CHAPTER: 5

Linear Inequalities and Linear Programming

Linear Inequalities

For example:
(i) $ax < b$; (ii) $ax + b \geq c$; (iii) $ax + by > c$; (iv) $ax + by \leq c$
are linear inequalities.
(i) and (ii) are linear inequalities in one variable x.
(iii) and (iv) are linear inequalities in two variables x and y.

The following operations will not affect the order (or sense) of inequality while changing it to simpler equivalent form.

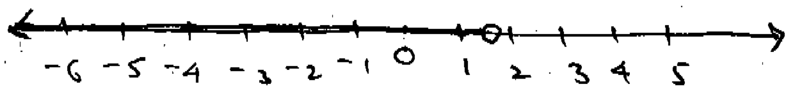
(i) Adding or subtracting a constant to each side of it.
(ii) Multiplying or dividing each side of it by a positive constant.

Note: The order (or sense) of an inequality is changed by multiplying or dividing its each side by a negative constant.

Now Consider an inequality, $x < \frac{3}{2} \longrightarrow (1)$

All real numbers $< \frac{3}{2}$ are in the sol. set of (1).

Thus the interval $(-\infty, \frac{3}{2})$ or $-\infty < x < \frac{3}{2}$ is the sol. set of (1), which is shown in the figure



Graphing of a Linear Inequality in Two variables

Generally a linear inequality in two variables x and y can be one of the following forms

$ax + by < c$; $ax + by > c$; $ax + by \leq c$; $ax + by \geq c$

where a, b, c are constants and a, b are not both zero.

We know that the graph of linear equation $ax + by = c$ is a straight line which divides the plane into two disjoint regions as stated below;

(1) The set of ordered pairs (x, y) such that $ax + by < c$
(2) The set of ordered pairs (x, y) such that $ax + by > c$

The regions (1) and (2) are called half planes and the line $ax+by=c$ is called the boundary of each half plane.

Note: Note that a vertical line divides the plane into left and right half planes while a non-vertical line divides the plane into upper and lower half planes.

Solution of a linear inequality in x and y

A solution of a linear inequality in x and y is an ordered pair of number which satisfy the inequality.

Example: The order pair $(1,1)$ is a solution of the inequality $x+2y<6$

because $1+2(1)=3<6$

There are infinitely many ordered pairs that satisfy the inequality $x+2y<6$, so its graph will be a half plane.

Associated or Corresponding equation.

The linear equation $ax+by=c$ is called " associated or corresponding equation of each of inequality

$ax+by<c$; $ax+by>c$; $ax+by\leq c$; $ax+by\geq c$.

Procedure for Graphing a linear Inequality in two variables.

(i) The associated or corresponding equation of inequality is first graphed by using "dashes' if the inequality involves the symbols $>$ or $<$ and a solid line is drawn if the inequality involves the symbols $\geq$ or $\leq$.

(ii) A test point (not on the graph of the corresponding equation) is chosen which determines that half plane is on which side of the boundary line.

Example: 1 Graph the inequality $x+2y<6$

Solution: Given inequality is $x+2y<6 \longrightarrow (1)$

The associated equation of inequality (1) is

$$x+2y=6 \longrightarrow (2)$$

For $y=0$, (2) $\Rightarrow$ $x=6$

$\therefore$ (2) cuts x-axis at the point $(6,0)$

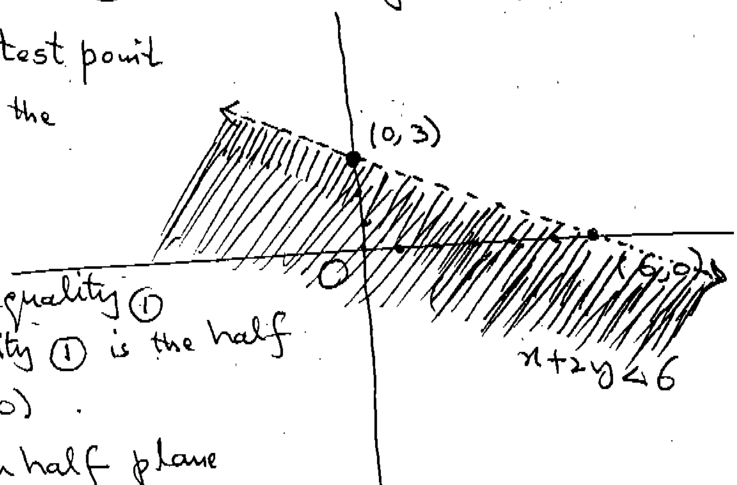
For $x=0$, (2) $\Rightarrow 0+2y=6$

$\therefore y=3$

$\therefore$ (2) cuts y-axis at the point $(0,3)$

As no point of line (2) is a solution of the inequality (1) so the graph of the line (2) is shown by using dashes.

We take $(0,0)$ as a test point because it is not on the line (2).



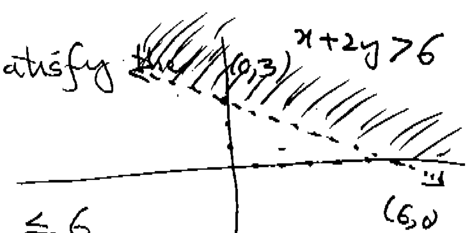
As $0+2(0)=0<6$

$\therefore (0,0)$ satisfies the inequality (1)

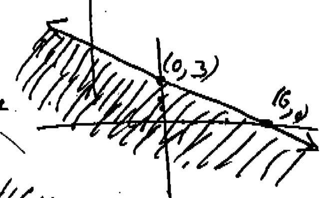
$\therefore$ Graph of the inequality (1) is the half plane containing $(0,0)$.

A portion of the open half plane below the line (2) is shown as shaded region.

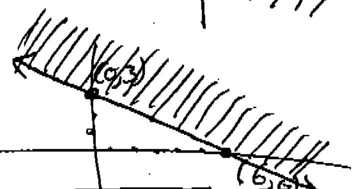
Note: 1 All points above the dashed line satisfy the inequality $x+2y>6$



Note: 2 The graph of the inequality $x+2y\le 6$ includes the graph of line (2), and open half plane below the line (2) as shown in figure by shading a portion of the graph.



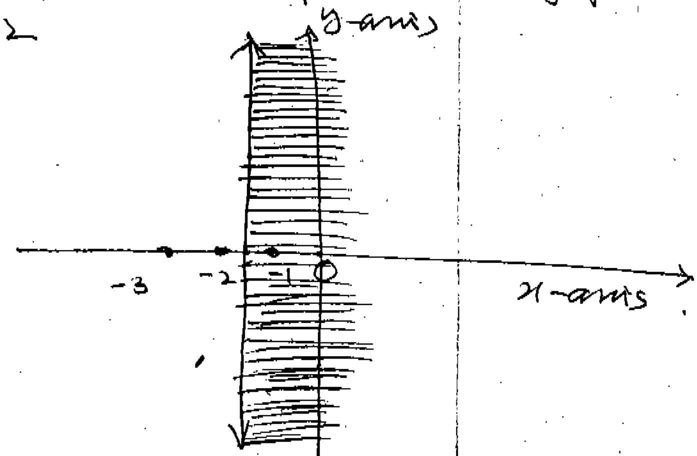
Note: 3 The graph of the inequality $x+2y\ge 6$ includes the graph of line (2) and open half plane above the line (2) as shown in figure by shading a portion of the graph.



Note: 4: The graphs of $x+2y\le 6$ and $x+2y\ge 6$ are closed half planes.

Example:2 Graph the following linear inequalities in xy-plane.

(i) $2x \geq -3$ (ii) $y \leq 2$



(i) Given inequality is

$2x \geq -3 \rightarrow (1)$

Associated equation is

$2x = -3 \rightarrow (2)$

or $x = -\frac{3}{2}$

which is a line parallel to y-axis (i.e the vertical line)

$\therefore$ The graph of inequality $2x > -3$ is the open half plane to the right of line (2)

Hence the graph of inequality $2x \geq -3$ is the closed half plane to the right of the line (2).

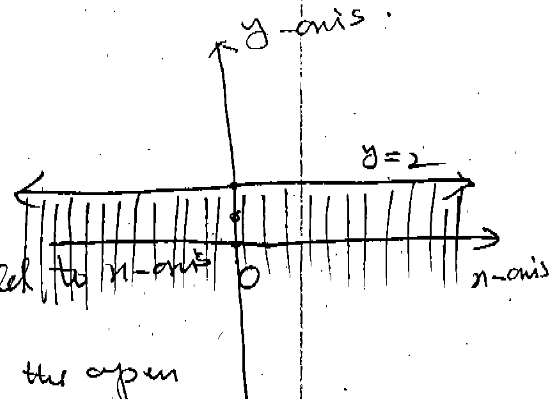
(ii) Given inequality is

$y \leq 2 \rightarrow (1)$

Associated equation is

$y = 2 \rightarrow (2)$

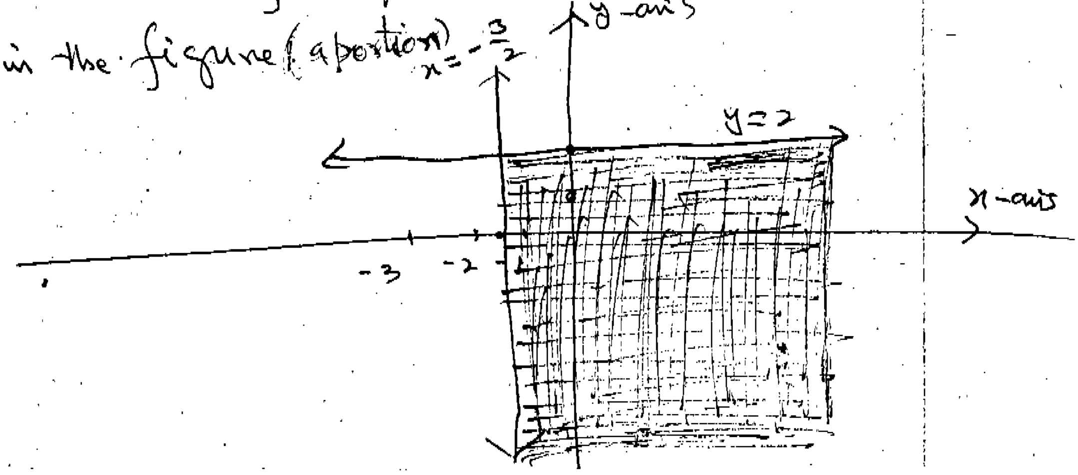
which is a (horizontal line) line parallel to x-axis



$\therefore$ The graph of inequality $y < 2$ is the open half plane below the line $y = 2$.

Hence graph of inequality $y \leq 2$ is the closed half plane below the line $y = 2$

Note: The intersection of inequalities (i) and (ii) is as shown in the figure (a portion)



Region bounded by 2 or 3 simultaneous inequalities.

The graph of a system of inequalities consists of the set of all ordered pairs (x, y) in the xy-plane which simultaneously satisfy all the inequalities in the system. We draw the graph of each inequality in the system on the same co-ordinate axes and then take intersection of all the graph. The common region so obtained is called the solution region for the system of inequalities.

Example: 1 Graph the system of inequalities

$$x - 2y \leq 6 \rightarrow (1)$$
$$2x + y \leq 2 \rightarrow (2)$$

Solution: Given inequalities are

$$x - 2y \leq 6 \rightarrow (1)$$
$$2x + y \leq 2 \rightarrow (2)$$

The associated equation of (1) is

$$x - 2y = 6 \rightarrow (3)$$

For $y = 0$, (3) $\Rightarrow x - 0 = 6 \quad \therefore x = 6$

$\therefore$ (3) cuts the x-axis at $(6, 0)$

For $x = 0$, (3) $\Rightarrow 0 - 2y = 6 \quad \therefore y = \frac{6}{-2} = -3$

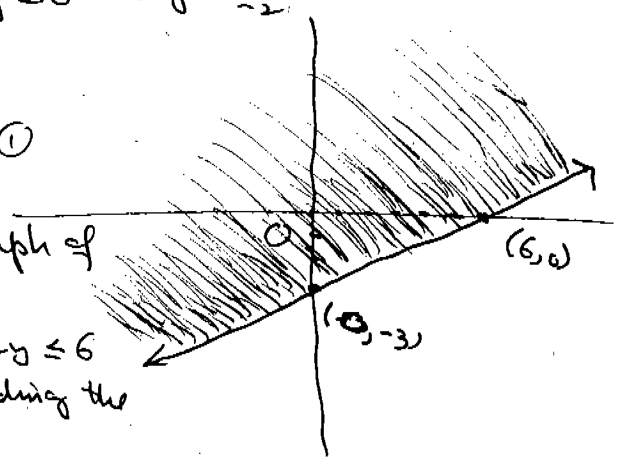
$\therefore$ (3) cuts y-axis at $(0, -3)$

putting $(0,0)$ on the L.H.S of (1)

$0 - 2(0) = 0 < 6$

$\therefore (0,0)$ lies on the side of graph of inequality (1).

Thus graph of inequality $x - 2y \leq 6$ is the upper half-plane including the graph of line (3).



