Question # 1
Let $A=(2,5), B(-1,1)$ and $C(2,-6)$, find
(i) $\overrightarrow{AB}$ (ii) $2\overrightarrow{AB}-\overrightarrow{CB}$
(iii) $2\overrightarrow{CB}-2\overrightarrow{CA}$

Solution
Given $A(2,5)$, $B(-1,1)$ and $C(2,-6)$

(i) $\overrightarrow{AB}=(-1-2)\hat{\underline{i}}+(1-5)\hat{\underline{j}}=-3\hat{\underline{i}}-4\hat{\underline{j}}$

(ii) From above $\overrightarrow{AB}=-3\hat{\underline{i}}-4\hat{\underline{j}}$

Also $\overrightarrow{CB}=(2+1)\hat{\underline{i}}+(-6-1)\hat{\underline{j}}=3\hat{\underline{i}}-7\hat{\underline{j}}$

Now
$$2\overrightarrow{AB}-\overrightarrow{CB}=2\left(-3\hat{\underline{i}}-4\hat{\underline{j}}\right)-\left(3\hat{\underline{i}}-7\hat{\underline{j}}\right)$$
$$=-6\hat{\underline{i}}-8\hat{\underline{j}}-3\hat{\underline{i}}+7\hat{\underline{j}}$$
$$=-9\hat{\underline{i}}-\hat{\underline{j}}$$

(iii) *Do yourself as above*

Question # 2
Let $\underline{u}=\hat{\underline{i}}+2\hat{\underline{j}}-\hat{\underline{k}}$, $\underline{v}=3\hat{\underline{i}}-2\hat{\underline{j}}+2\hat{\underline{k}}$, $\underline{w}=5\hat{\underline{i}}-\hat{\underline{j}}+3\hat{\underline{k}}$, Find the indicated vector or number.

Solution
(i) $\underline{u}=\hat{\underline{i}}+2\hat{\underline{j}}-\hat{\underline{k}}$
$\underline{v}=3\hat{\underline{i}}-2\hat{\underline{j}}+2\hat{\underline{k}}$
$\underline{w}=5\hat{\underline{i}}-\hat{\underline{j}}+3\hat{\underline{k}}$

$$\underline{u}+2\underline{v}+\underline{w}=\hat{\underline{i}}+2\hat{\underline{j}}-\hat{\underline{k}}+2\left(3\hat{\underline{i}}-2\hat{\underline{j}}+2\hat{\underline{k}}\right)+\left(5\hat{\underline{i}}-\hat{\underline{j}}+3\hat{\underline{k}}\right)$$
$$=\hat{\underline{i}}+2\hat{\underline{j}}-\hat{\underline{k}}+6\hat{\underline{i}}-4\hat{\underline{j}}+4\hat{\underline{k}}+5\hat{\underline{i}}-\hat{\underline{j}}+3\hat{\underline{k}}$$
$$=12\hat{\underline{i}}-3\hat{\underline{j}}+6\hat{\underline{k}}$$

(ii) *Do yourself*

(iii) $$3\underline{v}+\underline{w}=3\left(3\hat{\underline{i}}-2\hat{\underline{j}}+2\hat{\underline{k}}\right)+5\hat{\underline{i}}-\hat{\underline{j}}+3\hat{\underline{k}}$$
$$=9\hat{\underline{i}}-6\hat{\underline{j}}+6\hat{\underline{k}}+5\hat{\underline{i}}-\hat{\underline{j}}+3\hat{\underline{k}}$$
$$=14\hat{\underline{i}}-7\hat{\underline{j}}+9\hat{\underline{k}}$$

Now $|3\underline{v}+\underline{w}|=\sqrt{(14)^2+(-7)^2+(9)^2}$
$$=\sqrt{196+49+81}=\sqrt{326}$$

Question # 3
Find the magnitude of the vector $\underline{v}$ and write the direction cosines of $\underline{v}$
(i) $\underline{v}=2\hat{\underline{i}}+3\hat{\underline{j}}+4\hat{\underline{k}}$ (ii) $\underline{v}=\hat{\underline{i}}-\hat{\underline{j}}-\hat{\underline{k}}$
(iii) $\underline{v}=4\hat{\underline{i}}-5\hat{\underline{j}}$.

