Problem Constraints

We have described the linear inequalities prescribe limitations and restrictions in allocation of available sources. While tackling a certain problem from every day life each linear inequality concerning the problem is named as problem constraint. The system of linear inequalities involved in the problem concerned are called problem constraints. The variables used in the system of linear inequalities relating to the problems of every day life are non-negative and are called non-negative constraints.

These non-negative constraints play an important role for taking decision. So these variables are called decision variables

Feasible Solution set

The solution region (which is restricted to the first quadrant) is referred to as a feasible region for the set of given constraints (i.e the solution region for which $x \geq 0$, $y \geq 0$ is called feasible region).

Feasible solution: Each point of the feasible region is called a feasible solution of the system of linear inequalities (or for the set of given constraints).

Feasible solution set

A set consisting of all the feasible solutions of the system of linear inequalities is called a feasible solution set.

Example: 1: Graph the feasible region and find the corner points for the following system of inequalities (or subject to the following constraints). $x-y \leq 3$, $x+2y \leq 6$, $x \geq 0$, $y \geq 0$

Solution: Given system of inequalities (or constraints) is

$$\left.\begin{array}{l} x-y \leq 3 \rightarrow \text{(i)} \\ x+2y \leq 6 \rightarrow \text{(ii)} \\ x \geq 0 \rightarrow \text{(iii)} \\ y \geq 0 \rightarrow \text{(iv)} \end{array}\right\} \rightarrow \text{(I)}$$

The associated equations of inequalities (i) and (ii) are

$$x-y=3 \longrightarrow (1)$$
$$x+2y=6 \longrightarrow (2)$$

For $y=0$, (1) $\Rightarrow$ $x-0=3$ $\therefore x=3$

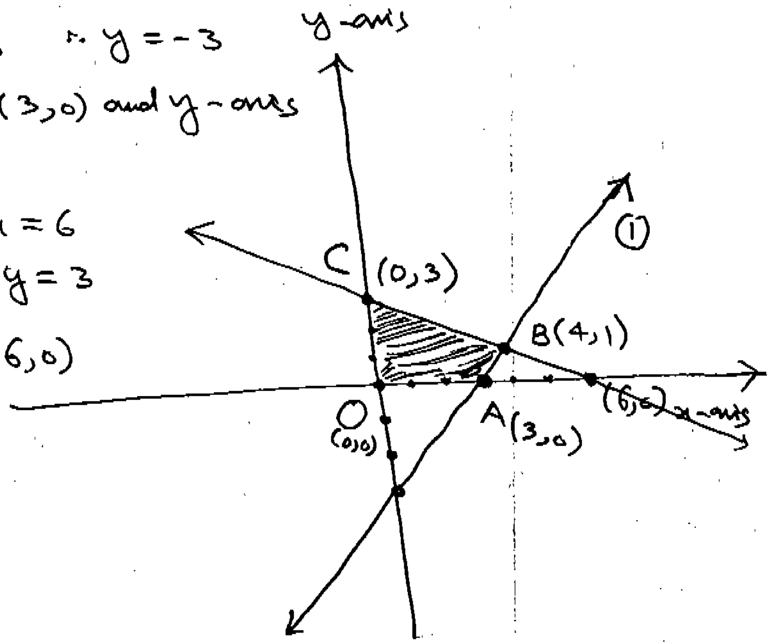
For $x=0$, (1) $\Rightarrow$ $0-y=3$ $\therefore y=-3$

$\therefore$ Line (1) cuts the x-axis at $(3,0)$ and y-axis at $(0,-3)$

For $y=0$, (2) $\Rightarrow$ $x+0=6$ $\therefore x=6$

for $x=0$, (2) $\Rightarrow$ $0+2y=6$ $\therefore y=3$

$\therefore$ Line (2) cuts the x-axis at $(6,0)$ and y-axis at $(0,3)$



We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (i)

$0-0=0<3$

$\therefore (0,0)$ satisfies (i)

$\therefore$ Graph of inequality (i) is the closed half plane (formed by line (1)) on the side of origin $(0,0)$.

Putting $(0,0)$ in the L.H.S of (ii)

$0+2(0)=0+0=0<6$

$\therefore (0,0)$ satisfies inequality (ii)

$\therefore$ Graph of inequality (ii) is the closed half plane (made by line (2)) on the side of origin $(0,0)$

The graph of $x\geq 0$ is the closed right half plane of xy-plane.

and the graph of $y\geq 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region for the given system of linear inequalities and is shown by shading.

Here feasible region is OABC.

$O(0,0)$, $A(3,0)$, $B(4,1)$ and $C(0,3)$ are corner points of feasible region.

$B(4,1)$ is the point of intersection of lines (1) and (2).

③

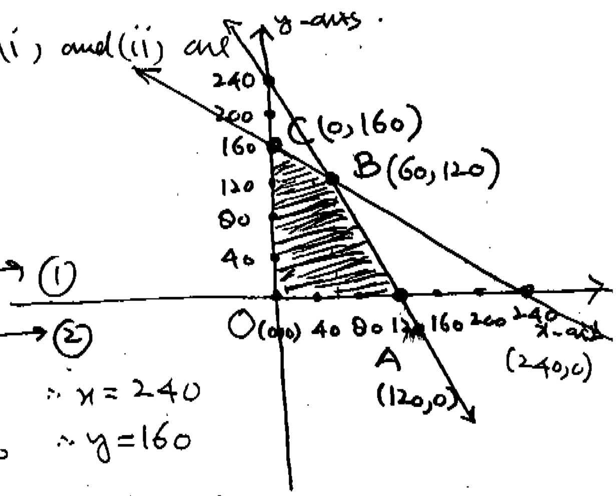
Example: 2. A manufacturer wants to make two types of concrete. Each bag of A-grade concrete contains 8 kilograms of gravel (small pebbles with coarse sand) and 4 kilograms of cement, which each bag of B-grade concrete contains 12 kilograms of gravel and two kilograms of cement. If there are 1920 kilograms of gravel and 480 kilograms of cement, then graph the feasible region under the given restrictions and find corner points of the feasible region.

Solution: Let x be the number of bags of A-grade concrete produced and y be the number of bags of B-grade concrete produced.

$$\therefore \left.\begin{aligned} 8x + 12y &\le 1920 \rightarrow (i) \\ 4x + 2y &\le 480 \rightarrow (iii) \\ x \geq 0,\ y &\geq 0 \rightarrow (iv) \\ &\rightarrow (iii) \end{aligned}\right]$$

is system of linear inequalities (or constraints).

The associated equations of (i) and (ii) are



$$8x + 12y = 1920$$

$$4x + 2y = 480$$

or $\quad 2x + 3y = 480 \longrightarrow ①$

$\quad 2x + y = 240 \longrightarrow ②$

For $y = 0$, $① \Rightarrow 2x + 0 = 480 \quad \therefore x = 240$

For $x = 0$, $① \Rightarrow 0 + 3y = 480 \quad \therefore y = 160$

$\therefore$ Line ① cuts x-axis at $(240, 0)$ and y-axis at $(0, 160)$

For $y = 0$, $② \Rightarrow 2x + 0 = 240 \quad \therefore x = 120$

for $x = 0$, $② \Rightarrow 0 + y = 240 \quad \therefore y = 240$

$\therefore$ Line ② cuts x-axis at $(120, 0)$ and y-axis at $(0, 240)$

We take test point as $(0,0)$.