The associated equation of (2) is

$$2x + y = 2 \rightarrow (4)$$

For $y = 0$, (4) $\Rightarrow 2x + 0 = 2 \quad \therefore x = 1$

$\therefore$ line (4) cuts x-axis at $(1, 0)$

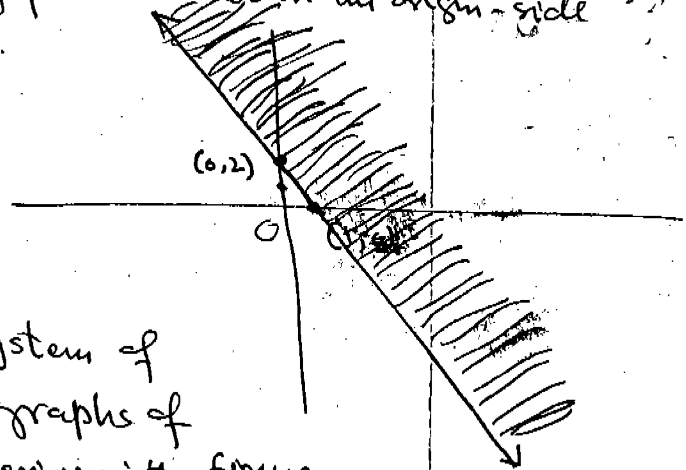
for $x = 0$, (4) $\Rightarrow 0 + y = 2 \quad \therefore y = 2$

$\therefore$ line (4) cuts y-axis at $(0, 2)$

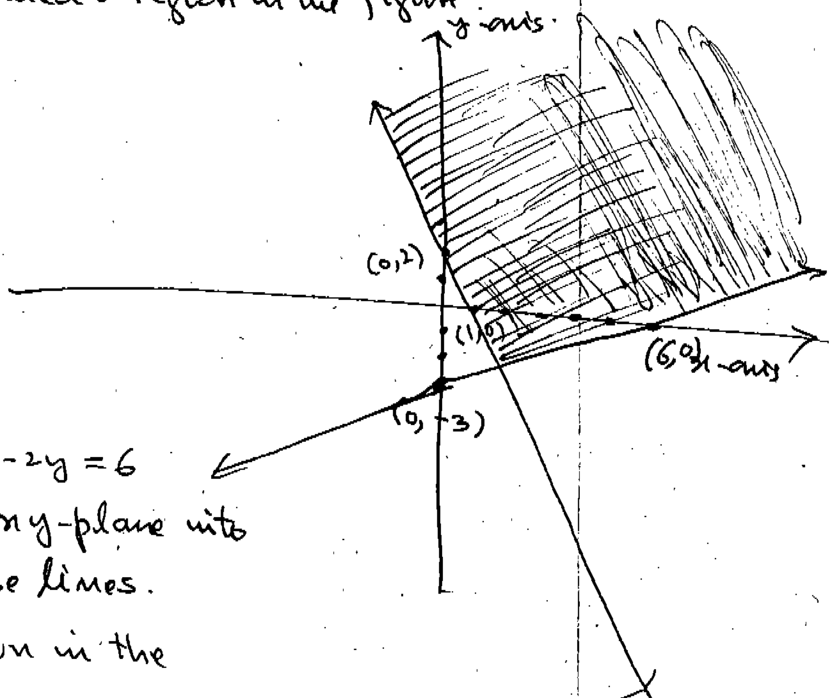
Putting $(0,0)$ in L.H.S of (2)

$2(0) + 0 = 0 < 2$ i.e $(0,0)$ does not satisfy (2)

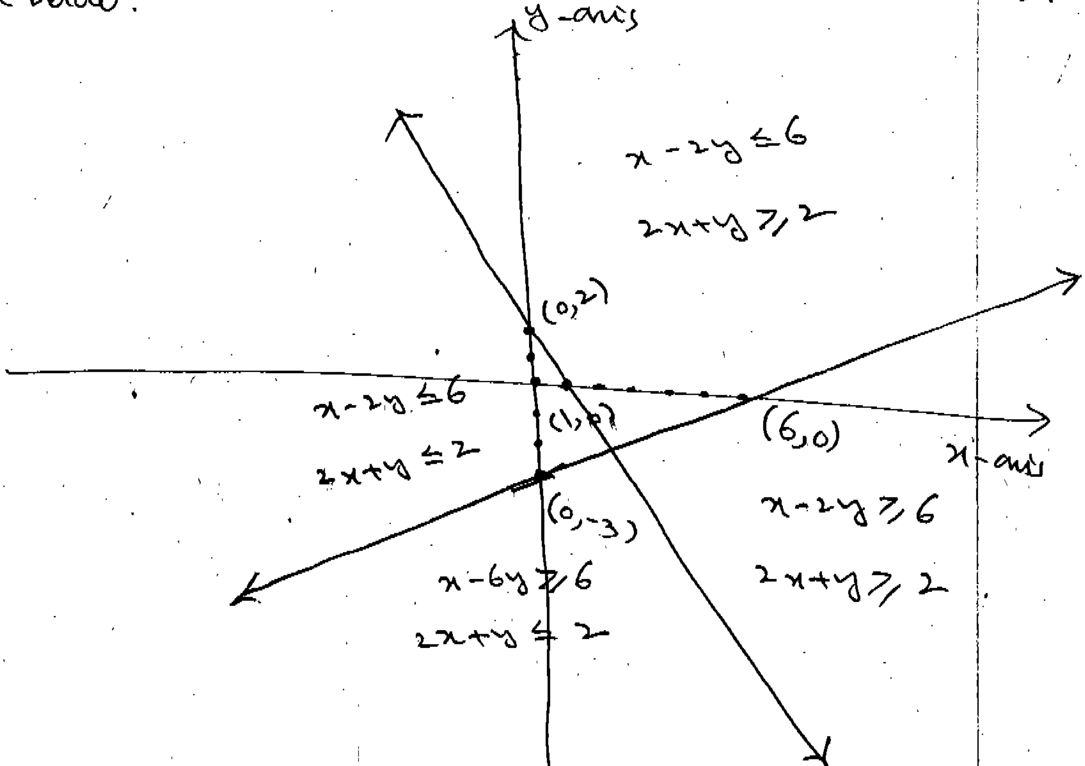
$\therefore (0,0)$ does not lie on the side of graph of (2) $\therefore$ The graph of inequality (2) is the closed half plane not on the origin-side of the line (4), as shown in the figure.



The solution region of the given system of inequalities is the intersection of the graphs of (1) and (2) as shown by shaded region in the figure.



<u>Note:</u> Note that the lines $x-2y=6$ and $2x+y=2$ divide the xy-plane into four regions bounded by these lines. These four regions are shown in the figure below.



Example: 2: Graph the solution region for the following system of inequalities

$$x - 2y \le 6 \to (1)$$
$$2x + y \ge 2 \to (2)$$
$$x + 2y \le 10 \to (3)$$

Solution: Given inequalities are

$$x - 2y \le 6 \to (1)$$
$$2x + y \ge 2 \to (2)$$
$$x + 2y \le 10 \to (3)$$

The graphs of (1) and (2) are already shown in example: 1

Now the associated equation of (3) is

$$x + 2y = 10 \to (4)$$

For $y = 0$, (4) $\Rightarrow x = 10$

$\therefore$ (4) cuts x-axis at the point $(10, 0)$

For $x = 0$, (4) $\Rightarrow 0 + 2y = 10$

$\therefore y = 5$

$\therefore$ (4) cuts y-axis at the point $(0, 5)$

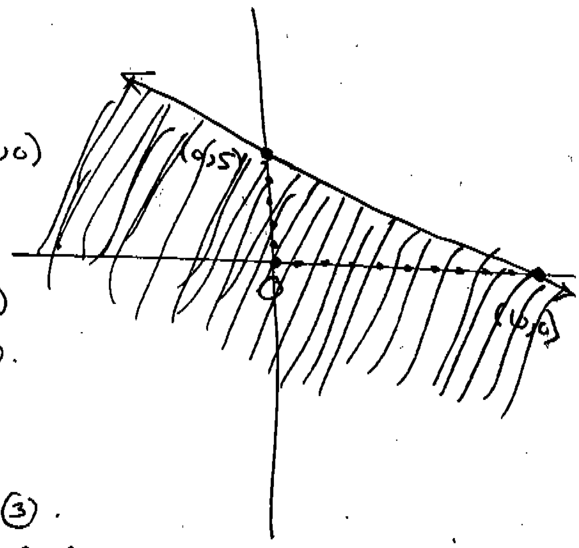
Now putting $(0,0)$ in L.H.S of (3).

$$0 + 2(0) = 0 < 10$$

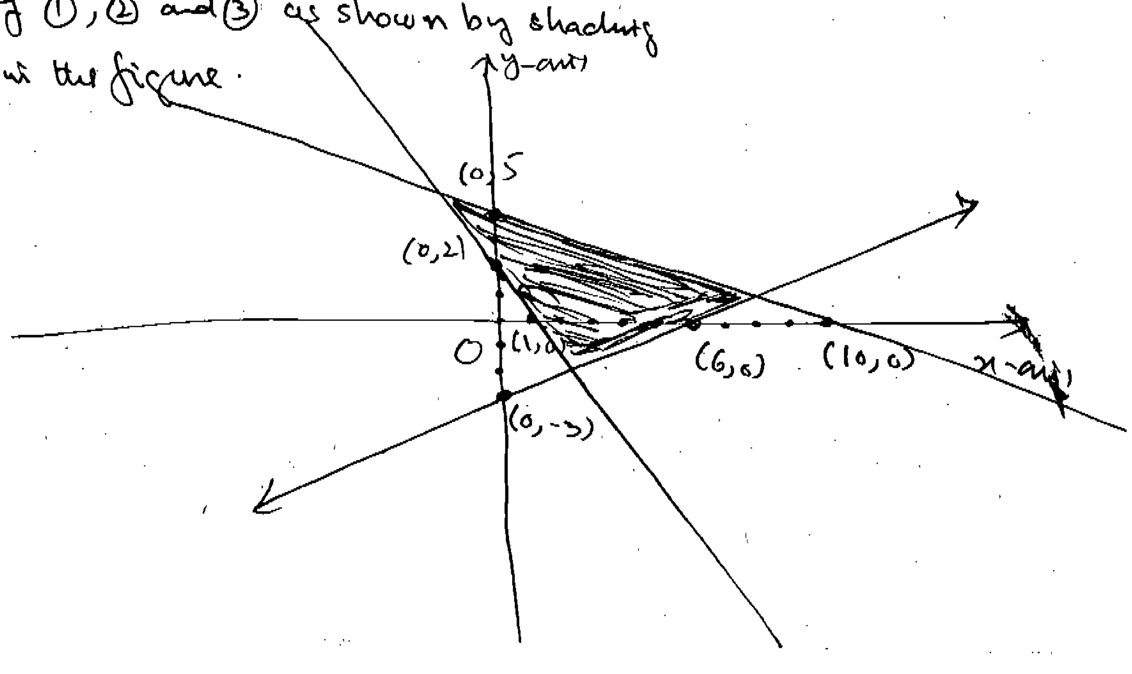
i.e (3) is satisfied.

$\therefore (0,0)$ lies on the side of graph (3).

$\therefore$ The graph of (3) is the closed half plane on the side of origin.



The solution region of system of inequalities (1), (2), (3) is the intersection of three graphs of (1), (2) and (3) as shown by shading in the figure.



Corner point of a Solution Region.

A point of a solution region where two of its boundary lines intersect, is called a corner point or vertex of the solution region

Example: 3 Graph the following systems of inequalities

(i) $2x+y \geq 2$, $x+2y \leq 10$, $y \geq 0$

(ii) $2x+y \geq 2$, $x+2y \leq 10$, $x \geq 0$

(iii) $2x+y \geq 2$, $x+2y \leq 10$, $x \geq 0, y \geq 0$

(i) Given system is

$$\left.\begin{aligned} 2x+y &\geq 2 \rightarrow (1) \\ x+2y &\leq 10 \rightarrow (2) \\ y &\geq 0 \rightarrow (3) \end{aligned}\right\} \rightarrow I$$

The corresponding equations of (1) and (2) are

$2x+y=2 \longrightarrow (3)$

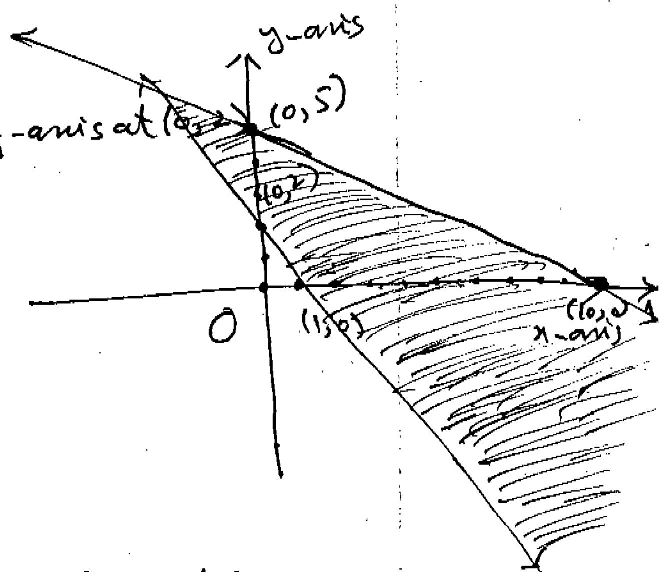
$x+2y=10 \longrightarrow (4)$

For the graph of (3),

For $y=0$, (3) $\Rightarrow 2x+0=2 \therefore x=1$

For $x=0$, (3) $\Rightarrow 0+y=2 \therefore y=2$

$\therefore$ Line (3) cuts x-axis at (1,0) and y-axis at (0,2)



For the graph of (4)

For $y=0$, $x+0=10 \therefore x=10$

For $x=0$, (4) $\Rightarrow 0+2y=10 \therefore y=5$

$\therefore$ Line (4) cuts x-axis at (10,0) and y-axis at (0,5)

Putting (0,0) on L.H.S of (1), $2(0)+0=0<2$

$\therefore$ (0,0) does not satisfy (1) $\therefore$ Graph of (1) does not on the side of origin.