Solution
(i) $\underline{v}=2\hat{\underline{i}}+3\hat{\underline{j}}+4\hat{\underline{k}}$
$$\Rightarrow |\underline{v}|=\sqrt{(2)^2+(3)^2+(4)^2}$$
$$=\sqrt{4+9+16}=\sqrt{29}$$

Unit vector of $\underline{v}=\hat{\underline{v}}=\dfrac{\underline{v}}{|\underline{v}|}=\dfrac{2\hat{\underline{i}}+3\hat{\underline{j}}+4\hat{\underline{k}}}{\sqrt{29}}$
$$=\frac{2}{\sqrt{29}}\hat{\underline{i}}+\frac{3}{\sqrt{29}}\hat{\underline{j}}+\frac{4}{\sqrt{29}}\hat{\underline{k}}$$

Hence direction cosines of $\underline{v}$ are
$$\frac{2}{\sqrt{29}},\frac{3}{\sqrt{29}},\frac{4}{\sqrt{29}}.$$

(ii) *Do yourself as above.*

(iii) *Do yourself as (i)*

Question # 4
Find α, so that $\left|\alpha\hat{\underline{i}}+(\alpha+1)\hat{\underline{j}}+2\hat{\underline{k}}\right|=3$

Solution
Since $\left|\alpha\hat{\underline{i}}+(\alpha+1)\hat{\underline{j}}+2\hat{\underline{k}}\right|=3$
$$\Rightarrow \sqrt{\alpha^2+(\alpha+1)^2+(2)^2}=3$$
$$\Rightarrow \sqrt{\alpha^2+\alpha^2+2\alpha+1+4}=3$$

On squaring both sides
$$2\alpha^2+2\alpha+5=9$$
$$\Rightarrow 2\alpha^2+2\alpha+5-9=0$$
$$\Rightarrow 2\alpha^2+2\alpha-4=0$$
$$\Rightarrow \alpha^2+\alpha-2=0$$
$$\Rightarrow \alpha^2+2\alpha-\alpha-2=0$$
$$\Rightarrow \alpha(\alpha+2)-1(\alpha+2)=0$$
$$\Rightarrow (\alpha+2)(\alpha-1)=0$$

$\Rightarrow \ \alpha+2=0$ or $\alpha-1=0$

$\Rightarrow \ \alpha=-2$ or $\alpha=1$

Question # 5

Find a unit vector in the direction of

$\underline{v}=\underline{\hat{i}}+2\underline{\hat{j}}-\underline{\hat{k}}$

Solution

Given $\underline{v}=\underline{\hat{i}}+2\underline{\hat{j}}-\underline{\hat{k}}$

$$|\underline{v}|=\sqrt{(1)^2+(2)^2+(-1)^2}$$
$$=\sqrt{1+4+1}=\sqrt{6}$$

Now

$$\underline{\hat{v}}=\frac{\underline{v}}{|\underline{v}|}=\frac{\underline{\hat{i}}+2\underline{\hat{j}}-\underline{\hat{k}}}{\sqrt{6}}$$
$$=\frac{1}{\sqrt{6}}\underline{\hat{i}}+\frac{2}{\sqrt{6}}\underline{\hat{j}}-\frac{1}{\sqrt{6}}\underline{\hat{k}}$$

Question # 6

If $\underline{a}=3\underline{\hat{i}}-\underline{\hat{j}}-4\underline{\hat{k}}$, $\underline{b}=-2\underline{\hat{i}}-4\underline{\hat{j}}-3\underline{\hat{k}}$ and $\underline{c}=\underline{\hat{i}}+2\underline{\hat{j}}-\underline{\hat{k}}$. Find a unit vector parallel to $3\underline{a}-2\underline{b}+4\underline{c}$.

Solution

Given $\underline{a}=3\underline{\hat{i}}-\underline{\hat{j}}-4\underline{\hat{k}}$

$\underline{b}=-2\underline{\hat{i}}-4\underline{\hat{j}}-3\underline{\hat{k}}$

$\underline{c}=\underline{\hat{i}}+2\underline{\hat{j}}-\underline{\hat{k}}$

Suppose that

$\underline{d}=3\underline{a}-2\underline{b}+4\underline{c}$

$$\Rightarrow \ \underline{d}=3\left(3\underline{\hat{i}}-\underline{\hat{j}}-4\underline{\hat{k}}\right)-2\left(-2\underline{\hat{i}}-4\underline{\hat{j}}-3\underline{\hat{k}}\right)+4\left(\underline{\hat{i}}+2\underline{\hat{j}}-\underline{\hat{k}}\right)$$
$$=9\underline{\hat{i}}-3\underline{\hat{j}}-12\underline{\hat{k}}+4\underline{\hat{i}}+8\underline{\hat{j}}+6\underline{\hat{k}}+4\underline{\hat{i}}+8\underline{\hat{j}}-4\underline{\hat{k}}$$
$$=17\underline{\hat{i}}+13\underline{\hat{j}}-10\underline{\hat{k}}$$