Putting $(0,0)$ in L.H.S of (i), (i) is satisfied

also putting $(0,0)$ in L.H.S of (ii), (ii) is satisfied.

$\therefore$ Graphs of (i) and (ii) are the closed half planes on the side of origin.

The graph of $x \geq 0$ is the closed right half plane and graph of $y \geq 0$ is the closed upper half plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of system of linear inequalities as shown by shading.
Here feasible region is OABC.
where $O(0,0)$, $A(120,0)$, $B(60,120)$ and $C(0,160)$ are corner points of feasible region.
$B(60,120)$ is the intersection of lines ① and ②.

Example: 3 Graph the feasible regions subject to the following constraints.

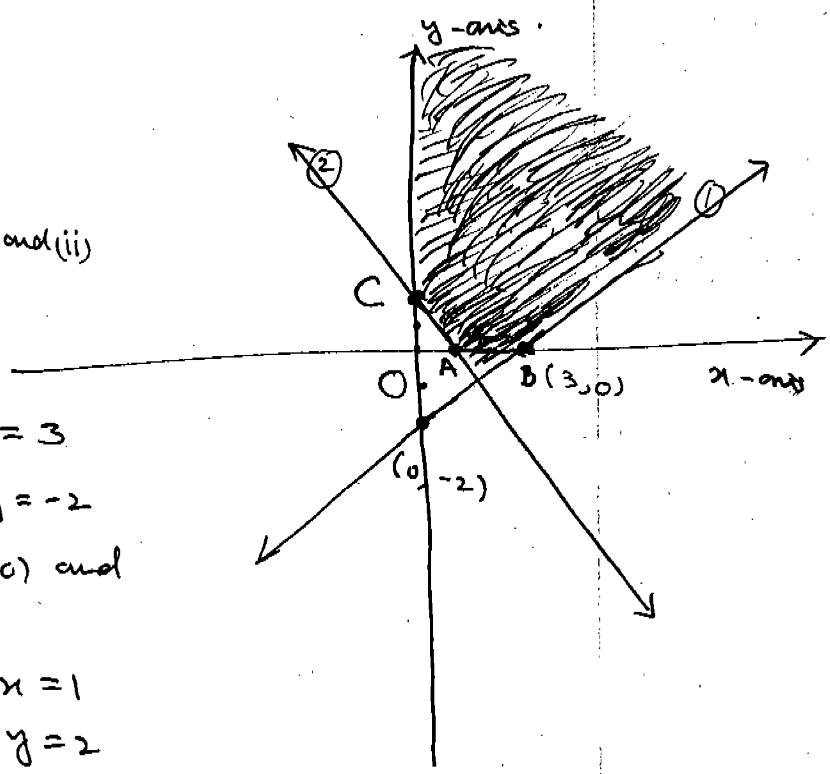
(a) $2x - 3y \leq 6$
$2x + y \geq 2$
$x \geq 0, y \geq 0$

(b) $2x - 3y \leq 6$
$2x + y \geq 2$
$x + 2y \leq 8$, $x \geq 0, y \geq 0$

(a) Given constraints are
$2x - 3y \leq 6 \rightarrow$ (i)
$2x + y \geq 2 \rightarrow$ (ii)
$x \geq 0, y \geq 0 \rightarrow$ (iv)
$\rightarrow$ (iii)

Associated equations of (i) and (ii) are
$2x - 3y = 6 \longrightarrow$ ①
$2x + y = 2 \longrightarrow$ ②

for $y = 0$, ① $\Rightarrow 2x - 0 = 6 \therefore x = 3$
for $x = 0$, ① $\Rightarrow 0 - 3y = 6 \therefore y = -2$
$\therefore$ Line ① cuts x-axis at $(3,0)$ and y-axis at $(0,-2)$
for $y = 0$, ② $\Rightarrow 2x + 0 = 2 \therefore x = 1$
for $x = 0$, ② $\Rightarrow 0 + y = 2 \therefore y = 2$
$\therefore$ Line ② cuts x-axis at $(1,0)$ and y-axis at $(0,2)$.
We take $(0,0)$ as test point.
$(0,0)$ satisfies (i)
but $(0,0)$ does not satisfy (ii)



$\therefore$ Graph of (i) is the closed half plane on the side of $(0,0)$ and graph of (ii) is the closed half plane which is not on the side of $(0,0)$.

The graph of $x \geq 0$ is the closed right half plane and the graph of $y \geq 0$ is the closed upper half plane.

$\therefore$ The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region subject to the given constraints, is partially shown in the figure by shading. There three corner points of feasible region.

The three corner points are $A(1,0)$, $B(3,0)$, $C(0,2)$

Here the feasible region is unbounded as it cannot be enclosed in any circle how large it may be.

If a line segment obtained by joining any two points of a region lies entirely within the region then region is called convex.

Here line segment AB and AC lie entirely within the region, therefore this region is convex.

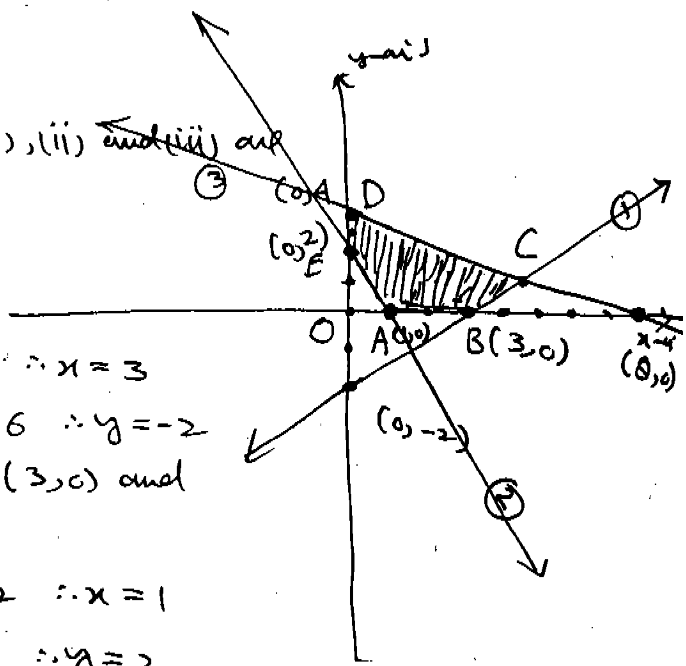
(b) Given constraints are

$2x - 3y \leq 6 \rightarrow$ (i)

$2x + y \geq 2 \rightarrow$ (ii)

$x + 2y \leq 8 \rightarrow$ (iii)

$x \geq 0,\ y \geq 0 \rightarrow$ (v)
$\rightarrow$ (iv)