Putting (0,0) in L.H.S of (2) $0+2(0)=0 \leq 10$ which is satisfied

$\therefore$ Graph of (2) is on the side of origin.

∴ The intersection of graphs of (1) and (2) is as shown in the figure 1.

Associated equation of (3) is $y = 0$ ∴ graph of (3) will be closed upper half plane.

The solution region of system (I) is as shown in figure: 2 by shading the region. The boundary of shaded region is also included in the solution region.

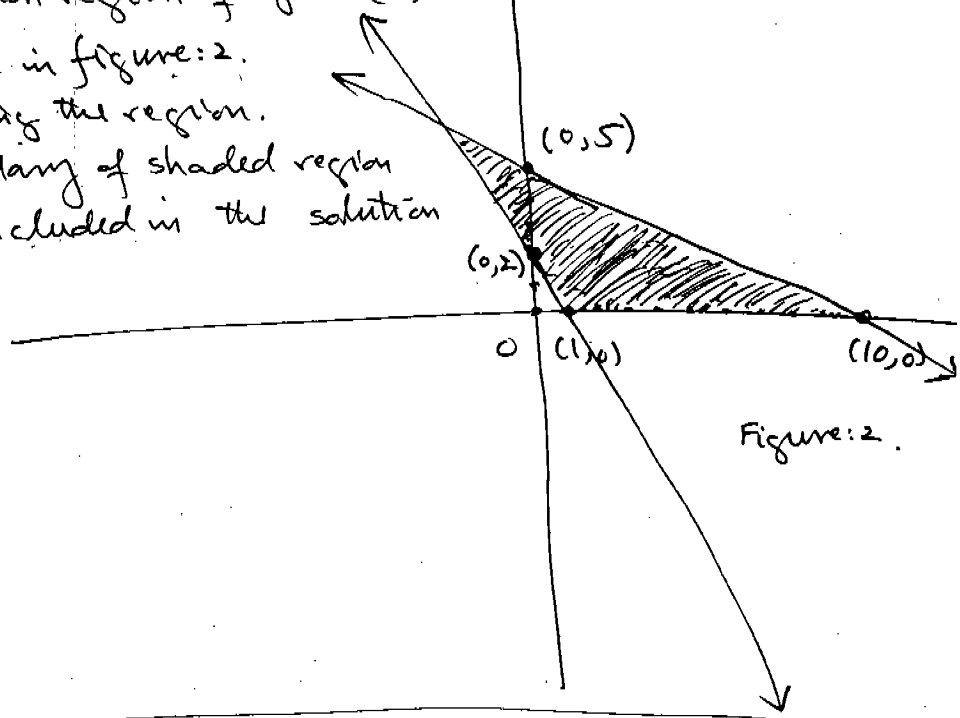


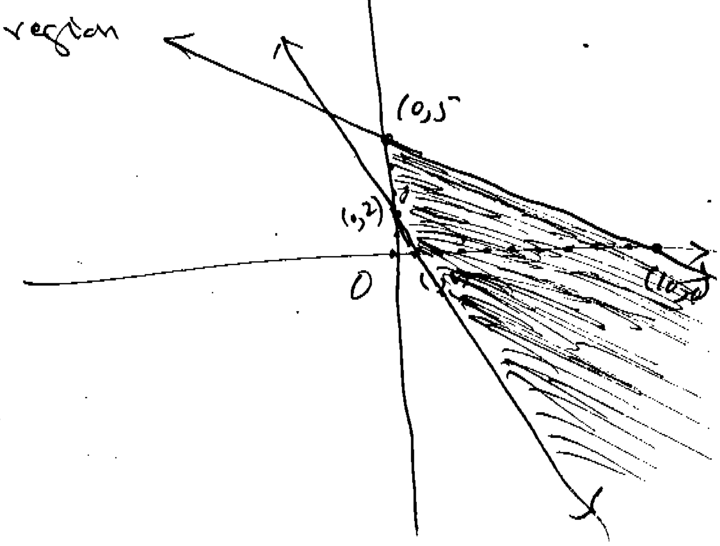
Figure: 2.

(ii)
$$\left.\begin{array}{l} 2x + y \geq 2 \rightarrow (1) \\ x + 2y \leq 10 \rightarrow (2) \\ x \geq 0 \rightarrow (3) \end{array}\right] \rightarrow (II)$$

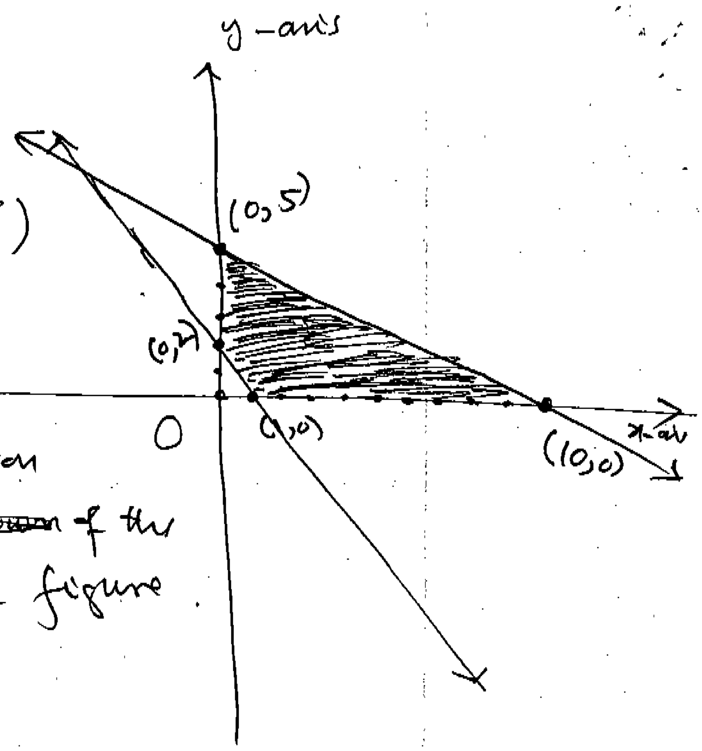
The intersection of (1) and (2) is as shown in figure: 1. of system (I)

Now (3) ⇒ the graph of (3) is the closed right half plane.

∴ The ~~S~~ solution region of system (II) has been shown in figure: 4: by shaded region



(iii) $\left.\begin{aligned} 2x+y &\geq 2 \\ x+2y &\leq 10 \\ x\geq 0,\ y &\geq 0 \end{aligned}\right\} \longrightarrow (\text{III})$



The solution region of system (III) is the shaded region including the boundary ~~shown~~ of the shaded region as shown in the figure.

Exercise: 5.1

Q.1 (i) Given inequality is

$2x+y \leq 6 \rightarrow (1)$

Associated equation of (1) is

$2x+y=6 \rightarrow (2)$

for $y=0$, (2) $\Rightarrow 2x+0=6 \quad \therefore x=3$

for $x=0$, (2) $\Rightarrow 0+y=6 \quad \therefore y=6$

$\therefore$ line (2) cuts x-axis at $(3,0)$ and y-axis at $(0,6)$

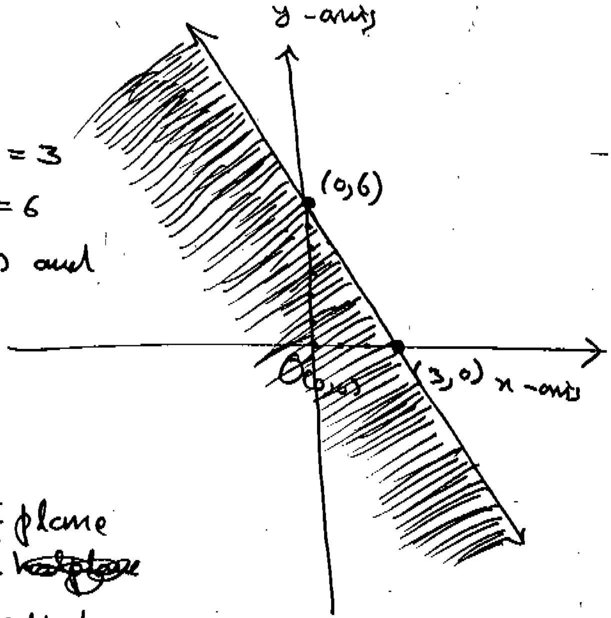
Putting $(0,0)$ in L.H.S of (1)

$2(0)+0=0+0=0<6$

$\therefore$ (1) is satisfied by $(0,0)$

$\therefore$ Graph of (1) is the closed half plane on the side of $(0,0)$, or closed ~~half plane~~ half plane containing $(0,0)$

The ~~graph~~ shaded portion part of graph is as shown in the figure.



(ii) Given inequality is

$3x+7y \geq 21 \rightarrow (1)$

The corresponding or associated equation of (1) is

$3x+7y=21 \rightarrow (2)$

For $y=0$, (2) $\Rightarrow 3x+0=21 \quad \therefore x=7$

For $x=0$, (2) $\Rightarrow 0+7y=21 \quad \therefore y=3$

$\therefore$ line (2) cuts x-axis at $(0,7)$ and y-axis at $(0,3)$

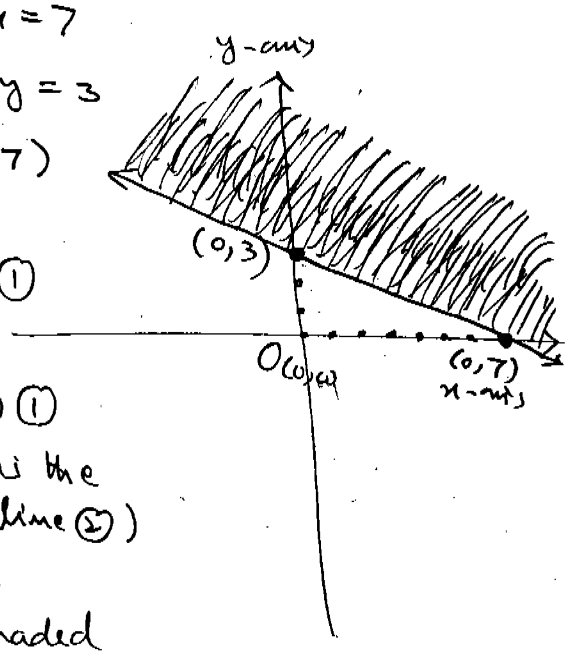
Now putting $(0,0)$ in L.H.S of (1)

$3(0)+7(0)=0+0=0<21$

i.e $(0,0)$ does not satisfy (1)

$\therefore$ The graph of inequality (1) is the closed half plane (made by line (2)) which does not contain $(0,0)$.

$\therefore$ The part of graph is the shaded portion as shown in the figure.



(2)

(iii) Given inequality is

$$3x - 2y \geq 6 \rightarrow (1)$$

Associated equation of (1) is

$$3x - 2y = 6 \rightarrow (2)$$

For $y = 0$, (1) $\Rightarrow 3x - 0 = 6 \quad \therefore x = 2$

For $x = 0$, (1) $\Rightarrow 0 - 2y = 6 \quad \therefore y = -3$

$\therefore$ line (2) cuts x-axis at $(2, 0)$ and y-axis at $(0, -3)$

The line (2) divides xy-plane into two half planes.

Now we take $(0,0)$ as test point.