Now

$$|\underline{d}|=\sqrt{(17)^2+(-13)^2+(-10)^2}$$
$$=\sqrt{289+169+100}=\sqrt{558}=3\sqrt{62}$$

Now

$$\underline{\hat{d}}=\frac{\underline{d}}{|\underline{d}|}=\frac{17\underline{\hat{i}}+13\underline{\hat{j}}-10\underline{\hat{k}}}{3\sqrt{62}}$$
$$=\frac{17}{3\sqrt{62}}\underline{\hat{i}}+\frac{13}{3\sqrt{62}}\underline{\hat{j}}-\frac{10}{3\sqrt{62}}\underline{\hat{k}}.$$

Question # 7

Find a vector whose

(i) Magnitude is 4 and is parallel to $\underline{a}=2\underline{\hat{i}}-3\underline{\hat{j}}+6\underline{\hat{k}}$

(ii) Magnitude is 2 and is parallel to $\underline{a}=-\underline{\hat{i}}+\underline{\hat{j}}+\underline{\hat{k}}$

Solution

Consider $\underline{a}=2\underline{\hat{i}}-3\underline{\hat{j}}+6\underline{\hat{k}}$

$$|\underline{a}|=\sqrt{(2)^2+(-3)^2+(6)^2}$$
$$=\sqrt{4+9+36}=\sqrt{49}=7$$

Now

$$\underline{\hat{a}}=\frac{\underline{a}}{|\underline{a}|}=\frac{2\underline{\hat{i}}-3\underline{\hat{j}}+6\underline{\hat{k}}}{7}$$
$$=\frac{2}{7}\underline{\hat{i}}-\frac{3}{7}\underline{\hat{j}}+\frac{6}{7}\underline{\hat{k}}$$

Let $\underline{b}$ be a vector having magnitude 4

i.e. $|\underline{b}|=4$

Since $\underline{b}$ is parallel to $\underline{a}$

1. therefore $\underline{\hat{b}}=\underline{\hat{a}}=\frac{2}{7}\underline{\hat{i}}-\frac{3}{7}\underline{\hat{j}}+\frac{6}{7}\underline{\hat{k}}$

Now $\underline{b}=|\underline{b}|\underline{\hat{b}}=4\left(\frac{2}{7}\underline{\hat{i}}-\frac{3}{7}\underline{\hat{j}}+\frac{6}{7}\underline{\hat{k}}\right)$

$$=\frac{8}{7}\underline{\hat{i}}-\frac{12}{7}\underline{\hat{j}}+\frac{24}{7}\underline{\hat{k}}$$

(ii) *Do yourself.*

Question # 8

If $\underline{u}=2\underline{\hat{i}}+3\underline{\hat{j}}+4\underline{\hat{k}}$, $\underline{v}=-\underline{\hat{i}}+3\underline{\hat{j}}-\underline{\hat{k}}$ and $\underline{w}=\underline{\hat{i}}+6\underline{\hat{j}}+z\underline{\hat{k}}$ represent the sides of a triangle. Find the value of z.

Solution

Given $\underline{u}=2\underline{\hat{i}}+3\underline{\hat{j}}+4\underline{\hat{k}}$

$\underline{v}=-\underline{\hat{i}}+3\underline{\hat{j}}-\underline{\hat{k}}$

$\underline{w}=\underline{\hat{i}}+6\underline{\hat{j}}+z\underline{\hat{k}}$

Since $\underline{u}$, $\underline{v}$ and $\underline{w}$ are sides of triangle therefore

$\underline{u}+\underline{v}=\underline{w}$

$$\Rightarrow \ 2\underline{\hat{i}}+3\underline{\hat{j}}+4\underline{\hat{k}}-\underline{\hat{i}}+3\underline{\hat{j}}-\underline{\hat{k}}=\underline{\hat{i}}+6\underline{\hat{j}}+z\underline{\hat{k}}$$
$$\Rightarrow \ \underline{\hat{i}}+6\underline{\hat{j}}+3\underline{\hat{k}}=\underline{\hat{i}}+6\underline{\hat{j}}+z\underline{\hat{k}}$$

Equating coefficient of $\underline{\hat{k}}$ only, we have

$3=z$ i.e. $\boxed{z=3}$

Question # 9
The position vectors of the points A, B, C and D are $2\hat{\underline{i}} - \hat{\underline{j}} + \hat{\underline{k}}$, $3\hat{\underline{i}} + \hat{\underline{j}}$, $2\hat{\underline{i}} + 4\hat{\underline{j}} - 2\hat{\underline{k}}$ and $-\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}}$ respectively. Show that $\left|\overline{AB}\right|$ is parallel to $\left|\overline{CD}\right|$.