Associated equations of (i), (ii) ~~and (iii)~~ are

$2x - 3y = 6 \rightarrow$ (1)

$2x + y = 2 \rightarrow$ (2)

$x + 2y = 8 \rightarrow$ (3)

$4x - 6y = 12$
$3x + 6y = 24$
$\therefore 7x = 36$
$x = \frac{36}{7}$

For $y = 0$, (1) $\Rightarrow 2x - 0 = 6 \quad \therefore x = 3$

for $x = 0$, (1) $\Rightarrow 0 - 3y = 6 \quad \therefore y = -2$

$\therefore$ Line (1) cuts x-axis at $(3,0)$ and y-axis at $(0,-2)$

For $y = 0$, (2) $\Rightarrow 2x + 0 = 2 \quad \therefore x = 1$

for $x = 0$, (2) $\Rightarrow 0 + y = 2 \quad \therefore y = 2$

$\therefore$ Line (2) cuts x-axis at $(1,0)$ and y-axis at $(0,2)$

For $y = 0$, (3) $\Rightarrow x + 0 = 8 \quad \therefore x = 8$

For $x=0$, (3) $\Rightarrow$ $0+2y=8$ $\therefore y=4$.

$\therefore$ Line (3) cuts x-axis at $(8,0)$ and y-axis at $(0,4)$.

We take $(0,0)$ as test point.

$(0,0)$ satisfies (i)

$(0,0)$ does not satisfy (ii)

$(0,0)$ satisfies (iii)

$\therefore$ Graph of (i) is the closed half plane on the side of $(0,0)$

" " (ii) is " " " " not on the side of $(0,0)$

" " (iii) " " " " " on the side of $(0,0)$.

The graph of $x \geq 0$ is the closed right half plane

The graph of $y \geq 0$ is the closed upper half plane

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region subject to given constraints

Here feasible region is ABCDE.

where $A(1,0)$, $B(3,0)$, $C(\frac{36}{7}, \frac{10}{7})$, $D(0,4)$, $E(0,2)$

are corners of feasible region.

$C(\frac{36}{7}, \frac{10}{7})$ is the point of intersection of lines (1) and (3).

Exercise: 5.2

Q.1:

Given inequalities are

$2x - 3y \le 6 \rightarrow$ (i)

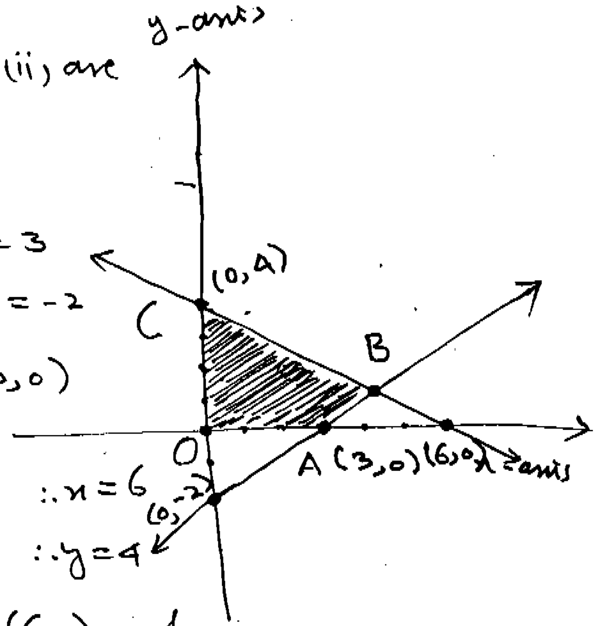
$2x + 3y \le 12 \rightarrow$ (ii)

$x \ge 0 \rightarrow$ (iii), $y \ge 0 \rightarrow$ (iv)

Associated equations of (i) and (ii) are

$2x - 3y = 6 \rightarrow$ ①

$2x + 3y = 12 \rightarrow$ ②



For $y = 0$, ① $\Rightarrow 2x - 0 = 6 \quad \therefore x = 3$

for $x = 0$, ① $\Rightarrow 0 - 3y = 6 \quad \therefore y = -2$

$\therefore$ The line ① cuts x-axis at $(3, 0)$

and y-axis at $(0, -2)$.

Now for $y = 0$, ② $\Rightarrow 2x + 0 = 12 \quad \therefore x = 6$

for $x = 0$, ② $\Rightarrow 0 + 3y = 12 \quad \therefore y = 4$

$\therefore$ The line ② cuts x-axis at $(6, 0)$ and

y-axis at $(0, 4)$

We take $(0,0)$ as test point.

$(0,0)$ satisfy both inequalities (i) and (ii).

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of $(0,0)$.

The graph of $x \ge 0$ is the closed right half plane of xy-plane and the graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the given system of linear inequalities as shown in the figure by shading the region.

Here OABC is the feasible region.

where $O(0,0)$, $A(3,0)$, $B(4.5, 1)$ or $B(\frac{9}{2}, 1)$ and $C(0, 4)$ are corner points of feasible region.

$B(\frac{9}{2}, 1)$ is the point of intersection of lines ① and ②.

(ii) Given inequalities are

$$x + y \leq 5 \rightarrow (i)$$

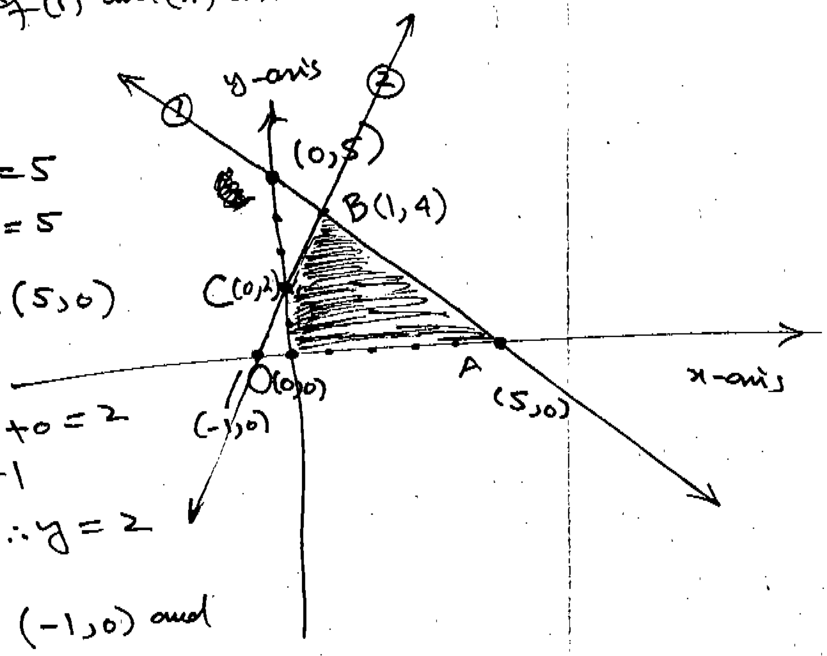
$$-2x + y \leq 2 \rightarrow (ii)$$

$$x \geq 0 \rightarrow (iii), \quad y \geq 0 \rightarrow (iv)$$

The associated equations of (i) and (ii) are

$$x + y = 5 \rightarrow ①$$

$$-2x + y = 2 \rightarrow ②$$



For $y = 0$, ① $\Rightarrow x + 0 = 5 \quad \therefore x = 5$

for $x = 0$, ① $\Rightarrow 0 + y = 5 \quad \therefore y = 5$

$\therefore$ The line ① cuts x-axis at $(5, 0)$

and y-axis at $(0, 5)$.