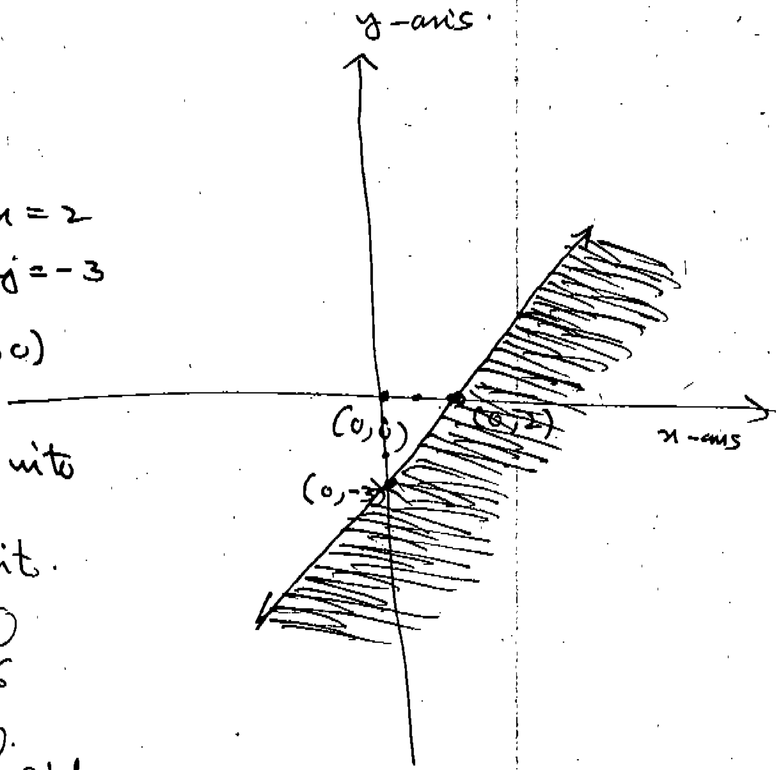
Putting $(0,0)$ in the L.H.S of (1)

$$3(0) - 2(0) = 0 - 0 = 0 < 6$$

i.e. $(0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane which does not contain $(0,0)$

$\therefore$ The part of graph is shown as shaded region as shown in the figure.



(iv) Given inequality is

$$5x - 4y \leq 20 \rightarrow (1)$$

Associated equation of (1) is

$$5x - 4y = 20 \rightarrow (2)$$

For $y = 0$, (2) $\Rightarrow 5x - 0 = 20 \quad \therefore x = 4$

For $x = 0$, (2) $\Rightarrow 0 - 4y = 20 \quad \therefore y = -5$

$\therefore$ The line (2) cuts the x-axis at $(4, 0)$ and y-axis at $(0, -5)$

The line (2) divides the xy-plane into two half planes.

Now we take $(0,0)$ as the test point.

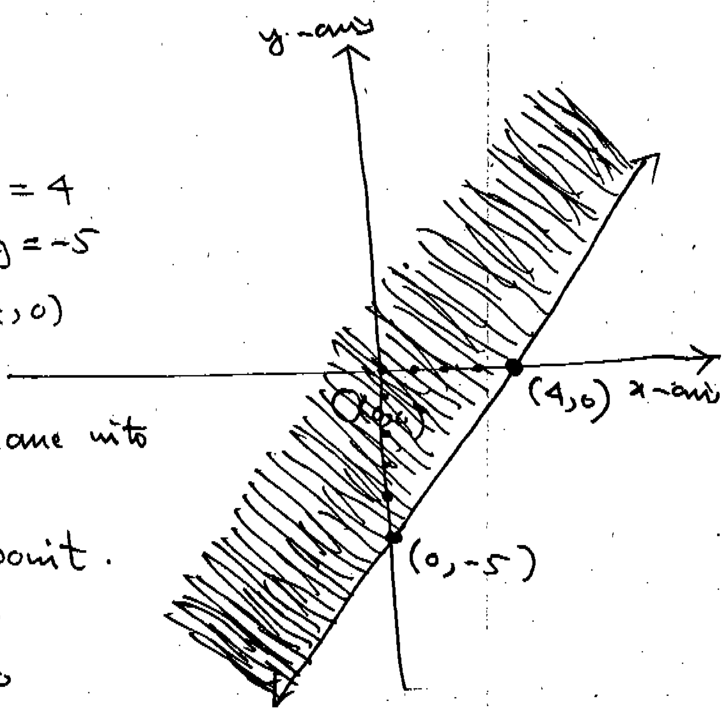
Putting $(0,0)$ in the L.H.S of (1)

$$5(0) - 4(0) = 0 - 0 = 0 < 20$$

i.e. $(0,0)$ satisfies (1)

$\therefore$ The graph of (1) is the closed half plane containing $(0,0)$ or on the side of origin $(0,0)$.

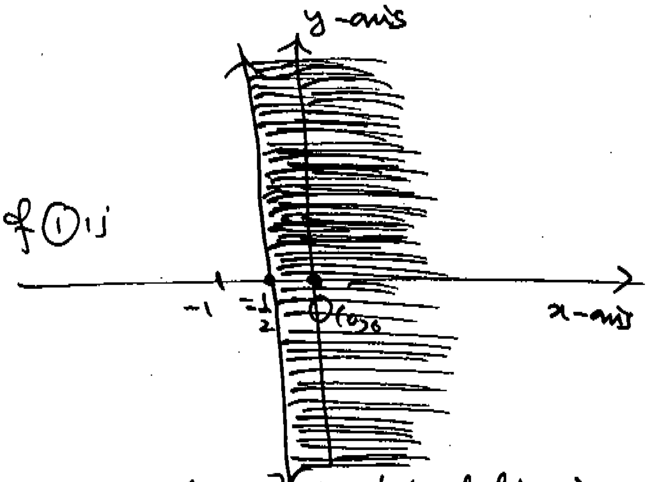
$\therefore$ The part of graph of (1) is the shaded region as shown in the figure.



③

(v) Given inequality is

$$2x+1 \geq 0 \rightarrow ①$$



The associated equation of ① is

$$2x+1=0 \rightarrow ②$$

② can be written as

$$2x=-1$$

or $x=-\frac{1}{2}$

which is a line parallel to y-axis (i.e. the vertical line) at a distance $\frac{1}{2}$ on the left side of y-axis.

The line ② divides the xy-plane into two half planes, i.e. into right half plane and left half plane.

① ⇒ $2x \geq -1$

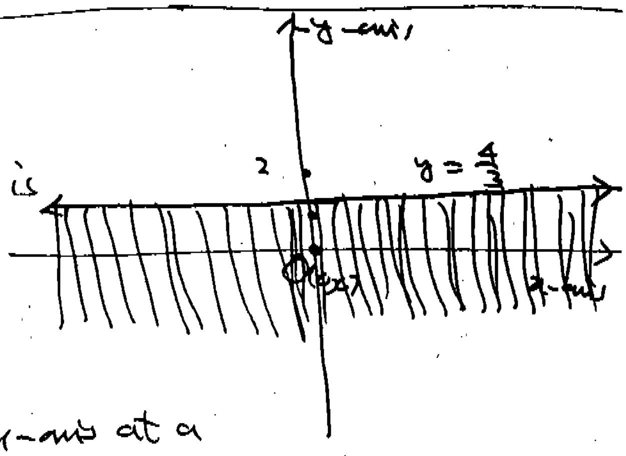
i.e. $x \geq -\frac{1}{2}$

∴ The graph of the inequality ① is the closed right half plane.

The part of graph of ① is the shaded region as shown in the figure.

(vi) Given inequality is

$$3y-4 \leq 0 \rightarrow ①$$



Associated equation of ① is

$$3y-4=0 \rightarrow ②$$

② ⇒ $3y=4$

∴ $y=\frac{4}{3}$

i.e. ② is a line parallel to x-axis at a distance of $\frac{4}{3}$ from the origin above x-axis

The line ② divides the xy-plane into two half planes, upper half plane and lower half plane.

As ① ⇒ $3y \leq 4$

∴ $y \leq \frac{4}{3}$

∴ The graph of the ① is the closed low plane.

The part of the graph of ① is the shaded region as shown in the figure.

Q.(2) ii) Given inequalities are

$$\left.\begin{aligned} 2x-3y &\le 6 \rightarrow (1) \\ 2x+3y &\le 12 \rightarrow (2) \end{aligned}\right] \rightarrow (I)$$

Associated equation of (1) is

$$2x-3y=6 \rightarrow (3)$$

Associated equation of (2) is

$$2x+3y=12 \rightarrow (4)$$

For $y=0$, (3) $\Rightarrow$ $2x-0=6$ $\therefore x=3$

For $x=0$, (3) $\Rightarrow$ $0-3y=6$ $\therefore y=-2$

$\therefore$ Line (3) cuts x-axis at $(3,0)$ and y-axis at $(0,-2)$

Now for $y=0$, (4) $\Rightarrow$ $2x+0=12$ $\therefore x=6$

for $x=0$, (4) $\Rightarrow$ $0+3y=12$ $\therefore y=4$

$\therefore$ Line (4) cuts x-axis at $(6,0)$ and y-axis $(0,4)$

Now we take $(0,0)$ as test point.

Putting $(0,0)$ in the L.H.S of (1)

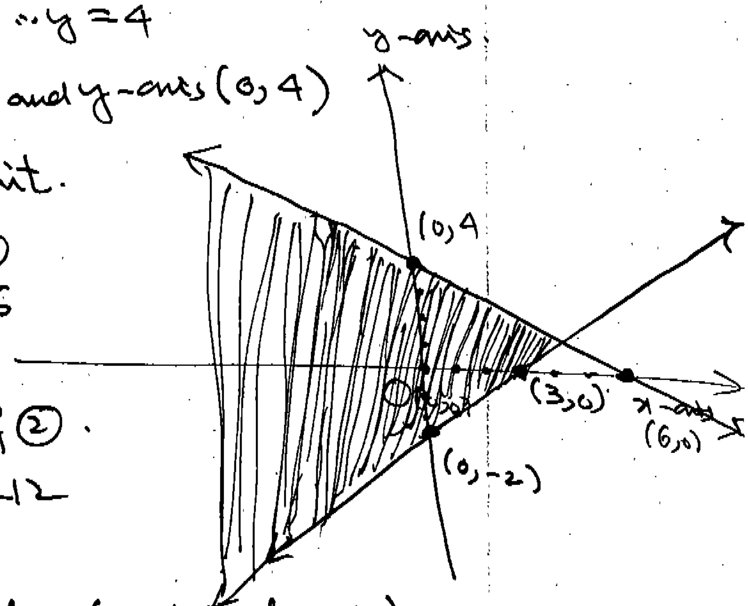
$$2(0)-3(0)=0-0=0<6$$

i.e $(0,0)$ satisfies (1)

and putting $(0,0)$ in the L.H.S of (2).

$$2(0)+3(0)=0+0=0<12$$

$\therefore$ $(0,0)$ also satisfies (2)



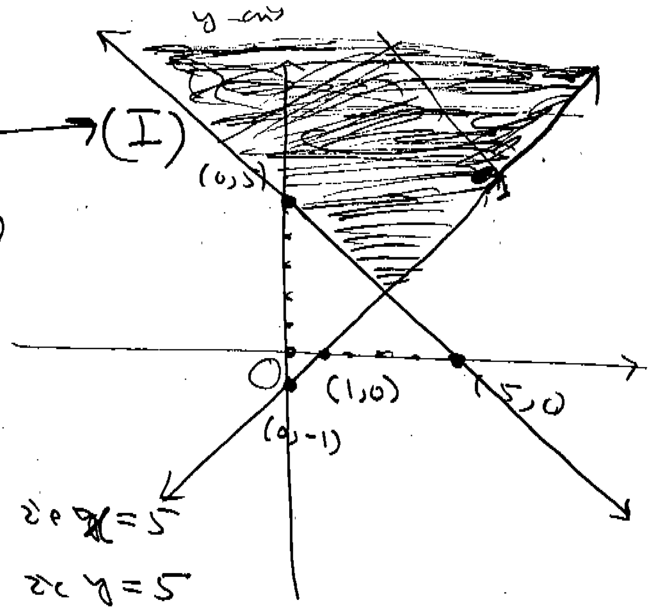
$\therefore$ Graph of (1) is the closed half plane (made by line (3)) containing $(0,0)$

Also graph of (2) is the closed half plane (mad by line (4)) containing $(0,0)$.

The intersection of these two graphs is the solution set of system (I) as shown by shading.

(ii) Given inequalities are

$$\left.\begin{array}{l} x+y \geq 5 \rightarrow (1) \\ -y+x \leq 1 \rightarrow (2) \end{array}\right] \longrightarrow (I)$$



Associated equation of (1)
is $x+y=5 \longrightarrow (3)$

Associated equation of (2)
is $-y+x=1 \longrightarrow (4)$

For $y=0$, (3) $\Rightarrow x+0=5$ i.e $x=5$

For $x=0$, (3) $\Rightarrow 0+y=5$ i.e $y=5$

$\therefore$ Line (3) cuts x-axis at (5,0) and y-axis at (0,5)

Now for $y=0$, (4) $\Rightarrow -0+x=1 \quad \therefore x=1$

for $x=0$, (4) $\Rightarrow -y+0=1 \quad \therefore y=-1$

$\therefore$ (4) cuts x-axis at (1,0) and y-axis at (0,-1)

Now we take (0,0) at the test point.

Putting (0,0) in the L.H.S of (1)

$0+0=0<5$

i.e (0,0) does not satisfy (1).

$\therefore$ The graph of (1) is the closed half plane (made by line (3)) not on the side of origin.

Now putting (0,0) in the L.H.S of (2)

$-0+0=0<1$

$\therefore$ (0,0) satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (4)) on the side of origin.

The intersection of graphs of (1) and (2) is the solution set of system (I) as shown by shading in the figure.