Solution

Position vector (p.v) of point $A = 2\hat{\underline{i}} - \hat{\underline{j}} + \hat{\underline{k}}$

p.v of point $B = 3\hat{\underline{i}} + \hat{\underline{j}}$

p.v. of point $C = 2\hat{\underline{i}} + 4\hat{\underline{j}} - 2\hat{\underline{k}}$

p.v. of point $D = -\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}}$

$\overrightarrow{AB} =$ p.v. of $B -$ p.v. of A

$= 3\hat{\underline{i}} + \hat{\underline{j}} - 2\hat{\underline{i}} + \hat{\underline{j}} - \hat{\underline{k}} = \hat{\underline{i}} + 2\hat{\underline{j}} - \hat{\underline{k}}$

$\overrightarrow{CD}$ = p.v. of $D -$ p.v. of C

$= -\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}} - 2\hat{\underline{i}} - 4\hat{\underline{j}} + 2\hat{\underline{k}}$

$= -3\hat{\underline{i}} - 6\hat{\underline{j}} + 3\hat{\underline{k}}$

$= -3\left(\hat{\underline{i}} + 2\hat{\underline{j}} - \hat{\underline{k}}\right) = -3\overrightarrow{AB}$

i.e. $\overrightarrow{CD} = \lambda\overrightarrow{AB}$ where $\lambda = -3$

Hence $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

Question # 10
We say that two vectors $\underline{v}$ and $\underline{w}$ in space are parallel if there is a scalar c such that $\underline{v} = c\underline{w}$. The vector point in the same direction if $c > 0$ and the vectors point in the opposite direction if $c < 0$

(a) Find two vectors of length 2 parallel to the vector $\underline{v} = 2\hat{\underline{i}} - 4\hat{\underline{j}} + 4\hat{\underline{k}}$

(b) Find the constant a so that the vectors $\underline{v} = \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}}$ and $\underline{w} = a\hat{\underline{i}} + 9\hat{\underline{j}} - 12\hat{\underline{k}}$ are parallel.

(c) Find a vector of length 5 in the direction opposite that of $\underline{v} = \hat{\underline{i}} - 2\hat{\underline{j}} + 3\hat{\underline{k}}$

(d) Find a and b so that the vectors $3\hat{\underline{i}} - \hat{\underline{j}} + 4\hat{\underline{k}}$ and $a\hat{\underline{i}} + b\hat{\underline{j}} - 2\hat{\underline{k}}$ are parallel.

Solution

(a) $\underline{v} = 2\hat{\underline{i}} - 4\hat{\underline{j}} + 4\hat{\underline{k}}$

$$|\underline{v}| = \sqrt{(2)^2 + (-4)^2 + (4)^2}$$
$$= \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

Now $\hat{\underline{v}} = \dfrac{\underline{v}}{|\underline{v}|} = \dfrac{2\hat{\underline{i}} - 4\hat{\underline{j}} + 4\hat{\underline{k}}}{6}$

$$= \frac{1}{3}\hat{\underline{i}} - \frac{2}{3}\hat{\underline{j}} + \frac{2}{3}\hat{\underline{k}}$$

The two vectors of length 2 and parallel to $\underline{v}$ are $2\hat{\underline{v}}$ and $-2\hat{\underline{v}}$.

$$2\hat{\underline{v}} = 2\left(\frac{1}{3}\hat{\underline{i}} - \frac{2}{3}\hat{\underline{j}} + \frac{2}{3}\hat{\underline{k}}\right) = \frac{2}{3}\hat{\underline{i}} - \frac{4}{3}\hat{\underline{j}} + \frac{4}{3}\hat{\underline{k}}$$
$$-2\hat{\underline{v}} = -2\left(\frac{1}{3}\hat{\underline{i}} - \frac{2}{3}\hat{\underline{j}} + \frac{2}{3}\hat{\underline{k}}\right) = -\frac{2}{3}\hat{\underline{i}} + \frac{4}{3}\hat{\underline{j}} - \frac{4}{3}\hat{\underline{k}}$$