Now for $y = 0$, (ii) $\Rightarrow -2x + 0 = 2$

$\therefore x = -1$

for $x = 0$, ② $\Rightarrow -0 + y = 2 \quad \therefore y = 2$

$\therefore$ The line ② cuts x-axis at $(-1, 0)$ and

y-axis at $(0, 2)$

We take $(0, 0)$ as test point.

As $(0, 0)$ satisfies both (i) and (ii)

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of $(0, 0)$.

The graph of $x \geq 0$ is the closed right half plane of xy-plane

and the graph of $y \geq 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the given system of linear inequalities and is shown in the diagram by shading the region.

Here OABC is the feasible region.

where $O(0,0)$, $A(5,0)$, $B(1,4)$, $C(0,2)$ are corner point of feasible region OABC.

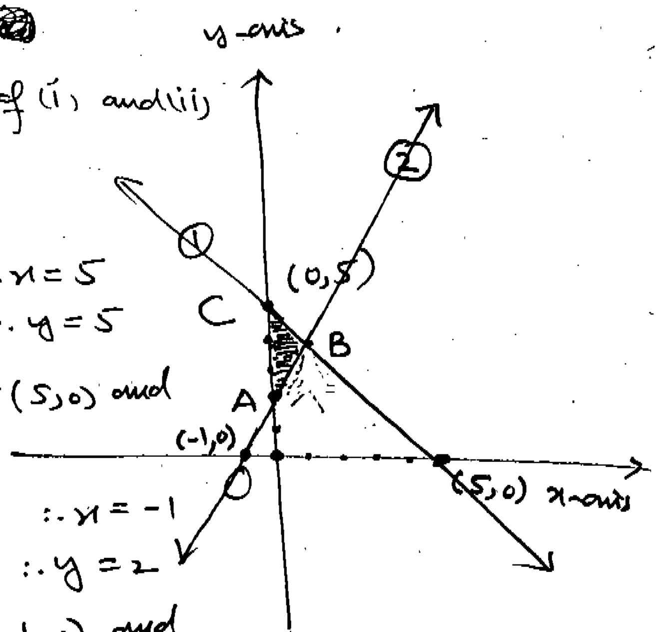
B is point of intersection of lines ① and ②

(iii) Given inequalities are

$x+y \le 5 \rightarrow (i)$

$-2x+y \ge 2 \rightarrow (ii)$

$x \ge 0 \rightarrow (iii)$,



The associated equations of (i) and (ii) are $x+y=5 \longrightarrow (1)$

$-2x+y=2 \longrightarrow (2)$

For $y=0$, $(1) \Rightarrow x+0=5$ $\therefore x=5$

for $x=0$, $(1) \Rightarrow 0+y=5$ $\therefore y=5$

$\therefore$ The line (1) cuts x-axis at $(5,0)$ and y-axis at $(0,5)$.

For $y=0$, $(2) \Rightarrow -2x+0=2$ $\therefore x=-1$

for $x=0$, $(2) \Rightarrow 0+y=2$ $\therefore y=2$

$\therefore$ Line (2) cuts x-axis at $(-1,0)$ and y-axis at $(0,2)$.

We take test point as origin $(0,0)$

$(0,0)$ satisfies (i).

$\therefore$ Graph of (i) is the closed half plane on the side of $(0,0)$

As $(0,0)$ does not satisfy (ii)

$\therefore$ Graph of (ii) is the closed half plane (made by line (2)) not on the side of $(0,0)$.

Graph of (iii) or graph of $x \ge 0$ is the closed right half plane of xy-plane.

~~Graph of (iv) or graph of y ≥ 0 is the closed upper half plane.~~

The intersection of graphs of (i), (ii), (iii) is the feasible region ABC as shown in the figure by shading the region.

$A(0,0)$, $B(1,4)$, $C(0,5)$ are corner points of the feasible region ABC.

$3x+7y=21$
$7x-7y=21$
$10x=42$
$x=\frac{21}{5}$
$y=x-3=\frac{21}{5}-3$
$=\frac{6}{5}$

(iv) Given inequalities are

$3x+7y \le 21 \rightarrow$ (i)
$x-y \le 3 \rightarrow$ (ii)
$x \ge 0 \rightarrow$ (iii)
$y \ge 0 \rightarrow$ (iv)

The associated equations of (i) and (ii) are

$3x+7y=21 \rightarrow$ ①
$x-y=3 \rightarrow$ ②

For $y=0$, ① $\Rightarrow 3x+0=21$ $\therefore x=7$

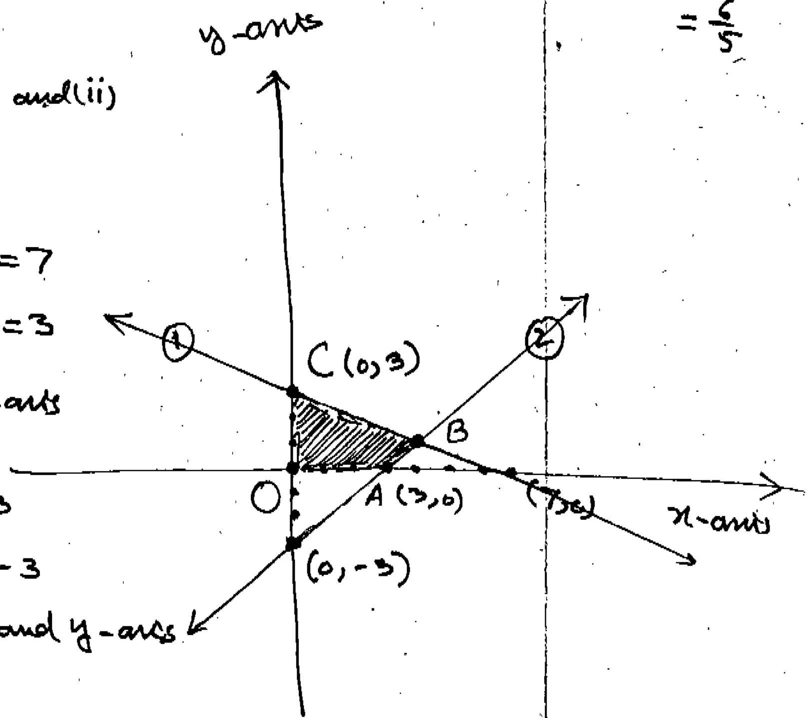
for $x=0$, ① $\Rightarrow 0+7y=21$ $\therefore y=3$

$\therefore$ ① cuts x-axis at $(7,0)$ and y-axis at $(0,3)$.