(iii) Given inequalities are

$$\left.\begin{array}{l} 3x+7y \geq 21 \rightarrow (1) \\ x-y \leq 2 \rightarrow (2) \end{array}\right] \rightarrow (I)$$

The associated equation of (1) is

$$3x+7y=21 \rightarrow (3)$$

The associated equation of (2) is

$$x-y=2 \rightarrow (4)$$

For $y=0$, (3) $\Rightarrow 3x+0=21 \quad \therefore x=7$

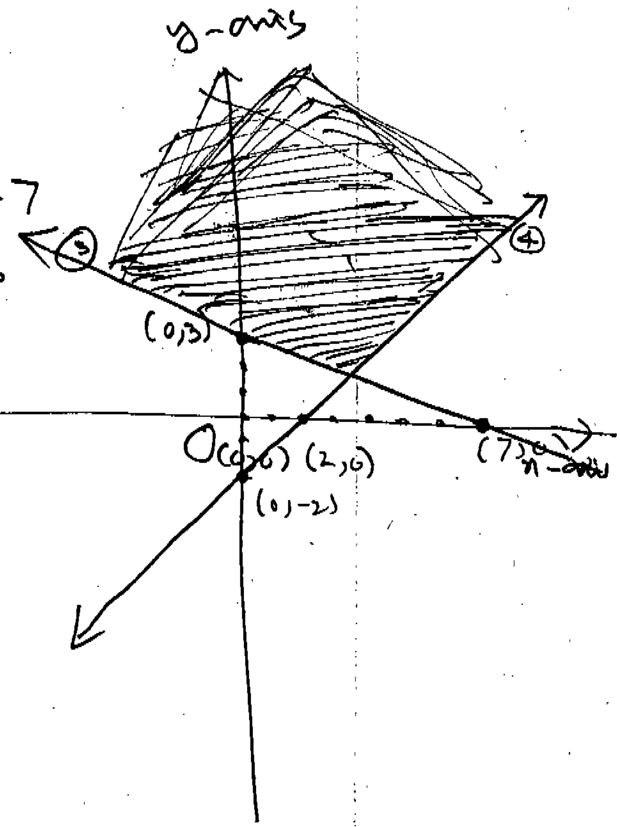
For $x=0$, (3) $\Rightarrow 0+7y=21 \quad \therefore y=3$

$\therefore$ The line (3) cuts x-axis at (7, 0) and y-axis at (0, 3)

For $y=0$, (4) $\Rightarrow x-0=2 \quad \therefore x=2$

For $x=0$, (4) $\Rightarrow 0-y=2 \quad \therefore y=-2$

$\therefore$ The line (4) cuts x-axis at (2, 0) and y-axis at (0, −2).



Now we take (0,0) as test point.

Putting (0,0) in the L.H.S of (1)

$$3(0)+7(0)=0+0=0<21$$

$\therefore$ (0,0) does not satisfy (1).

$\therefore$ The graph of (1) is the closed half plane (made by line (3)) not on the side of origin.

Now putting (0,0) in the L.H.S of (2).

$$0-0=0<2$$

i.e (0,0) satisfies (2).

$\therefore$ The graph of (2) is the closed half plane (made by line (4)) on the side of origin.

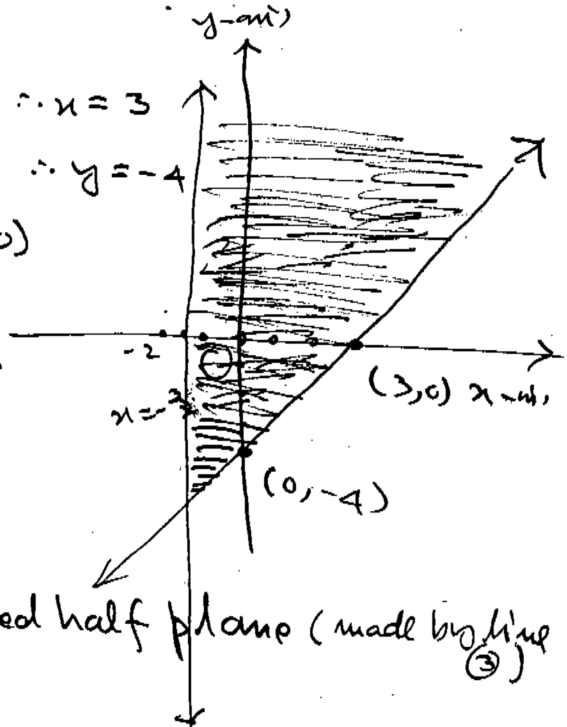
The intersection of graphs of (1) and (2) is the solution set of system (I) as shown in the diagram by shading.

(iv) Given inequalities are

$4x-3y \le 12 \longrightarrow (1)$
$x \geq -\frac{3}{2} \longrightarrow (2)$ $\Big] \longrightarrow (I)$

Associated equation of (1) is
$4x-3y=12 \longrightarrow (3)$
Associated equation of (2) is
$x=-\frac{3}{2} \longrightarrow (4)$

For $y=0$, (3) $\Rightarrow$ $4x-0=12$ $\therefore x=3$
For $x=0$, (3) $\Rightarrow$ $0-3y=12$ $\therefore y=-4$
$\therefore$ Line (3) cuts x-axis at $(3,0)$
and y-axis at $(0,-4)$
we take $(0,0)$ as test point.
Putting $(0,0)$ in L.H.S of (1)
$4(0)-3(0)=0-0=0<12$
$\therefore (0,0)$ satisfies (1)
$\therefore$ The graph of (1) is the closed half plane (made by line (3))
on the side of origin.



(4) is the line ∥ to y-axis at a distance of $\frac{3}{2}$ from
the origin on the Left side of y-axis.
Now the line (4) divides xy-plane into right half plane
and left half plane.

as $x \geq -\frac{3}{2}$
$\therefore$ Graph of (2) is closed right half plane.

The intersection of graphs of (1) and (2) is the
solution set of system (I) as shown in the figure
by shading.

(8)

(v) Given inequalities are

$$\left.\begin{array}{l} 3x+7y \geq 21 \longrightarrow (1) \\ y \leq 4 \longrightarrow (2) \end{array}\right] \rightarrow (I)$$

The associated (or corresponding) equation of (1) is

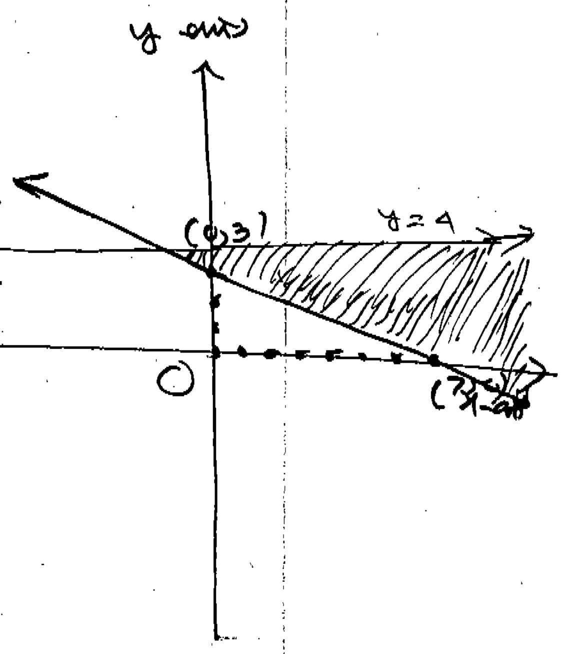
$$3x+7y=21 \longrightarrow (3)$$

Associated equation of (2) is

$$y=4 \longrightarrow (4)$$

For $y=0$, (3) $\Rightarrow 3x+0=21 \quad \therefore x=7$

For $x=0$, (3) $\Rightarrow 0+7y=21 \quad \therefore y=3$.



$\therefore$ The line (3) cuts x-axis at (7,0) and y-axis at (0,3)

We take (0,0) as test point.

putting (0,0) in the left side of (1)

$$3(0)+7(0)=0+0=0<21$$

$\therefore$ (0,0) does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (3)) not on the side of (0,0)

line (4) is a line parallel to x-axis.

and is at distance of 4 units from the origin and above x-axis.

~~The graph of (2) is the~~

as $y \leq 4$

$\therefore$ graph of (2) is the closed lower half plane (made by line (4))

The intersection of graphs (1) and (2) is the solution set of system (I) as shown in the figure by shading.

Q.3: 1: Given inequalities are

$$\left.\begin{array}{l} 2x-3y \le 6 \rightarrow (1) \\ 2x+3y \le 12 \rightarrow (2) \\ y \ge 0 \rightarrow (3) \end{array}\right\} \rightarrow (I)$$

Associated equations of (1), (2) and (3) are

$2x-3y = 6 \rightarrow (4)$

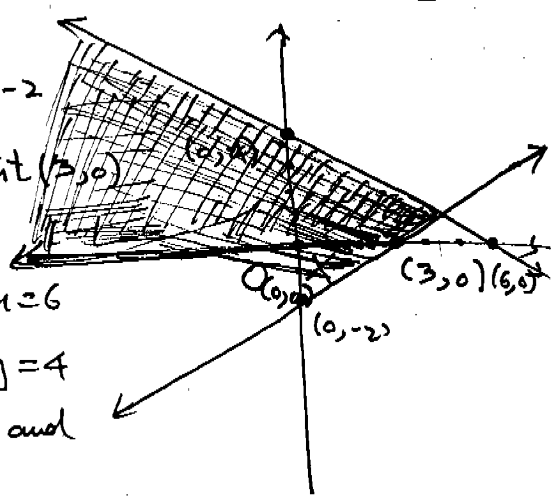
$2x+3y = 12 \rightarrow (5)$

$y = 0 \rightarrow (6)$

For $y=0$, $(4) \Rightarrow 2x-0=6 \quad \therefore x=3$

For $x=0$, $(4) \Rightarrow 0-3y=6 \quad \therefore y=-2$

$\therefore$ Line (4) cuts x-axis at the point $(3,0)$ and y-axis at the point $(0,-2)$



Now for $y=0$, $(5) \Rightarrow 2x+0=12 \quad \therefore x=6$

for $x=0$, $(5) \Rightarrow 0+3y=12 \quad \therefore y=4$

$\therefore$ Line (5) cuts x-axis at $(6,0)$ and y-axis at $(0,4)$.

Now we take test point $(0,0)$.

Putting $(0,0)$ in the L.H.S of (1)

$2(0)-3(0) = 0-0 = 0 < 6$

$\therefore (0,0)$ satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin.

Now putting $(0,0)$ in the L.H.S of (2)

$2(0)+3(0) = 0+0 = 0 < 12$

$\therefore$ (2) is also satisfied by $(0,0)$.

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

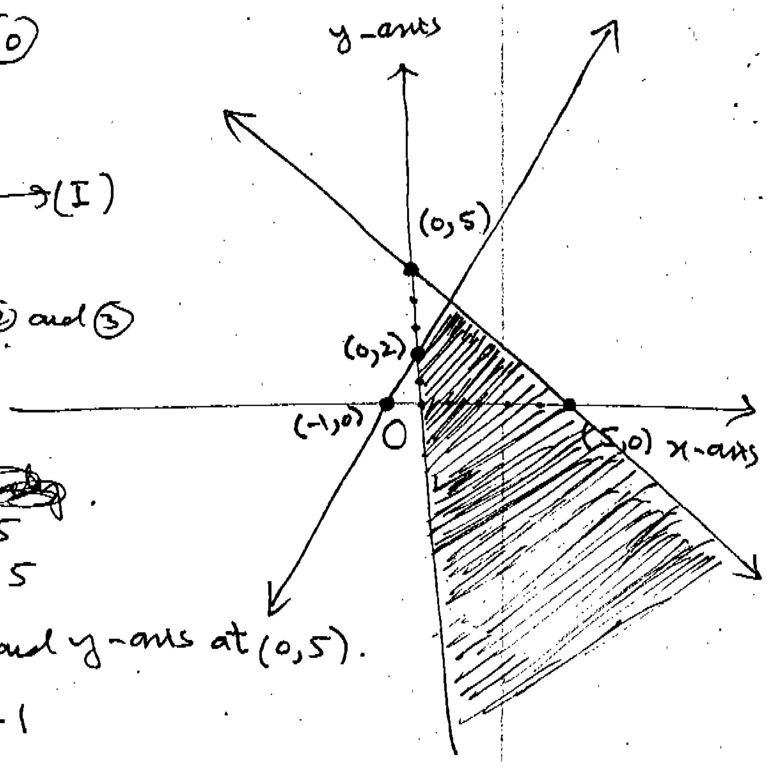
(6) is the equation of x-axis.

Graph of (3) is the closed upper half plane.

The intersection of the graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.

(ii) Given inequalities are

$$\left.\begin{array}{l} x+y \le 5 \rightarrow (1) \\ y-2x \le 2 \rightarrow (2) \\ x \geq 0 \rightarrow (3) \end{array}\right] \rightarrow (I)$$



Associated equations of (1), (2) and (3)

are $x+y=5 \rightarrow (4)$

$y-2x=2 \rightarrow (5)$

$x = 0 \rightarrow (6)$ ~~respectively~~.