(b) Given $\underline{v} = \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}}$, $\underline{w} = a\hat{\underline{i}} + 9\hat{\underline{j}} - 12\hat{\underline{k}}$

Since $\underline{v}$ and $\underline{w}$ are parallel therefore there exists $\lambda \in \mathbb{R}$ such that

$$\underline{v} = \lambda\underline{w}$$
$$\Rightarrow \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}} = \lambda\left(a\hat{\underline{i}} + 9\hat{\underline{j}} - 12\hat{\underline{k}}\right)$$
$$\Rightarrow \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}} = a\lambda\hat{\underline{i}} + 9\lambda\hat{\underline{j}} - 12\lambda\hat{\underline{k}}$$

Comparing coefficients of $\hat{\underline{i}}$, $\hat{\underline{j}}$ and $\hat{\underline{k}}$

$1 = a\lambda$ ... (i)

$-3 = 9\lambda$ ... (ii)

$4 = -12\lambda$ ... (iii)

From (ii) $\lambda = -\dfrac{3}{9} \Rightarrow \lambda = -\dfrac{1}{3}$

Putting in equation (i)

$$1 = a\left(-\frac{1}{3}\right) \Rightarrow -3 = a \quad \text{i.e.} \quad \boxed{a = -3}$$

(c) Consider $\underline{v} = \hat{\underline{i}} - 2\hat{\underline{j}} + 3\hat{\underline{k}}$

$$|\underline{v}| = \sqrt{(1)^2 + (-2)^2 + (3)^2}$$
$$= \sqrt{1 + 4 + 9} = \sqrt{14}$$

Now

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{\hat{\underline{i}} - 2\hat{\underline{j}} + 3\hat{\underline{k}}}{\sqrt{14}}$$
$$= \frac{1}{\sqrt{14}}\hat{\underline{i}} - \frac{2}{\sqrt{14}}\hat{\underline{j}} + \frac{3}{\sqrt{14}}\hat{\underline{k}}$$

Let $\underline{a}$ be a vector having magnitude 5 i.e. $|\underline{a}| = 5$

Since $\underline{a}$ is parallel to $\underline{v}$ but opposite in direction,
therefore

$$\hat{\underline{a}} = -\hat{\underline{v}} = -\frac{1}{\sqrt{14}}\hat{\underline{i}} + \frac{2}{\sqrt{14}}\hat{\underline{j}} - \frac{3}{\sqrt{14}}\hat{\underline{k}}$$

Now $\underline{a} = |\underline{a}|\hat{\underline{a}}$

$= 5\left(-\frac{1}{\sqrt{14}}\hat{\underline{i}} + \frac{2}{\sqrt{14}}\hat{\underline{j}} - \frac{3}{\sqrt{14}}\hat{\underline{k}}\right)$

$= -\frac{5}{\sqrt{14}}\hat{\underline{i}} + \frac{10}{\sqrt{14}}\hat{\underline{j}} - \frac{15}{\sqrt{14}}\hat{\underline{k}}.$

(d) Suppose that $\underline{v} = 3\hat{\underline{i}} - \hat{\underline{j}} + 4\hat{\underline{k}}$ and $\underline{w} = a\hat{\underline{i}} + b\hat{\underline{j}} - 2\hat{\underline{k}}$

$\because$ $\underline{v}$ and $\underline{w}$ are parallel

$\therefore$ there exists $\lambda \in \mathbb{R}$ such that

$\underline{v} = \lambda \underline{w}$

$\Rightarrow 3\hat{\underline{i}} - \hat{\underline{j}} + 4\hat{\underline{k}} = \lambda\left(a\hat{\underline{i}} + b\hat{\underline{j}} - 2\hat{\underline{k}}\right)$

$\Rightarrow 3\hat{\underline{i}} - \hat{\underline{j}} + 4\hat{\underline{k}} = a\lambda\hat{\underline{i}} + b\lambda\hat{\underline{j}} - 2\lambda\hat{\underline{k}}$

Comparing coefficients of $\hat{\underline{i}}$, $\hat{\underline{j}}$ and $\hat{\underline{k}}$

$3 = a\lambda \ldots$ (i)

$-1 = b\lambda \ldots$ (ii)

$4 = -2\lambda \ldots$ (iii)

From equation (iii)