For $y=0$, ② $\Rightarrow x-0=3$ $\therefore x=3$

For $x=0$, ② $\Rightarrow 0-y=3$ $\therefore y=-3$

$\therefore$ Line ② cuts x-axis at $(3,0)$ and y-axis at $(0,-3)$.



We take $(0,0)$ as test point.

As $(0,0)$ satisfies (i) and (ii)

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of origin $(0,0)$.

The graph of $x \ge 0$ is the closed right half plane of xy-plane.

and the graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the given system of linear inequalities ~~as sto~~ and is shown in the figure by shading the region.

Here OABC is the feasible region.

where $O(0,0)$, $A(3,0)$, $B(\frac{21}{5}, \frac{6}{5})$, $C(0,3)$ are the corner of feasible region.

(v) Given inequalities are

$$\left.\begin{array}{l} 3x+2y \geq 6 \rightarrow (i) \\ x+y \leq 4 \rightarrow (ii) \\ x \geq 0 \rightarrow (iii) \\ y \geq 0 \rightarrow (iv) \end{array}\right] \rightarrow (I)$$

The associated equations of (i) and (ii) are

$3x+2y=6 \rightarrow$ ①

$x+y=4 \rightarrow$ ②

For $y=0$, ① $\Rightarrow 3x+0=6 \therefore x=2$

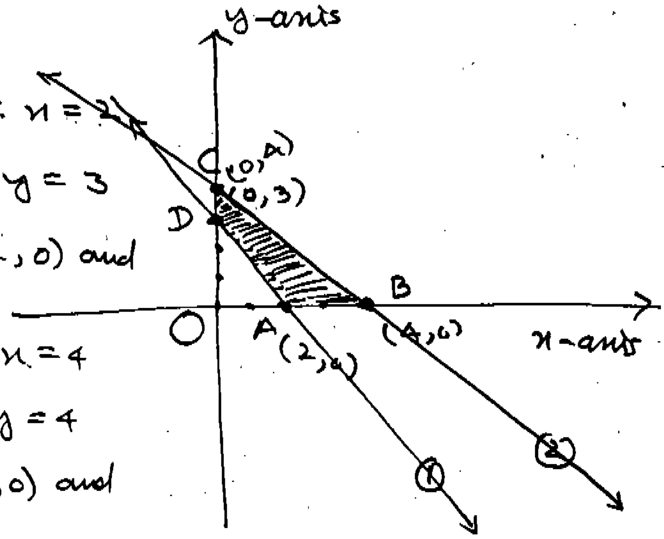
for $x=0$, ① $\Rightarrow 0+2y=6 \therefore y=3$

$\therefore$ Line ① cuts x-axis at $(2,0)$ and y-axis at $(0,3)$.

For $y=0$, ② $\Rightarrow x+0=4 \therefore x=4$

for $x=0$, ② $\Rightarrow 0+y=4 \therefore y=4$

$\therefore$ Line ② cuts x-axis at $(4,0)$ and y-axis at $(0,4)$.



We take $(0,0)$ as test point.

$(0,0)$ does not satisfy (i)

$\therefore$ The graph of (i) is the closed half plane (made by line ①) not on the side of $(0,0)$.

$(0,0)$ satisfies (ii)

$\therefore$ graph of (ii) is the closed half plane (made by line ②) on the side of $(0,0)$.

Graph of (iii) is the closed right half plane of xy-plane and graph of (iv) is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region ABCD as shown in the figure by shading the region.

$A(2,0)$, $B(4,0)$, $C(0,4)$, $D(0,3)$ are the corners of the feasible region.

(vi) Given inequalities are

$$\left.\begin{array}{l} 5x+7y \le 35 \rightarrow (i) \\ x-2y \le 4 \rightarrow (ii) \\ x \ge 0 \rightarrow (iii), \quad y \ge 0 \rightarrow (iv) \end{array}\right\} \rightarrow (I)$$

$$\begin{array}{r} 5x+7y = 35 \\ 5x-10y = 20 \\ \hline 17y = 15 \end{array}$$

$\therefore y = \frac{15}{17}$

$x = 4 + \frac{30}{17} = \frac{98}{17}$

The associated equations of (i) and (ii) are

$5x+7y = 35 \rightarrow ①$

$x-2y = 4 \rightarrow ②$

For $y=0$, ① $\Rightarrow 5x+0 = 35 \quad \therefore x = 7$

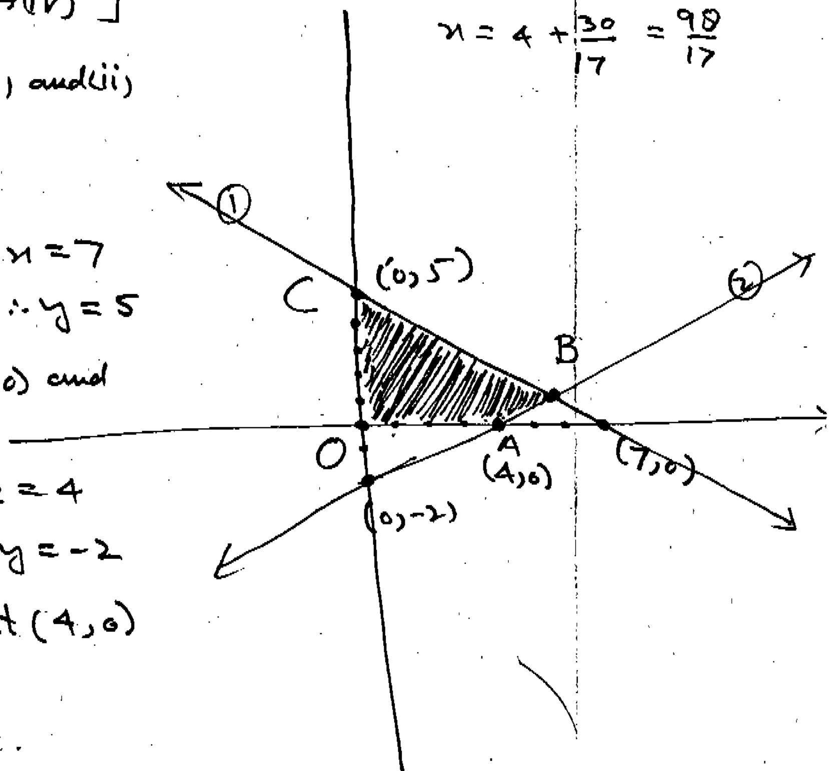
for $x=0$, ① $\Rightarrow 0+7y = 35 \quad \therefore y = 5$

$\therefore$ Line ① cuts x-axis at $(7,0)$ and y-axis at $(0,5)$.

For $y=0$, ② $\Rightarrow x-0 = 4 \quad \therefore x = 4$

for $x=0$, ② $\Rightarrow 0-2y = 4 \quad \therefore y = -2$

$\therefore$ The line ② cuts x-axis at $(4,0)$ and y-axis at $(0,-2)$



We take $(0,0)$ as the test point.

As $(0,0)$ satisfies both inequalities (i) and (ii)

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of $(0,0)$.

