

## EXERCISE 7.1

① (i)  $P(2,3), Q(6,-2)$   
 $\vec{PQ} = \vec{OQ} - \vec{OP} = [6, -2] - [2, 3]$   
 $= [6-2, -2-3]$   
 $= [4, -5] = 4\hat{i} - 5\hat{j}$  Ans.

Method II

$\vec{PQ} = (6-2)\hat{i} + (-2-3)\hat{j}$   
 $\vec{PQ} = 4\hat{i} - 5\hat{j}$  Ans.

(ii)  $P(0,5), Q(-1,-6)$

$\vec{PQ} = (-1-0)\hat{i} + (-6-5)\hat{j}$   
 $\vec{PQ} = -\hat{i} - 11\hat{j}$  Ans.

② (i) Given that  $\underline{u} = 2\hat{i} - 7\hat{j}$   
 $|\underline{u}| = \sqrt{(2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$  Ans.

(ii)  $\underline{u} = \hat{i} + \hat{j}$

$|\underline{u}| = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$  Ans.

(iii)  $\underline{u} = [3, -4]$

$|\underline{u}| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$  Ans.

③ Given that  $\underline{u} = 2\hat{i} - 7\hat{j}, \underline{v} = \hat{i} - 6\hat{j}$

$\underline{w} = -\hat{i} + \hat{j}$

(i)  $\underline{u} + \underline{v} - \underline{w} = (2\hat{i} - 7\hat{j}) + (\hat{i} - 6\hat{j}) - (-\hat{i} + \hat{j})$   
 $= 2\hat{i} - 7\hat{j} + \hat{i} - 6\hat{j} + \hat{i} - \hat{j}$   
 $= 4\hat{i} - 14\hat{j}$  Ans.

(ii)  $2\underline{u} - 3\underline{v} + 4\underline{w}$

$= 2(2\hat{i} - 7\hat{j}) - 3(\hat{i} - 6\hat{j}) + 4(-\hat{i} + \hat{j})$   
 $= 4\hat{i} - 14\hat{j} - 3\hat{i} + 18\hat{j} - 4\hat{i} + 4\hat{j}$   
 $= -3\hat{i} + 8\hat{j}$  Ans.

(iii)  $\frac{1}{2}\underline{u} + \frac{1}{2}\underline{v} + \frac{1}{2}\underline{w}$

$= \frac{1}{2}(2\hat{i} - 7\hat{j}) + \frac{1}{2}(\hat{i} - 6\hat{j}) + \frac{1}{2}(-\hat{i} + \hat{j})$   
 $= \hat{i} - \frac{7}{2}\hat{j} + \frac{1}{2}\hat{i} - 3\hat{j} - \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$   
 $= (1 + \frac{1}{2} - \frac{1}{2})\hat{i} + (-\frac{7}{2} - 3 + \frac{1}{2})\hat{j}$   
 $= \hat{i} - 6\hat{j}$  Ans.

④ Given that  $A(1,-1), B(2,0)$   
 $C(-1,3)$  and  $D(2,2)$

$\vec{AB} + \vec{CD} = (2-1)\hat{i} + (0+1)\hat{j} + (-2+1)\hat{i} + (2-3)\hat{j}$   
 $= \hat{i} + \hat{j} - \hat{i} - \hat{j} = 0\hat{i} + 0\hat{j} = \underline{0}$

⑤ Given that  $\vec{AB} = 4\hat{i} - 2\hat{j}$

$B(-2,5), O(0,0)$

$\therefore \vec{AB} = \vec{OB} - \vec{OA}$

$\Rightarrow \vec{AB} = \vec{OB} + \vec{AO} \Rightarrow \vec{AB} - \vec{OB} = \vec{AO}$

$\Rightarrow \vec{AO} = \vec{AB} - \vec{OB}$

$= (4\hat{i} - 2\hat{j}) - (-2\hat{i} + 5\hat{j})$

$= 4\hat{i} - 2\hat{j} + 2\hat{i} - 5\hat{j}$

$\vec{AO} = 6\hat{i} - 7\hat{j}$  Ans.