$-\frac{4}{2} = \lambda \quad \Rightarrow \quad \lambda = -2$

Putting value of λ in equation (i)

$3 = a(-2) \quad \Rightarrow \quad \boxed{a = -\frac{3}{2}}$

Putting value of λ in equation (ii)

$-1 = b(-2) \quad \Rightarrow \quad \boxed{b = \frac{1}{2}}$

Question # 11

Find the direction cosines for the given vector:

(i) $\underline{v} = 3\hat{\underline{i}} - \hat{\underline{j}} + 2\hat{\underline{k}}$ (ii) $\underline{v} = 6\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}}$

(iii) $\overrightarrow{PQ}$, where $P = (2,1,5)$ and $Q = (1,3,1)$.

Solution

(i) $\underline{v} = 3\hat{\underline{i}} - \hat{\underline{j}} + 2\hat{\underline{k}}$

$|\underline{v}| = \sqrt{(3)^2 + (-1)^2 + (2)^2}$

$= \sqrt{9+1+4} = \sqrt{14}$

Let $\hat{\underline{v}}$ be unit vector along $\underline{v}$. Then

$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{3\hat{\underline{i}} - \hat{\underline{j}} + 2\hat{\underline{k}}}{\sqrt{14}}$

$= \frac{3}{\sqrt{14}}\hat{\underline{i}} - \frac{1}{\sqrt{14}}\hat{\underline{j}} + \frac{2}{\sqrt{14}}\hat{\underline{k}}$

$\hat{\underline{v}} = \left[\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right]$

Hence the direction cosines of $\underline{v}$ are

$\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}.$

(ii) $\underline{v} = 6\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}}$

$|\underline{v}| = \sqrt{(6)^2 + (-2)^2 + (1)^2}$

$= \sqrt{36+4+1} = \sqrt{41}$

Let $\hat{\underline{v}}$ be unit vector along $\underline{v}$. Then

$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{6\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}}}{\sqrt{41}}$

$= \frac{6}{\sqrt{41}}\hat{\underline{i}} - \frac{2}{\sqrt{41}}\hat{\underline{j}} + \frac{1}{\sqrt{41}}\hat{\underline{k}}$

$\hat{\underline{v}} = \left[\frac{6}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{1}{\sqrt{41}}\right]$

Hence the direction cosines of $\underline{v}$ are

$\frac{6}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{1}{\sqrt{41}}.$

(iii) $P = (2,1,5)$, $Q = (1,3,1)$

$\overrightarrow{PQ} = (1-2)\hat{\underline{i}} + (3-1)\hat{\underline{j}} + (1-5)\hat{\underline{k}}$

$= -\hat{\underline{i}} + 2\hat{\underline{j}} - 4\hat{\underline{k}}$

$\left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + (2)^2 + (-4)^2}$

$= \sqrt{1+4+16} = \sqrt{21}$

Let $\hat{\underline{v}}$ be unit vector along $\overrightarrow{PQ}$. Then

$\hat{\underline{v}} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{-\hat{\underline{i}} + 2\hat{\underline{j}} - 4\hat{\underline{k}}}{\sqrt{21}}$

$= \frac{-1}{\sqrt{21}}\hat{\underline{i}} + \frac{2}{\sqrt{21}}\hat{\underline{j}} - \frac{4}{\sqrt{21}}\hat{\underline{k}}$

$\hat{\underline{v}} = \left[\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}\right]$

Hence the direction cosines of $\overrightarrow{PQ}$ are

$\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}.$

Question # 12

Which of the following triples can be the direction angles of a single vector:

(i) $45^\circ, 45^\circ, 60^\circ$ (ii) $30^\circ, 45^\circ, 60^\circ$

(iii) $45^\circ, 60^\circ, 60^\circ$

Solution

(i)

$45^\circ, 45^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ = 1$$

$$\text{L.H.S} = \cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = \frac{5}{4} \neq \text{R.H.S}$$

Therefore given angles are not direction angles.

(ii) $30^\circ, 45^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ = 1$$

$$\text{L.H.S} = \cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2} \neq \text{R.H.S}$$

Therefore given angles are not direction angles.

(iii) $30^\circ, 60^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 45^\circ + \cos^2 60^\circ + \cos^2 60^\circ = 1$$

$$\text{L.H.S} = \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 60^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1 = \text{R.H.S}$$

Therefore given angles are direction angles.

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Book: ***Exercise 7.2, page 341***
Calculus and Analytic Geometry
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