For $y=0$, (4) $\Rightarrow x+0=5 \therefore x=5$

For $x=0$, (4) $\Rightarrow 0+y=5 \therefore y=5$

$\therefore$ Line (1) cuts x-axis at (5,0) and y-axis at (0,5).

For $y=0$, (5) $\Rightarrow 0-2x=2 \therefore x=-1$

For $x=0$, (5) $\Rightarrow y-0=2 \therefore y=2$

$\therefore$ Line (2) cuts x-axis at (-1,0) and y-axis at (0,2)

Now we take (0,0) as test point.

Putting (0,0) in the L.H.S of (1)

$0+0=0<5$

i.e (1) is satisfied by (0,0).

$\therefore$ Graph of (1) is the closed half plane (intercepted by line (4))
on the sides of (0,0)

Putting (0,0) in the L.H.S of (2)

$0-2(0)=0-0=0<2$

$\therefore$ (2) is also satisfied by (0,0).

$\therefore$ Graph of (2) is the closed half plane (intercepted by line (5))
on the side of (0,0).

(6) is the equation of y-axis.

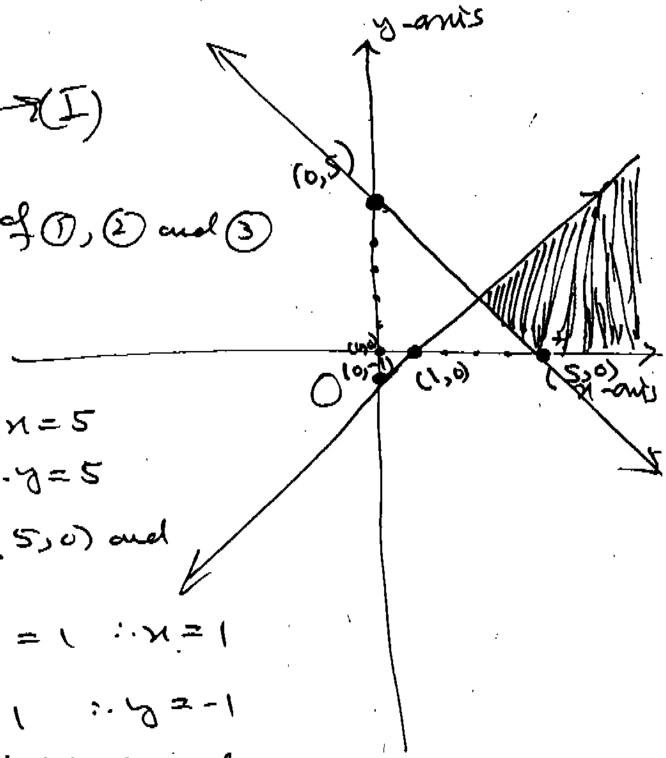
Now the line $x=0$, divides the xy-plane into right half plane and left half plane.

Now $x \geq 0 \Rightarrow$ the graph of (3) is the closed right half plane.

The intersection of graphs of (1), (2), (3) is the solution region of system (I) as shown in the figure by shading.

(iii) Given inequalities are

$x+y \geq 5 \rightarrow (1)$
$x-y \geq 1 \rightarrow (2)$
$y \geq 0 \rightarrow (3)$ $\Bigg] \rightarrow (I)$



The associated equations of (1), (2) and (3)

are $x+y = 5 \rightarrow (4)$
$x-y = 1 \rightarrow (5)$
$y = 0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow x+0=5$ $\therefore x=5$

For $x=0$, (4) $\Rightarrow 0+y=5$ $\therefore y=5$

$\therefore$ Line (4) cuts x-axis at $(5,0)$ and y-axis at $(0,5)$.

Now for $y=0$, (5) $\Rightarrow x-0=1$ $\therefore x=1$

for $x=0$, (5) $\Rightarrow 0-y=1$ $\therefore y=-1$

$\therefore$ Line (5) cuts x-axis at $(1,0)$ and y-axis at $(0,-1)$.

Now we take $(0,0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1)

$0+0=0<5$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (intersected by line (4)) not on the side of $(0,0)$.

Putting $(0,0)$ in the L.H.S of (2)

$0-0=0<1$

$\therefore (0,0)$ also does not satisfy (2).

$\therefore$ Graph of (2) is the closed half plane (by the division of xy-plane into two planes by line (5)) not on the side of $(0,0)$.

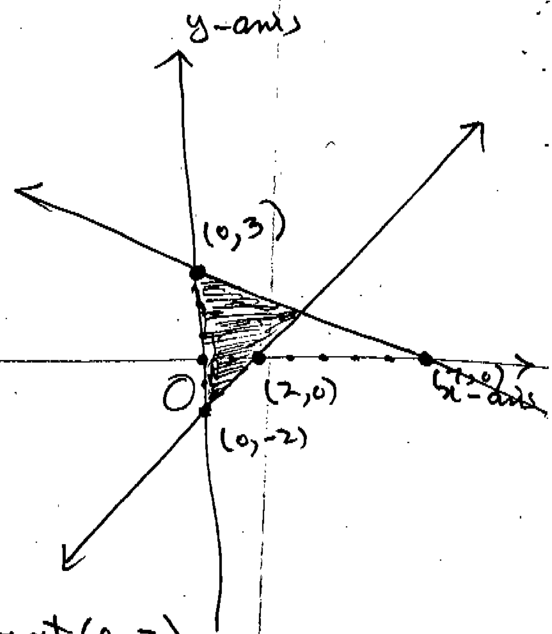
$y=0$ is the line of x-axis.

$\therefore$ The graph of $y \geq 0$ is the closed upper half plane of the xy-plane.

The intersection of the graphs of (1), (2), (3) is the solution region of the system (I) as shown in the figure by shading.

(iv) Given inequalities are

$$\left.\begin{array}{l} 3x+7y\le 21 \rightarrow (1) \\ x-y \le 2 \rightarrow (2) \\ x\ge 0 \rightarrow (3) \end{array}\right] \rightarrow (I)$$



The associated equations of (1), (2) and (3) are

$3x+7y=21 \rightarrow (4)$

$x-y=2 \rightarrow (5)$

$x=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow 3x+0=21$ $\therefore x=7$

For $x=0$, (4) $\Rightarrow 0+7y=21$ $\therefore y=3$

$\therefore$ Line (4) cuts x-axis at (7,0) and y-axis at (0,3)

For $y=0$, (5) $\Rightarrow x-0=2$ $\therefore x=2$

For $x=0$, (5) $\Rightarrow 0-y=2$ $\therefore y=-2$

$\therefore$ Line (5) cuts x-axis at (2,0) and y-axis (0, -2)

We take (0,0) as the test point.

Putting (0,0) in the L.H.S of (1).

$3(0)+7(0)=0+0=0<21$

$\therefore$ (0,0) satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of (0,0).

Putting (0,0) in the L.H.S of (2)

$0-0=0<2$

$\therefore$ Also (0,0) satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the side of (0,0).

and (6) is the equation of y-axis.

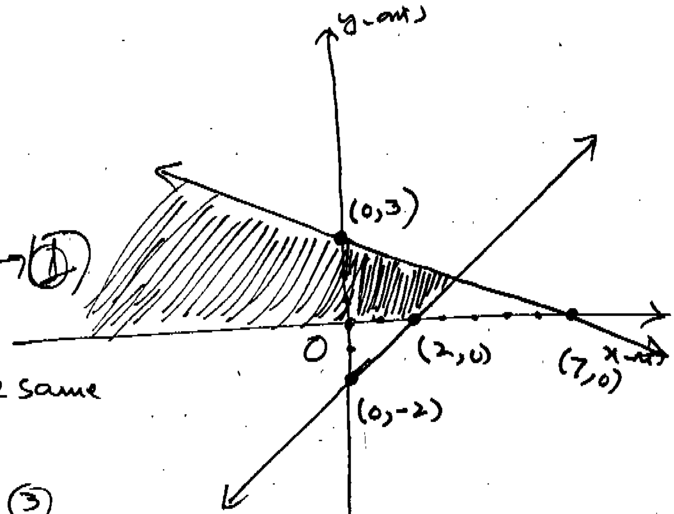
$\therefore$ The graph of $x\ge 0$, is the right closed half plane of xplane formed by y-axis.

The intersection of graphs of (1), (2), (3) is the solution region of system (I) as shown in the figure by shading.

(13)

(v) Given inequalities are

$$\left.\begin{aligned} 3x+7y &\le 21 \rightarrow (1) \\ x-y &\le 2 \rightarrow (2) \\ y &\ge 0 \rightarrow (3) \end{aligned}\right] \rightarrow (I)$$



The graphs of (1) and (2) are same as in part (iv).

Here associated equation of (3) is $y=0$

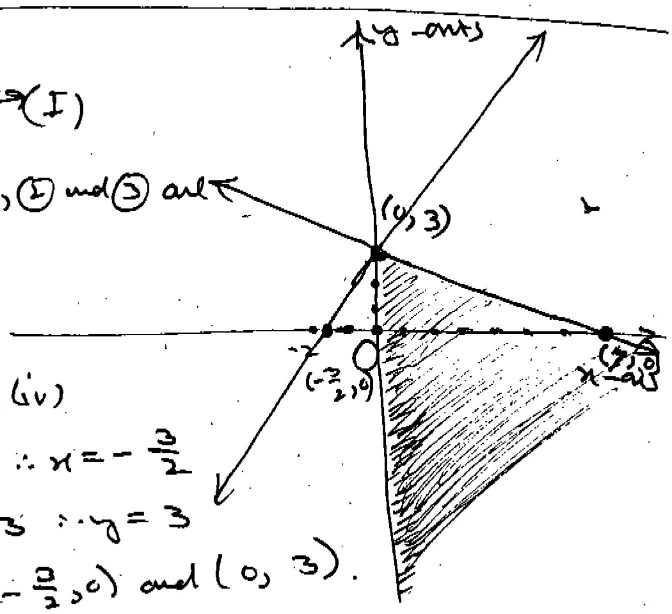
and $y=0$ is the equation of x-axis.

$\therefore$ Graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.

(vi)

$$\left.\begin{aligned} 3x+7y &\le 21 \rightarrow (1) \\ 2x-y &\ge -3 \rightarrow (2) \\ x &\ge 0 \rightarrow (3) \end{aligned}\right] \rightarrow (I)$$



Associated equations of (1), (2) and (3) are

$3x+7y = 21 \rightarrow (4)$

$2x-y = -3 \rightarrow (5)$

$x = 0 \rightarrow (6)$

Graph of (1) is as in part (iv)

For $y=0$, (5) $\Rightarrow 2x-0=-3 \quad \therefore x = -\frac{3}{2}$

For $x=0$, (5) $\Rightarrow 0-y=-3 \quad \therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(-\frac{3}{2},0)$ and $(0,3)$.

We take $(0,0)$ as the test point.

Putting $(0,0)$ in the L.H.S of (2).

$2(0)-0 = 0-0 = 0 > -3$

$\therefore (0,0)$ satisfies (2)

$\therefore$ The graph of (2) is the closed half plane (formed by line (5)) on the side of $(0,0)$.

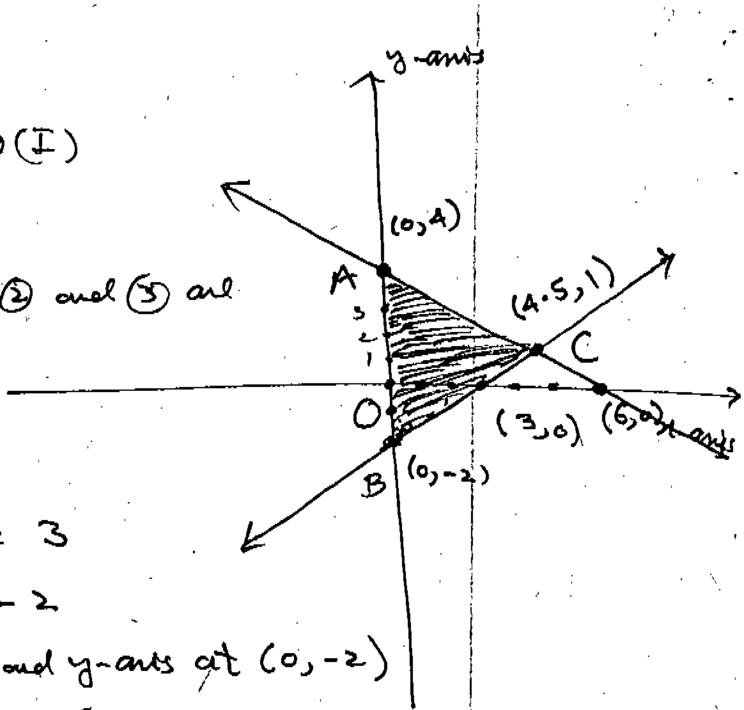
$x=0$ is the equation of y-axis.

$\therefore$ The graph of $x \ge 0$ is the closed right half plane of xy-plane.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.