⑥ (i) Given that  $\underline{v} = 2\hat{i} - \hat{j}$

$|\underline{v}| = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

Let  $\hat{v}$  be a unit vector along  $\underline{v}$

$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{2\hat{i} - \hat{j}}{\sqrt{5}} = \frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}$  Ans.

(ii)  $\underline{v} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

$|\underline{v}| = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{1+3}{4}}$

$|\underline{v}| = \sqrt{\frac{4}{4}} = \sqrt{1} = 1$

Let  $\hat{v}$  be a unit vector along  $\underline{v}$ , then

$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}}{1} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$  Ans.

(iii)  $\underline{v} = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$

$|\underline{v}| = \sqrt{(-\frac{\sqrt{3}}{2})^2 + (-\frac{1}{2})^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{\frac{4}{4}} = 1$

Let  $\hat{v}$  be a unit vector along  $\underline{v}$ , then

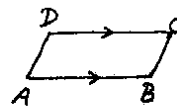
$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}}{1} = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$  Ans.

⑦ (i) Given that  $A(2,-4), B(4,0), C(1,6)$

Let  $D(x,y)$  be the required point.

(i) Given that ABCD is a  $\parallel\text{gm}$ .

$\therefore \vec{AB} = \vec{DC}$



$$\Rightarrow (4-2)\underline{i} + (0+4)\underline{j} = (1-x)\underline{i} + (6-y)\underline{j} \quad [6]$$

$$\Rightarrow 2\underline{i} + 4\underline{j} = (1-x)\underline{i} + (6-y)\underline{j}$$

$$\Rightarrow 2 = 1-x \quad \& \quad 4 = 6-y$$

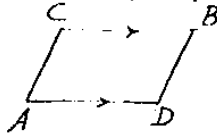
$$\Rightarrow x = 1-2 \quad \& \quad y = 6-4$$

$$\Rightarrow \boxed{x = -1} \quad \boxed{y = 2}$$

$\therefore D(-1, 2)$  Ans.

(ii) Given that ADBC is a  $\parallel gm$ .

$\therefore \vec{AD} = \vec{CB}$



$$\Rightarrow (x-2)\underline{i} + (y+4)\underline{j} = (4-1)\underline{i} + (0-6)\underline{j}$$

$$\Rightarrow (x-2)\underline{i} + (y+4)\underline{j} = 3\underline{i} + (-6)\underline{j}$$

$$\Rightarrow x-2 = 3 \quad \& \quad y+4 = -6$$

$$\Rightarrow x = 3+2 \quad \& \quad y = -6-4$$

$$\Rightarrow x = 5 \quad \& \quad y = -10$$

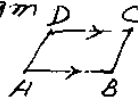
$\therefore D(5, -10)$  Ans.

(iii) Given that B(4,1), C(-2,3) & D(-8,0)

Let A(x,y) be the required point.

Given that ABCD is a  $\parallel gm$

$\therefore \vec{AB} = \vec{DC}$



$$\Rightarrow (4-x)\underline{i} + (1-y)\underline{j} = (-2+8)\underline{i} + (3-0)\underline{j}$$

$$\Rightarrow (4-x)\underline{i} + (1-y)\underline{j} = 6\underline{i} + 3\underline{j}$$

$$\Rightarrow 4-x = 6 \quad \& \quad 1-y = 3$$

$$\Rightarrow -x = 6-4 \quad , \quad -y = 3-1$$

$$\Rightarrow -x = 2 \quad , \quad -y = 2$$

$$\Rightarrow x = -2 \quad , \quad y = -2$$

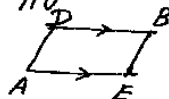
$\therefore A(-2, -2)$

(ii) Given that B(4,1), C(-2,3) and D(-8,0). A(-2,-2)

Let E(x,y) be the required point.

Given that AEBD is a  $\parallel gm$ .

$\therefore \vec{AE} = \vec{DB}$



$$\Rightarrow (x+2)\underline{i} + (y+2)\underline{j} = (4+8)\underline{i} + (1-0)\underline{j}$$

$$\Rightarrow (x+2)\underline{i} + (y+2)\underline{j} = 12\underline{i} + \underline{j}$$

$$\Rightarrow x+2 = 12 \quad \& \quad y+2 = 1$$

$$\Rightarrow x = 10 \quad , \quad y = -1$$

$\therefore E(10, -1)$  Ans.