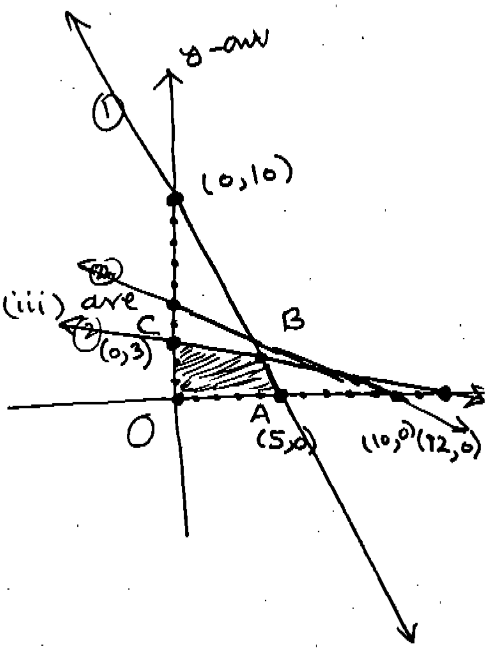
The graph of $x \ge 0$ is the closed right half plane and

the graph of $y \ge 0$ is the closed upper half plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the system of given inequalities as shown in the figure by shading the region.

Here OABC is the feasible region.

$O(0,0)$, $A(4,0)$, $B\left(\frac{98}{17}, \frac{15}{17}\right)$ and $C(0,5)$ are the corner points of the feasible region OABC.



Q.2 (i) Given inequalities are

$$\left.\begin{array}{l} 2x+y \le 10 \to (i) \\ x+4y \le 12 \to (ii) \\ x+2y \le 10 \to (iii) \\ x \ge 0 \to (iv),\quad y \ge 0 \to (v) \end{array}\right] \to (I)$$

The associated equations of (i), (ii) and (iii) are

$2x+y=10 \to ①$

$x+4y=12 \to ②$

$x+2y=10 \to ③$

For $y=0$, ① $\Rightarrow 2x+0=10 \quad \therefore x=5$

for $x=0$, ① $\Rightarrow 0+y=10 \quad \therefore y=10$

∴ Lines ① cuts x-axis at (5,0) and y-axis at (0,10).

for $y=0$, ② $\Rightarrow x+0=12 \quad \therefore x=12$

for $x=0$, ② $\Rightarrow 0+4y=12 \quad \therefore y=3$

∴ Line ② cuts x-axis at (12,0) and y-axis at (0, 3)

for $y=0$, ③ $\Rightarrow x+0=10 \quad \therefore x=10$

for $x=0$, ③ $\Rightarrow 0+2y=10 \quad \therefore y=5$

∴ Line ③ cuts x-axis at (10, 0) and y-axis at (0, 5).

We take origin (0,0) as test point.

As (0,0) satisfies (i), (ii) and (iii)

∴ The graphs of (i), (ii), (iii) are the closed half plane on the origin side of ①, ② and ③.

The graph of $x \ge 0$ is the closed right half plane and the graph of $y \ge 0$ is the closed upper half plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of the system of inequalities, and is as shown in the figure by shading the region.

The corners of feasible region are

O(0,0), A(5,0), B(4,2), C(0,3).

B(4,2) is point of intersection of lines ① and ②.

(ii) Given inequalities are

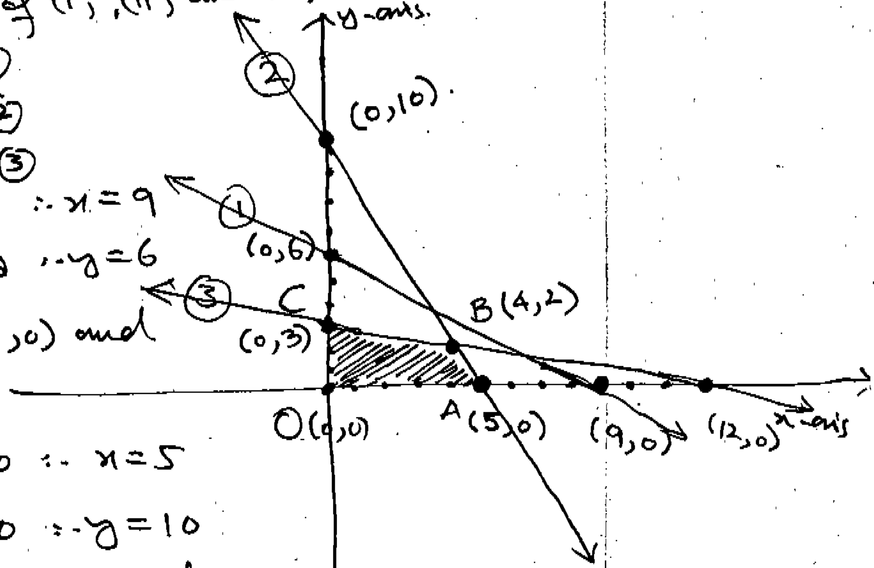
$$\left.\begin{array}{l} 2x+3y \le 18 \rightarrow (i) \\ 2x+y \le 10 \rightarrow (ii) \\ x+4y \le 12 \rightarrow (iii) \\ x \ge 0 \rightarrow (iv);\quad y \ge 0 \rightarrow (v) \end{array}\right\} \rightarrow (I)$$

The associated equations of (i), (ii) and (iii) are

$2x+3y = 18 \rightarrow (1)$

$2x+y = 10 \rightarrow (2)$

$x+4y = 12 \rightarrow (3)$



For $y=0$, (1) $\Rightarrow 2x+0=18 \therefore x=9$

for $x=0$, (1) $\Rightarrow 0+3y=18 \therefore y=6$

$\therefore$ Line (1) cuts x-axis at (9,0) and y-axis at (0,6).

for $y=0$, (2) $\Rightarrow 2x+0=10 \therefore x=5$

for $x=0$, (2) $\Rightarrow 0+y=10 \therefore y=10$

$\therefore$ Line (2) cuts x-axis at (5,0) and y-axis at (0,10)

for $y=0$, (3) $\Rightarrow x+0=12 \therefore x=12$

for $x=0$, (3) $\Rightarrow 0+4y=12 \therefore y=3$

$\therefore$ Line (3) cuts x-axis at (12,0) and y-axis at (0,3).

We take origin (0,0) as the test point.

As (0,0) satisfies (i), (ii) and (iii).

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the side of origin (0,0) of the lines (1), (2) and (3).

The graph of $x \ge 0$ is the closed right half plane of xy-plane.

The graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv), (v) is the feasible region OABC of the system of linear inequalities as shown in the figure by shading the region.

O(0,0), A(5,0), B(4,2), C(0,3) are corners of feasible region OABC.