(4) (i) Given inequalities are

$$\left.\begin{array}{l} 2x-3y \le 6 \rightarrow (1) \\ 2x+3y \le 12 \rightarrow (2) \\ x \geq 0 \rightarrow (3) \end{array}\right] \rightarrow (I)$$



The associated equations of (1), (2) and (3) are

$$2x-3y = 6 \rightarrow (4)$$
$$2x+3y = 12 \rightarrow (5)$$
$$x = 0 \rightarrow (6)$$

For $y=0$, (4) $\Rightarrow 2x-0=6 \therefore x=3$

For $x=0$, (4) $\Rightarrow 0-3y=6 \therefore y=-2$

$\therefore$ Line (1) cuts x-axis at $(3,0)$ and y-axis at $(0,-2)$

For $y=0$, (5) $\Rightarrow 2x+0=12 \therefore x=6$

For $x=0$, (5) $\Rightarrow 0+3y=12 \therefore y=4$

$\therefore$ Line (2) cuts x-axis at $(6,0)$ and y-axis at $(0,4)$

We take origin $(0,0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1).

$$2(0)-3(0)=0-0=0<6$$

$\therefore$ (1) is satisfied by $(0,0)$

Putting $(0,0)$ in L.H.S of (2)

$$2(0)+3(0)=0+0=0<12$$

$\therefore (0,0)$ satisfies (2).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$ and graph of (2) is the closed half plane (formed by line (5)) on the side of $(0,0)$.

$x=0$ is the equation of y-axis.

$\therefore$ Graph of $x \geq 0$ is the close right half plane of xy-plane.

The intersection of graphs of (1), (2), (3) is the solution region as shown in the figure by shading.

Here corner points of solution region are $A(0,4)$, $B(0,-2)$, $C(4.5,1)$

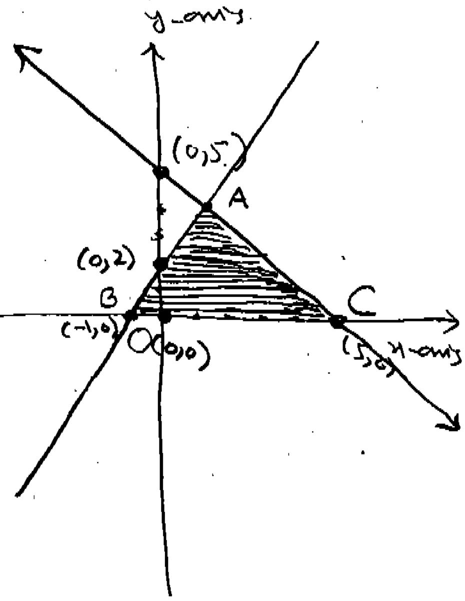
A is point of intersection of (5) and (6)

B is point of intersection of (4) and (6)

C is point of intersection of (4) and (5).

(ii) Given inequalities are

$$\left.\begin{array}{l} x+y \le 5 \rightarrow (1) \\ -2x+y \le 2 \rightarrow (2) \\ y \ge 0 \rightarrow (3) \end{array}\right\} \rightarrow (I)$$



The associated equation of (1), (2) and (3)

are $x+y=5 \rightarrow (4)$

$-2x+y = 2 \rightarrow (5)$

$y=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow x+0=5 \quad \therefore x=5$

for $x=0$, (4) $\Rightarrow 0+y=5 \quad \therefore y=5$

$\therefore$ Line (1) cuts x-axis at $(5,0)$ and y-axis at $(0,5)$

For $y=0$, (5) $\Rightarrow -2x+0=2 \quad \therefore x=-1$

For $x=0$, (5) $\Rightarrow -0+y=2 \quad \therefore y=2$

$\therefore$ Line (5) cuts x-axis at $(-1,0)$ and y-axis at $(0,2)$

We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$0+0=0<5$

$\therefore (0,0)$ satisfies (1)

Now putting $(0,0)$ in L.H.S of (2)

$-2(0)+0 = 0+0 = 0 < 2$

$\therefore (0,0)$ also satisfies (2).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin $(0,0)$.

Graph of (2) is the closed half plane (made by line (5)) on the side of origin $(0,0)$.

$y=0$ is x-axis.

$\therefore$ Graph of $y \ge 0$ is the closed upper half plane of the xy-plane.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.

Here A, B, C are the corner points of solution region.

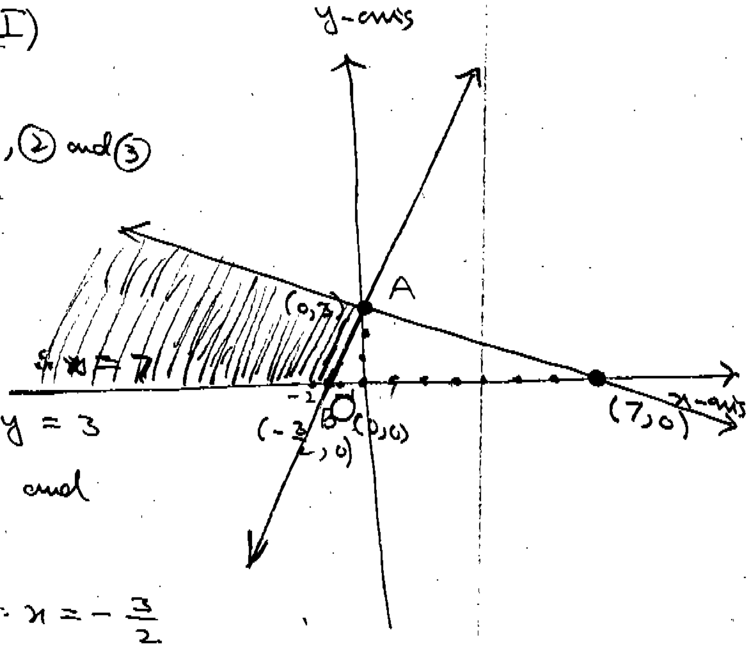
$A(1,4)$ is the point of intersection of (4) and (5)

$B(-1,0)$ is the point of intersection of (5) and (6)

$C(5,0)$ is the point of intersection of (4) and (6).

(iii) Given inequalities are

$$\left.\begin{array}{l} 3x+7y \le 21 \rightarrow (1) \\ 2x-y \le -3 \rightarrow (2) \\ y \ge 0 \rightarrow (3) \end{array}\right\} \rightarrow (I)$$



The associated equations of (1), (2) and (3) are $3x+7y=21 \rightarrow (4)$

$2x-y=-3 \rightarrow (5)$

$y=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow$ $3x+0=21$ ∴ $x=7$

For $x=0$, (4) $\Rightarrow$ $0+7y=21$ ∴ $y=3$

∴ Line (4) cuts x-axis at $(7,0)$ and y-axis at $(0,3)$.

For $y=0$, (5) $\Rightarrow$ $2x-0=-3$ ∴ $x=-\frac{3}{2}$

For $x=0$, (5) $\Rightarrow$ $0-y=-3$ ∴ $y=3$

∴ Line (5) cuts x-axis at $(-\frac{3}{2},0)$ and y-axis at $(0,3)$

We take the test point $(0,0)$

Putting $(0,0)$ in the L.H.S of (1)

$3(0)+7(0)=0+0=0<21$

∴ $(0,0)$ satisfies (1). ∴ The graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Now putting $(0,0)$ in the L.H.S of (2)

$2(0)-0=0-0=0 > -3$

∴ $(0,0)$ does not satisfy (2)

∴ The graph of (2) is the closed half plane (made by line (5)) not on the side of $(0,0)$.

$y=0$ is the x-axis.

∴ Graph of $y \ge 0$, is the upper half plane of xy-plane.

The intersection of graphs of (1), (2), (3) is the solution region ~~as shown~~ of system (I) as shown in the figure by shading the region.

Here A and B are only two corner points of solution region.

$A(0,3)$ is pt. of intersection of lines (4) and (5)

$B(-\frac{3}{2},0)$ is the pt of intersection of lines (5) and (6).

(iv) Given inequalities are

$$\left.\begin{array}{l} 3x+2y \geq 6 \to (1) \\ x+3y \leq 6 \to (2) \\ y \geq 0 \to (3) \end{array}\right] \to (I)$$

$9x+6y=18$
$2x+6y=12$
$7x=6$
$x=\frac{6}{7}$
$3y=6-\frac{6}{7}$
$=\frac{36}{7}$
$y=\frac{12}{7}$

The associated equations of (1), (2) and (3) are $3x+2y=6 \to (4)$

$x+3y=6 \to (5)$

$y=0 \to (6)$

For $y=0$, (4) $\Rightarrow 3x+0=6 \;\therefore x=2$

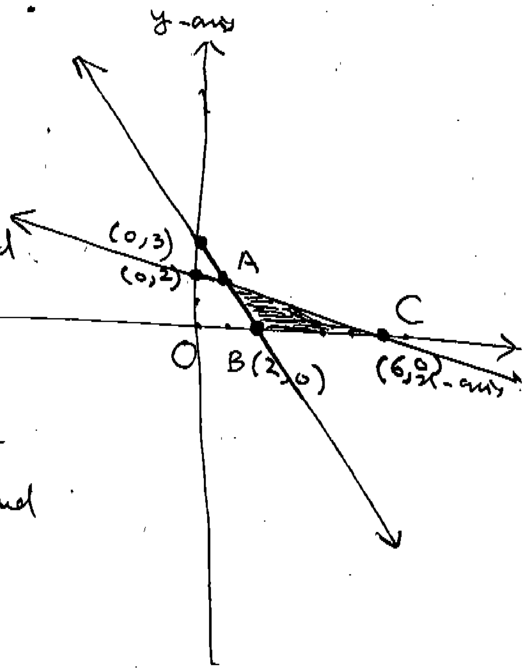
for $x=0$, (4) $\Rightarrow 0+2y=6 \;\therefore y=3$

$\therefore$ Line (4) cuts x-axis at $(2,0)$ and y-axis at $(0,3)$.

For $y=0$, (5) $\Rightarrow x+0=6 \;\therefore x=6$

for $x=0$, (5) $\Rightarrow 0+3y=6 \;\therefore y=2$

$\therefore$ Line (5) cuts x-axis at $(6,0)$ and y-axis at $(0,2)$



Now we take test point as $(0,0)$

Putting $(0,0)$ in L.H.S of (1)

$3(0)+2(0)=0+0=0<6$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) not on the side of $(0,0)$.

Now putting $(0,0)$ in L.H.S of (2)

$0+3(0)=0+0=0<6$

$\therefore (0,0)$ satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

$y=0$ is the x-axis.

$\therefore$ Graph of $y \geq 0$ is the closed upper half plane of xy-plane.

The intersection of graph of (1), (2), (3) is the solution region of system (I) as shown in the figure by shading the region.

Here corner points of solution region are A, B, C.

$A(\frac{6}{7}, \frac{12}{7})$ is the pt. of intersection of (4) and (5)

$B(2,0)$ is the point of intersection of (4) and (6)

$C(6,0)$ is the point of intersection of (5) and (6).

$5x+7y=35$
$-5x+15y=15$
$\therefore 22y=50 \quad \therefore y=\frac{25}{11}$
$x=3y-3=\frac{75}{11}-3=\frac{42}{11}$

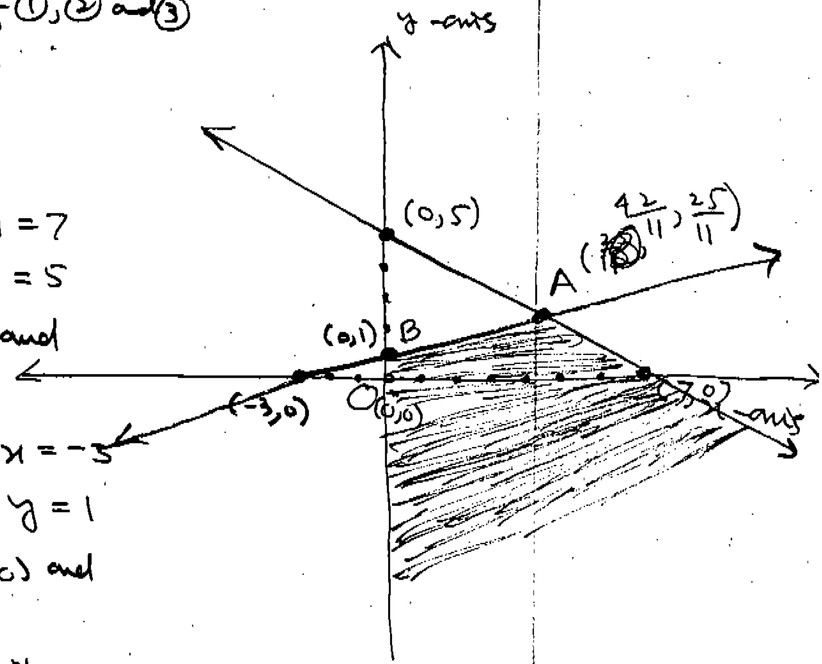
(v) Given inequalities are

$5x+7y \le 35 \rightarrow (1)$
$-x+3y \le 3 \rightarrow (2)$
$x \geq 0 \rightarrow (3)$

The associated equations of (1), (2) and (3) are $5x+7y=35 \rightarrow (4)$
$-x+3y=3 \rightarrow (5)$
$x=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow 5x+0=35 \quad \therefore x=7$
For $x=0$, (4) $\Rightarrow 0+7y=35 \quad \therefore y=5$
$\therefore$ Line (1) cuts x-axis at $(7,0)$ and y-axis at $(0,5)$

For $y=0$, (5) $\Rightarrow -x+0=3 \quad \therefore x=-3$
for $x=0$, (5) $\Rightarrow 0+3y=3 \quad \therefore y=1$
$\therefore$ Line (5) cuts x-axis at $(-3,0)$ and y-axis at $(0,1)$.