(9) Given that

O(0,0), A(-3,7), B(1,0)

Also  $\vec{OP} = \vec{AB}$

$\Rightarrow$  Let P(x,y) be the required point.

$\therefore \vec{OP} = \vec{AB}$

$$\Rightarrow (x-0)\underline{i} + (y-0)\underline{j} = (1+3)\underline{i} + (0-7)\underline{j}$$

$$\Rightarrow x\underline{i} + y\underline{j} = 4\underline{i} + (-7)\underline{j}$$

$$\Rightarrow x = 4 \quad \& \quad y = -7$$

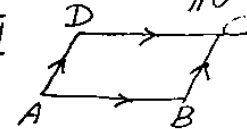
$\therefore P(4, -7)$  is the required point.

(10) Given that A(0,0), B(a,0), C(b,c)

and D(b-a, c)

To prove that ABCD is a  $\parallel gm$ .

$\vec{AB} = (a-0)\underline{i} + (0-0)\underline{j}$



$\vec{AB} = a\underline{i}$

$\vec{DC} = (b-b+a)\underline{i} + (c-c)\underline{j}$

$\Rightarrow \vec{DC} = a\underline{i}$

$\vec{AD} = (b-a-0)\underline{i} + (c-0)\underline{j}$

$\Rightarrow \vec{AD} = (b-a)\underline{i} + c\underline{j}$

$\vec{BC} = (b-a)\underline{i} + (c-0)\underline{j}$

$\Rightarrow \vec{BC} = (b-a)\underline{i} + c\underline{j}$

we see that

$\vec{AB} = \vec{DC}$  and  $\vec{AD} = \vec{BC}$

$\therefore ABCD$  is a  $\parallel gm$ .

(11) Given that B(1,2), C(-2,5) and

D(4,11). and  $\vec{AB} = \vec{CD}$

Let A(x,y).

$\therefore \vec{AB} = \vec{CD}$

$$\Rightarrow (1-x)\underline{i} + (2-y)\underline{j} = (4+2)\underline{i} + (11-5)\underline{j}$$

$$\Rightarrow (1-x)\underline{i} + (2-y)\underline{j} = 6\underline{i} + 6\underline{j}$$

$$\Rightarrow 1-x = 6 \quad \& \quad 2-y = 6$$

$$\Rightarrow -x = 6-1 \quad \& \quad -y = 6-2$$

$$\Rightarrow -x = 5 \quad \& \quad -y = 4$$

$$\Rightarrow x = -5 \quad \& \quad y = -4$$

$\therefore A(-5, -4)$  Ans.

⑫ (i) Given that

P.V. of C =  $2\hat{i} - 3\hat{j}$

P.V. of D =  $3\hat{i} + 2\hat{j}$

Let P.V. of P =  $\underline{r}$

Let P divides CD in the ratio 4:3

Then  $\underline{r} = \frac{4(3\hat{i} + 2\hat{j}) + 3(2\hat{i} - 3\hat{j})}{4+3}$

$\underline{r} = \frac{12\hat{i} + 8\hat{j} + 6\hat{i} - 9\hat{j}}{7}$

$\underline{r} = \frac{18\hat{i} - \hat{j}}{7}$

$\underline{r} = \frac{18}{7}\hat{i} - \frac{1}{7}\hat{j}$  Ans.

(ii) Given that

P.V. of E =  $5\hat{i}$

P.V. of F =  $4\hat{i} + \hat{j}$

Let P.V. of P =  $\underline{r}$

Let P divides EF in the ratio 2:5

Then  $\underline{r} = \frac{2(4\hat{i} + \hat{j}) + 5(5\hat{i})}{2+5}$

$\underline{r} = \frac{2(4\hat{i} + \hat{j}) + 5(5\hat{i})}{2+5}$

$\underline{r} = \frac{8\hat{i} + 2\hat{j} + 25\hat{i}}{7} = \frac{33\hat{i} + 2\hat{j}}{7}$

$\underline{r} = \frac{33}{7}\hat{i} + \frac{2}{7}\hat{j}$  Ans.

