + 2\underline{j} - \underline{k}$$

$$\underline{F}_3 = 5\underline{j} + 2\underline{k}$$

$$\therefore \text{net force} = \underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$\Rightarrow \underline{F} = 4\underline{i} + 5\underline{j} + \underline{k}$$

$$\therefore \underline{M} = \underline{r} \times \underline{F}$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 0 \\ 4 & 5 & 1 \end{vmatrix}$$

$$= \underline{i}(-1-0) - \underline{j}(1-0) + \underline{k}(5+4)$$

$$= -\underline{i} - \underline{j} + 9\underline{k} \quad \underline{\text{Ans.}}$$

(15) Given that

$$\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$$

$$P(1, -2, 3) \text{ and } Q(2, 1, 1) \text{ . Then}$$

$$\underline{r} = \overrightarrow{QP} = (1-2)\underline{i} + (-2-1)\underline{j} + (3-1)\underline{k}$$

$$\underline{r} = -\underline{i} - 3\underline{j} + 2\underline{k}$$

$$\therefore \underline{M} = \underline{r} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & -3 & 2 \\ 7 & 4 & -3 \end{vmatrix}$$

$$\underline{M} = \underline{i}(9-8) - \underline{j}(3-14) + \underline{k}(-4+21)$$

$$\underline{M} = \underline{i} + 11\underline{j} + 17\underline{k} \quad \underline{\text{Ans.}}$$

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