B(4,2) is the point of intersection of lines (2) and (3)

(iii) Given inequalities are

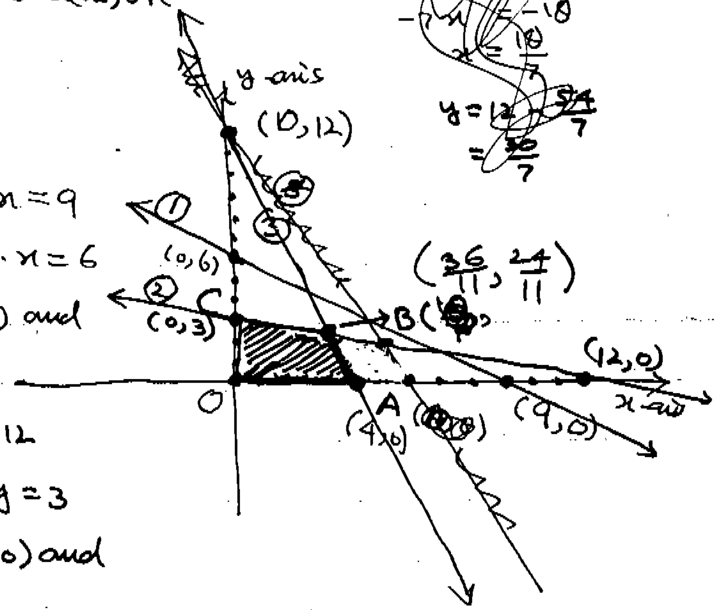
$$\left.\begin{aligned} 2x+3y &\le 18 \rightarrow (i) \\ x+4y &\le 12 \rightarrow (ii) \\ 3x+y &\le 12 \rightarrow (iii) \\ x \ge 0 \rightarrow (iv)&,\ y \ge 0 \rightarrow (v) \end{aligned}\right\} \rightarrow I$$

Associated equations of (i), (ii) and (iii) are

$2x+3y = 18 \rightarrow ①$

$x+4y = 12 \rightarrow ②$

$3x+y = 12 \rightarrow ③$



For $y=0$, ① $\Rightarrow 2x+0=18 \therefore x=9$

for $x=0$, ① $\Rightarrow 0+3y=18 \therefore x=6$

$\therefore$ Line ① cuts x-axis at (9, 0) and y-axis at (0, 6)

for $y=0$, ② $\Rightarrow x+0=12 \therefore x=12$

for $x=0$, ② $\Rightarrow 0+4y=12 \therefore y=3$

$\therefore$ Line ② cuts x-axis at (12, 0) and y-axis at (0, 3).

for $y=0$, ③ $\Rightarrow 3x+0=12 \therefore x=4$

for $x=0$, ③ $\Rightarrow 0+y=12 \therefore y=12$

$\therefore$ Line ③ cuts x-axis at (4, 0) and y-axis at (0, 12)

We take origin (0, 0) as test point.

As (0, 0) satisfies (i), (ii), (iii)

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the origin side of ①, ② and ③.

The graph of $x \ge 0$ is the closed right half plane of xy-plane.

and the graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of the system of linear inequalities as shown in the figure by shading the region.

O(0, 0), A(6, 0), $B(\frac{36}{11}, \frac{24}{11})$ and C(0, 3) are corners of feasible region.

$B(\frac{36}{11}, \frac{24}{11})$ is the point of intersection of ② and ③

(iv) Given inequalities are

$$\left.\begin{array}{l} x+2y \le 14 \to (i) \\ 3x+4y \le 36 \to (ii) \\ 2x+y \le 10 \to (iii) \\ x \ge 0 \to (iv),\ y \ge 0 \to (v) \end{array}\right\} \to (I)$$

The associated equations of (i), (ii) and (iii) are

$x+2y = 14 \to ①$

$3x+4y = 36 \to ②$

$2x+y = 10 \to ③$

~~For y = 0, ① ⇒ 0+2y = 14 ∴ y =~~

For $y=0$, ① $\Rightarrow x+0=14$ $\therefore x=14$

for $x=0$, ① $\Rightarrow 0+2y=14$ $\therefore y=7$

$\therefore$ Line ① cuts x-axis at $(14,0)$ and y-axis at $(0,7)$

For $y=0$, ② $\Rightarrow 3x+0=36$ $\therefore x=12$

for $x=0$, ② $\Rightarrow 0+4y=36$ $\therefore y=9$

$\therefore$ line ② cuts x-axis at $(12,0)$ and y-axis at $(0,9)$

For $y=0$, ③ $\Rightarrow 2x+0=10$ $\therefore x=5$

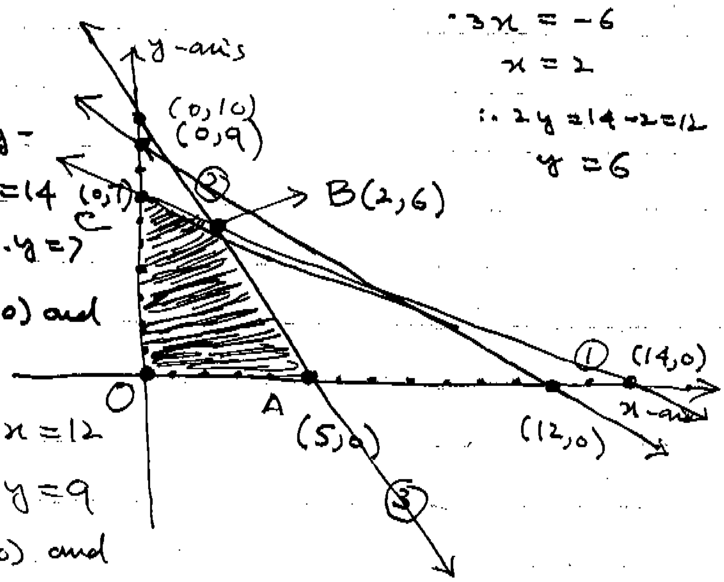
for $x=0$, ③ $\Rightarrow 0+y=10$ $\therefore y=10$

$\therefore$ line ③ cuts x-axis at $(5,0)$ and y-axis at $(0,10)$.

$$\begin{array}{r} x+2y=14 \\ 4x+2y=20 \\ \hline -3x=-6 \\ x=2 \end{array}$$

$\therefore 2y = 14-2 = 12$

$y = 6$



We take $(0,0)$ as test point.

As $(0,0)$ satisfies (i), (ii) and (iii)

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the origin side of ①, ②, ③.

Graph of $x \ge 0$ is the closed right half plane of xy-plane. and graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of system of linear inequalities as shown in the figure by shading the region.

$O(0,0)$, $A(5,0)$, $B(2,6)$, $C(0,7)$ are corners of feasible region.

$B(2,6)$ is point of intersection of lines ① and ③.