⑭ Let ABC be the triangle in which

$\vec{OA} = \underline{a}$

$\vec{OB} = \underline{b}$

$\vec{OC} = \underline{c}$ , where O is the origin.

Let D & E be the mid points of AB and AC respectively. Then

P.V. of D =  $\vec{OD} = \frac{\underline{a} + \underline{b}}{2}$  and

P.V. of E =  $\vec{OE} = \frac{\underline{a} + \underline{c}}{2}$

To prove that

$\vec{DE} \parallel \vec{BC}$  and  $|\vec{DE}| = \frac{1}{2} |\vec{BC}|$

Now  $\vec{DE} = \vec{OE} - \vec{OD} = \frac{\underline{a} + \underline{c}}{2} - \frac{\underline{a} + \underline{b}}{2}$

$\vec{DE} = \frac{\underline{a} + \underline{c} - \underline{a} - \underline{b}}{2} = \frac{\underline{c} - \underline{b}}{2}$

∴

$\vec{DE} = \frac{\underline{c} - \underline{b}}{2}$  — ①

$\vec{BC} = \vec{OC} - \vec{OB}$

$\Rightarrow \vec{BC} = \underline{c} - \underline{b}$  — ②

Using ② in ①, we get

$\vec{DE} = \frac{\vec{BC}}{2} \Rightarrow \vec{DE} = \frac{1}{2} \vec{BC}$

This shows that

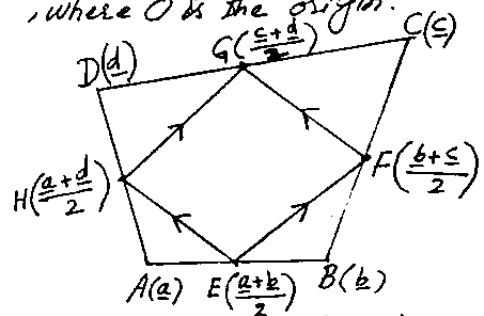
$\vec{DE} \parallel \vec{BC}$  and  $|\vec{DE}| = \frac{1}{2} |\vec{BC}|$   
(Proved)

⑮ Let ABCD be the quadrilateral in which  $\vec{OA} = \underline{a}$

$\vec{OB} = \underline{b}$

$\vec{OC} = \underline{c}$

$\vec{OD} = \underline{d}$ , where O is the origin.



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Let E, F, G and H be the mid points of the sides AB, BC, CD and AD respectively. Then  $\vec{OE} = \frac{\underline{a} + \underline{b}}{2}$

$\vec{OF} = \frac{\underline{b} + \underline{c}}{2}$ ,  $\vec{OG} = \frac{\underline{c} + \underline{d}}{2}$ ,  $\vec{OH} = \frac{\underline{a} + \underline{d}}{2}$

To prove that EFGH is a //gm.

$\vec{EF} = \vec{OF} - \vec{OE} = \frac{\underline{b} + \underline{c}}{2} - \frac{\underline{a} + \underline{b}}{2} = \frac{\underline{b} + \underline{c} - \underline{a} - \underline{b}}{2}$

$\vec{EF} = \frac{\underline{c} - \underline{a}}{2}$  — ①

$\vec{HG} = \vec{OG} - \vec{OH} = \frac{\underline{c} + \underline{d}}{2} - \frac{\underline{a} + \underline{d}}{2} = \frac{\underline{c} + \underline{d} - \underline{a} - \underline{d}}{2}$

$\vec{HG} = \frac{\underline{c} - \underline{a}}{2}$  — ②

$\vec{EH} = \vec{OH} - \vec{OE} = \frac{\underline{a} + \underline{d}}{2} - \frac{\underline{a} + \underline{b}}{2} = \frac{\underline{a} + \underline{d} - \underline{a} - \underline{b}}{2}$

$\vec{EH} = \frac{\underline{d} - \underline{b}}{2}$  — ③

$\vec{FG} = \vec{OG} - \vec{OF} = \frac{\underline{c} + \underline{d}}{2} - \frac{\underline{b} + \underline{c}}{2} = \frac{\underline{c} + \underline{d} - \underline{b} - \underline{c}}{2}$

$\vec{FG} = \frac{\underline{d} - \underline{b}}{2}$  — ④

∴ From ①, ②, ③ & ④, we get

$\vec{EF} = \vec{HG}$  and  $\vec{EH} = \vec{FG}$

∴ EFGH is a //gm. (Proved)

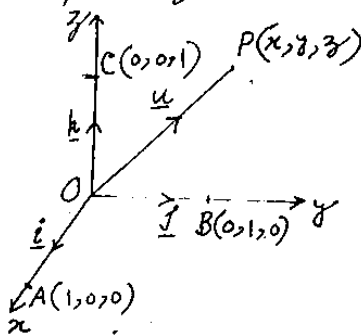
## Vectors in Space

Given a point  $P(x, y, z)$  in space there is a unique vector  $\underline{u}$  in the space such that

$$\vec{OP} = \underline{u} = [x, y, z] = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{u}| = \sqrt{x^2 + y^2 + z^2}$$

$\underline{i} = [1, 0, 0]$ ,  $\underline{j} = [0, 1, 0]$ ,  $\underline{k} = [0, 0, 1]$  are unit vectors along x-axis, y-axis and z-axis respectively.



## Properties of vectors

(i) Commutative Property:-

$$\underline{u} + \underline{v} = \underline{v} + \underline{u}$$

(ii) Associative Property:-

$$(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$$

(iii) Inverse for vector addition:-

$$\underline{u} + (-\underline{u}) = \underline{u} - \underline{u} = \underline{0}$$

(iv) Distributive Property:-

$$a(\underline{v} + \underline{w}) = a\underline{v} + a\underline{w} \text{ for } a \in \mathbb{R}$$

(v) Scalar Multiplication.

$$a(b\underline{u}) = (ab)\underline{u} \quad \forall a, b \in \mathbb{R}$$

## Distance Between Two Points in Space:-

Let  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  be the two points in space such that

$$\vec{OP}_1 = [x_1, y_1, z_1] \text{ and } \vec{OP}_2 = [x_2, y_2, z_2]$$

$$\text{Then } \vec{P_1P_2} = \vec{OP}_2 - \vec{OP}_1 = [x_2, y_2, z_2] - [x_1, y_1, z_1]$$

$$\vec{P_1P_2} = [x_2 - x_1, y_2 - y_1, z_2 - z_1]$$

$|\vec{P_1P_2}|$

$\therefore$  Distance between

$$P_1 \text{ and } P_2 = |\vec{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(Distance formula)

## Direction Angles and direction Cosines of a vector.

Let  $\vec{OP} = \underline{r} = [x, y, z]$

be a non-zero vector.

Let  $\underline{r}$  makes angles  $\alpha, \beta$

and  $\gamma$  with

x-axis, y-axis and z-axis respectively.

such that  $0 \leq \alpha \leq \pi$ ,  $0 \leq \beta \leq \pi$  and  $0 \leq \gamma \leq \pi$ . Then

(i) the angles  $\alpha, \beta$  and  $\gamma$  are called direction angles and

(ii) the numbers  $\cos \alpha, \cos \beta$  and  $\cos \gamma$  are called direction cosines of  $\underline{r}$ .

## Q. Prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Proof:- Let  $\vec{OP} = \underline{r} = [x, y, z]$

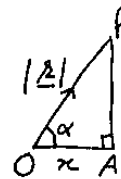
be a non-zero vector. Let  $\underline{r}$  makes angles  $\alpha, \beta$  and  $\gamma$  with x-axis, y-axis and z-axis respectively.

To prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

From the right  $\triangle OAP$

$$\cos \alpha = \frac{OA}{OP} = \frac{x}{|\underline{r}|}$$



Similarly

$$\cos \beta = \frac{y}{|\underline{r}|} \text{ and } \cos \gamma = \frac{z}{|\underline{r}|}$$

where  $|\underline{r}| = \sqrt{x^2 + y^2 + z^2}$

$$\text{Now } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{x^2}{|\underline{r}|^2} + \frac{y^2}{|\underline{r}|^2} + \frac{z^2}{|\underline{r}|^2}$$

$$= \frac{x^2 + y^2 + z^2}{|\underline{r}|^2} = \frac{|\underline{r}|^2}{|\underline{r}|^2} = 1$$