(v) Given inequalities are

$$\left.\begin{array}{l} x+3y \le 15 \to \text{(i)} \\ 2x+y \le 12 \to \text{(ii)} \\ 4x+3y \le 24 \to \text{(iii)} \\ x \ge 0 \to \text{(iv)},\ y \ge 0 \to \text{(v)} \end{array}\right\} \to \text{(I)}$$

The associated equations of (i), (ii) and (iii) are

$$x+3y = 15 \to (1)$$
$$2x+y = 12 \to (2)$$
$$4x+3y = 24 \to (3)$$

For $y=0$, (1) $\Rightarrow x+0=15 \therefore x=15$

for $x=0$, (1) $\Rightarrow 0+3y=15 \therefore y=5$

$\therefore$ Line (1) cuts x-axis at $(15,0)$ and y-axis at $(0,5)$

For $y=0$, (2) $\Rightarrow 2x+0=12 \therefore x=6$

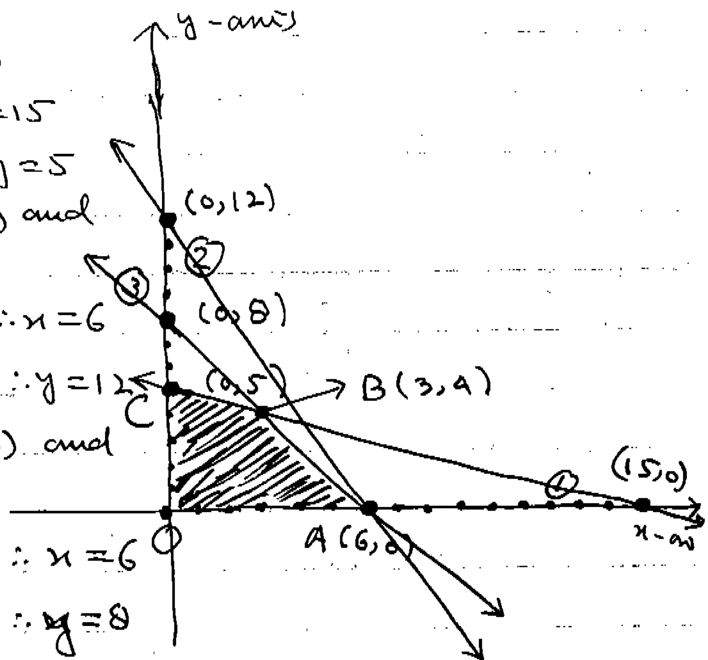
for $x=0$, (2) $\Rightarrow 0+y=12 \therefore y=12$

$\therefore$ Line (2) cuts x-axis at $(6,0)$ and y-axis at $(0,12)$

for $y=0$, (3) $\Rightarrow 4x+0=24 \therefore x=6$

for $x=0$, (3) $\Rightarrow 0+3y=24 \therefore y=8$

$\therefore$ Line (3) cuts x-axis at $(6,0)$ and y-axis at $(0,8)$



We take $(0,0)$ as test point.

As $(0,0)$ satisfies (i), (ii) and (iii)

$\therefore$ Graphs of (i), (ii) and (iii) are closed half plane on the origin side of (1), (2) and (3).

Graph of $x \ge 0$ is the closed right half plane of xy-plane.

and graph of $y \ge$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of system of linear inequalities as shown in the figure by shading the region.

$O(0,0)$, $A(6,0)$, $B(3,4)$ and $C(0,5)$ are corners of the feasible region.

The point $B(3,4)$ is the intersection of lines (1) and (3).

(vi) Given inequalities are

$$\left.\begin{array}{l} 2x+y \le 20 \to (i) \\ 8x+15y \le 120 \to (ii) \\ x+y \le 11 \to (iii) \\ x \ge 0 \to (iv),\ y \ge 0 \to (v) \end{array}\right\} \to (I)$$

The associated equations of (i), (ii) and (iii) are

$$2x+y = 20 \to (1)$$
$$8x+15y = 120 \to (2)$$
$$x+y = 11 \to (3)$$

$$\begin{array}{r} 8x+15y=120 \\ \underline{\underset{-}{8}x \underset{-}{+} 8y = \underset{-}{88}} \\ 7y = 32 \end{array}$$

$$x = 11 - \frac{32}{7} = \frac{45}{7}$$

For $y=0$, (1) $\Rightarrow 2x+0=20 \therefore x=10$

for $x=0$, (1) $\Rightarrow 0+y=20 \therefore y=20$

$\therefore$ Line (1) cuts x-axis at $(10,0)$ and y-axis at $(0,20)$.

For $y=0$, (2) $\Rightarrow 8x+0=120 \therefore x=15$

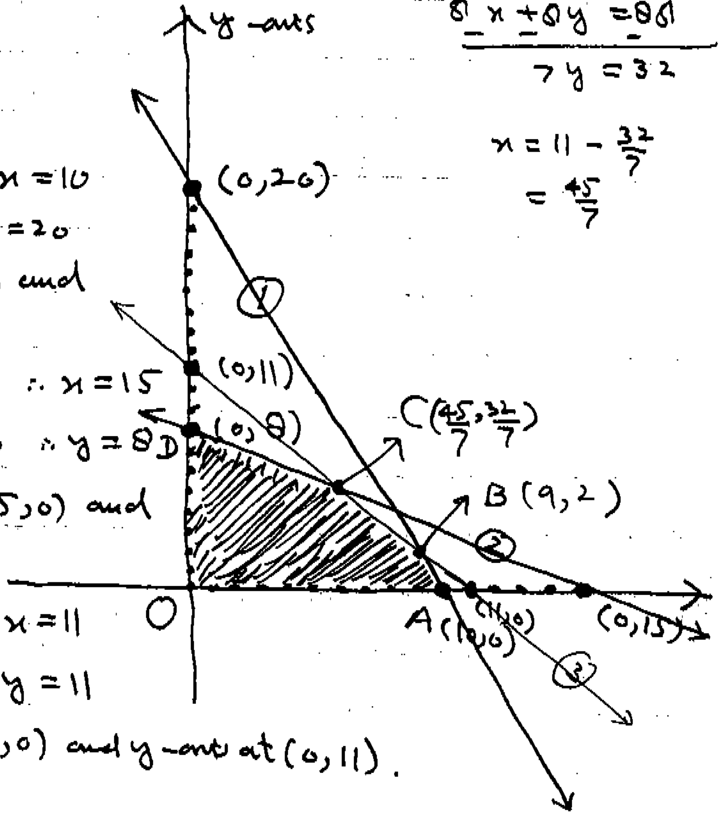
for $x=0$, (2) $\Rightarrow 0+15y=120 \therefore y=8$

$\therefore$ Line (2) cuts x-axis at $(15,0)$ and y-axis at $(0,8)$.

for $y=0$, (3) $\Rightarrow x+0=11 \therefore x=11$

for $x=0$, (3) $\Rightarrow 0+y=11 \therefore y=11$

$\therefore$ Line (3) cuts x-axis at $(11,0)$ and y-axis at $(0,11)$.



We take $(0,0)$ as test point.

As $(0,0)$ satisfies (i), (ii) and (iii).

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the origin sides of (1), (2) and (3).

The graph of $x \ge 0$ is the closed right half plane of xy-plane and the graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABCD of system of linear inequalities as shown in the figure by shading the region.

$O(0,0)$, $A(10,0)$, $B(9,2)$, $C(\frac{45}{7},\frac{32}{7})$, $D(0,8)$ are corners of feasible region OABCD.

$B(9,2)$ is point of intersection of lines (1) and (3)

and $C(\frac{45}{7},\frac{32}{7})$ is point of intersection of lines (2) and (3).