Now we take $(0,0)$ as test point.
Putting $(0,0)$ in the L.H.S of (1)
$5(0)+7(0)=0+0=0<35$
$\therefore (0,0)$ satisfies (1)
$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Now putting $(0,0)$ in the L.H.S of (2)
$-0+3(0)=-0+0=0<3$
$\therefore (0,0)$ also satisfies (2)
$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the the side of $(0,0)$.

$x=0$ is the y-axis.
$\therefore$ The graph of $x \geq 0$ is the closed right half plane of xy-plane.
The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading the region.
Here are only two corner points A and B.
$A\left(\frac{42}{11},\frac{25}{11}\right)$ is point of intersection of (4) and (5).
$B(0,1)$ is point of intersection of (5) and (6)

(vi)
$$\left.\begin{array}{l} 5x+7y \le 35 \rightarrow ① \\ x-2y \le 2 \rightarrow ② \\ x \ge 0 \rightarrow ③ \end{array}\right\} \rightarrow (I)$$

$$\begin{array}{r} 5x+7y = 35 \\ 5x-10y = 10 \\ - \quad + \quad\quad - \\ \hline 17y = 25 \end{array}$$

$y = \frac{25}{17}$

$x = 2+2y = 2+\frac{50}{17}$

$= \frac{84}{17}$

The associated equations of ①, ② and ③

are $5x+7y=35 \rightarrow ④$

$x-2y = 2 \rightarrow ⑤$

$x = 0 \rightarrow ⑥$

~~For y = 0, ④ ⇒~~

For $y=0$, ④ $\Rightarrow 5x+0=35 \quad \therefore x=7$

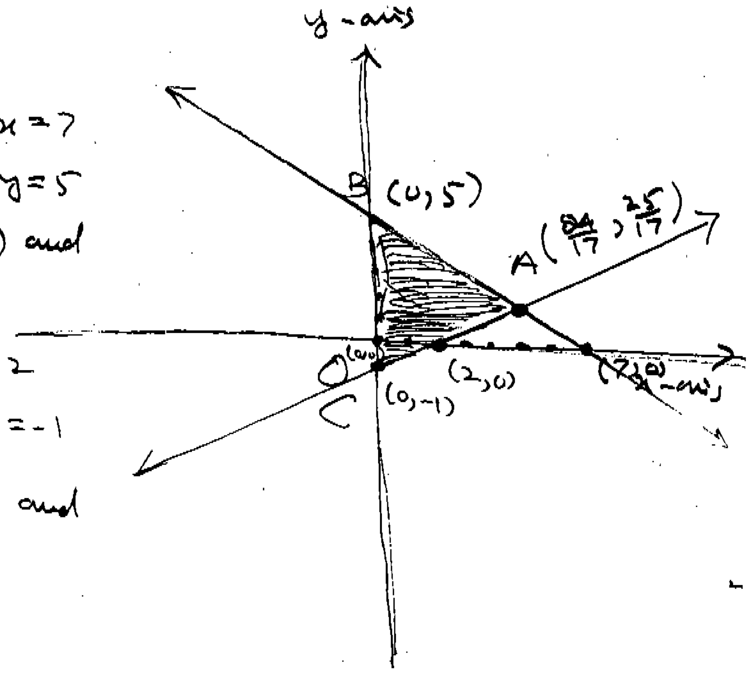
for $x=0$, ④ $\Rightarrow 0+7y=35 \quad \therefore y=5$

∴ Line ④ cuts x-axis at (7, 0) and y-axis at (0, 5)

For $y=0$, ⑤ $\Rightarrow x-0=2 \quad \therefore x=2$

for $x=0$, ⑤ $\Rightarrow 0-2y=2 \quad \therefore y=-1$

∴ Line ⑤ cuts x-axis at (2, 0) and y-axis at (0, −1)



We take (0, 0) as test point.

Putting (0, 0) in the L.H.S of ①.

$5(0)+7(0) = 0+0 = 0 < 35$

∴ (0, 0) satisfies ①

∴ Graph of ① is the closed half plane (made by line ④) on the side of (0, 0).

Putting (0, 0) in the L.H.S of ②

$0-2(0) = 0-0 = 0 < 2$

∴ (0, 0) also satisfies ②

∴ Graph of ② is the closed half plane (made by line ⑤) on the side of (0, 0)

$x=0$ is the y-axis.

∴ Graph of $x \ge 0$ is the closed is the right half plane of xy-plane.

The intersection of graphs of ① ② and ③ is the solution region of system (I) is as shown in the figure by shading the region.

Here A, B, C are the corner points of solution.

A $(\frac{84}{17}, \frac{25}{17})$ is the point of intersection of ④ and ⑤

B (0, 5) is the point of intersection of ④ and ⑥

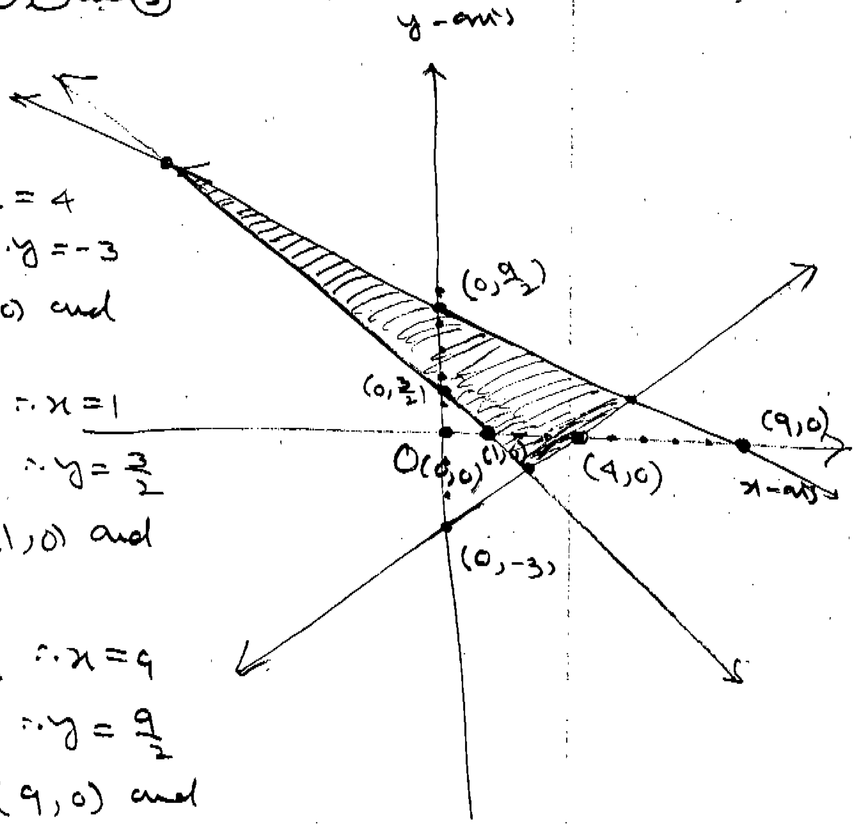
C (0, −1) is the point of intersection of ⑤ and ⑥.

Q.5:(i) Given inequalities are

$$\left.\begin{array}{l} 3x-4y \le 12 \rightarrow (1) \\ 3x+2y \ge 3 \rightarrow (2) \\ x+2y \le 9 \rightarrow (3) \end{array}\right] \rightarrow (I)$$

The associated equations of (1) (2) and (3) are

$$3x-4y=12 \rightarrow (4)$$
$$3x+2y=3 \rightarrow (5)$$
$$x+2y=9 \rightarrow (6)$$



For $y=0$, (4) $\Rightarrow 3x-0=12 \therefore x=4$

For $x=0$, (4) $\Rightarrow 0-4y=12 \therefore y=-3$

$\therefore$ Line (4) cuts x-axis at $(4,0)$ and y-axis at $(0,-3)$

For $y=0$, (5) $\Rightarrow 3x+0=3 \therefore x=1$

for $x=0$, (5) $\Rightarrow 0+2y=3 \therefore y=\frac{3}{2}$

$\therefore$ Line (5) cuts x-axis at $(1,0)$ and y-axis at $(0,\frac{3}{2})$.

For $y=0$, (6) $\Rightarrow x+0=9 \therefore x=9$

for $x=0$, (6) $\Rightarrow 0+2y=9 \therefore y=\frac{9}{2}$

$\therefore$ Line (6) cuts x-axis at $(9,0)$ and y-axis at $(0,\frac{9}{2})$

We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$$3(0)-4(0)=0-0=0<12$$

$\therefore (0,0)$ ~~does~~ satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Putting $(0,0)$ in the L.H.S of (2)

$$3(0)+2(0)=0+0=0<3$$

$\therefore (0,0)$ does not satisfy (2)

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) not on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (3)

$$0+2(0)=0+0=0<9$$

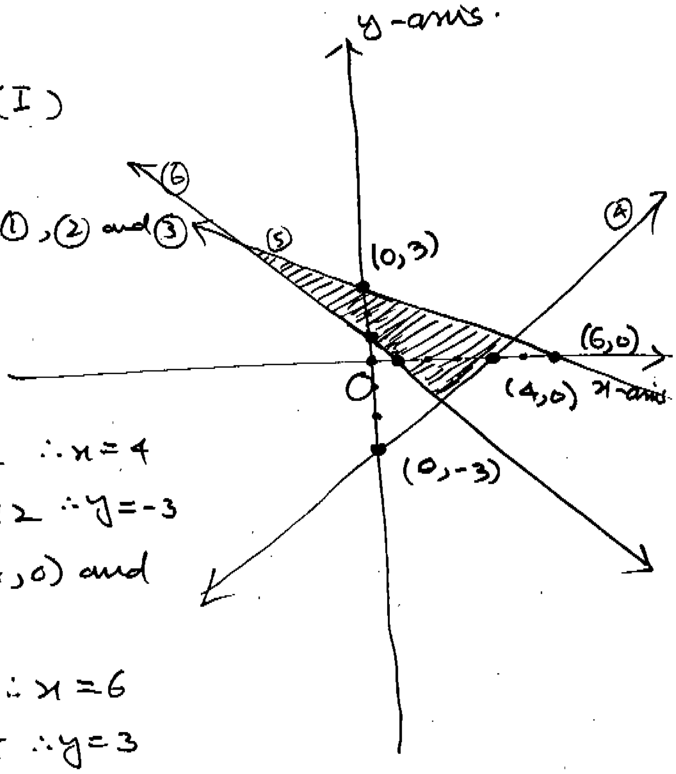
$\therefore (0,0)$ satisfies (3).

$\therefore$ Graph of (3) is that closed half plane (made by line (6)) on the side of $(0,0)$.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading the region.

(ii) Given inequalities are

$$\left.\begin{array}{l} 3x-4y \le 12 \rightarrow (1) \\ x+2y \le 6 \rightarrow (2) \\ x+y \ge 1 \rightarrow (3) \end{array}\right] \rightarrow (I)$$



Associated equations of (1), (2) and (3) are $3x-4y=12 \rightarrow (4)$

$x+2y=6 \rightarrow (5)$

$x+y=1 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow 3x-0=12$ $\therefore x=4$

for $x=0$, (4) $\Rightarrow 0-4y=12$ $\therefore y=-3$

$\therefore$ Line (4) cuts x-axis at $(4,0)$ and y-axis at $(0,-3)$

For $y=0$, (5) $\Rightarrow x+0=6$ $\therefore x=6$

for $x=0$, (5) $\Rightarrow 0+2y=6$ $\therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(6,0)$ and y-axis at $(0,3)$

For $y=0$, (6) $\Rightarrow x+0=1$ i.e $x=1$

for $x=0$, (6) $\Rightarrow 0+y=1$ i.e $y=1$

$\therefore$ Line (6) cuts x-axis at $(1,0)$ and y-axis at $(0,1)$

We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$3(0)-4(0)=0-0=0<12$

$\therefore (0,0)$ satisfies (1)

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin $(0,0)$.

Putting $(0,0)$ in L.H.S of (2)

$0+2(0)=0+0=0<6$

$\therefore (0,0)$ satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the side of origin $(0,0)$

Putting $(0,0)$ in the L.H.S of (3).

$0+0=0<1$

$\therefore (0,0)$ does not satisfy (3)

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) not on the side of origin $(0,0)$

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading the region.

(iii) Given inequalities are

$2x+y \le 4 \rightarrow (1)$

$2x-3y \geq 12 \rightarrow (2)$

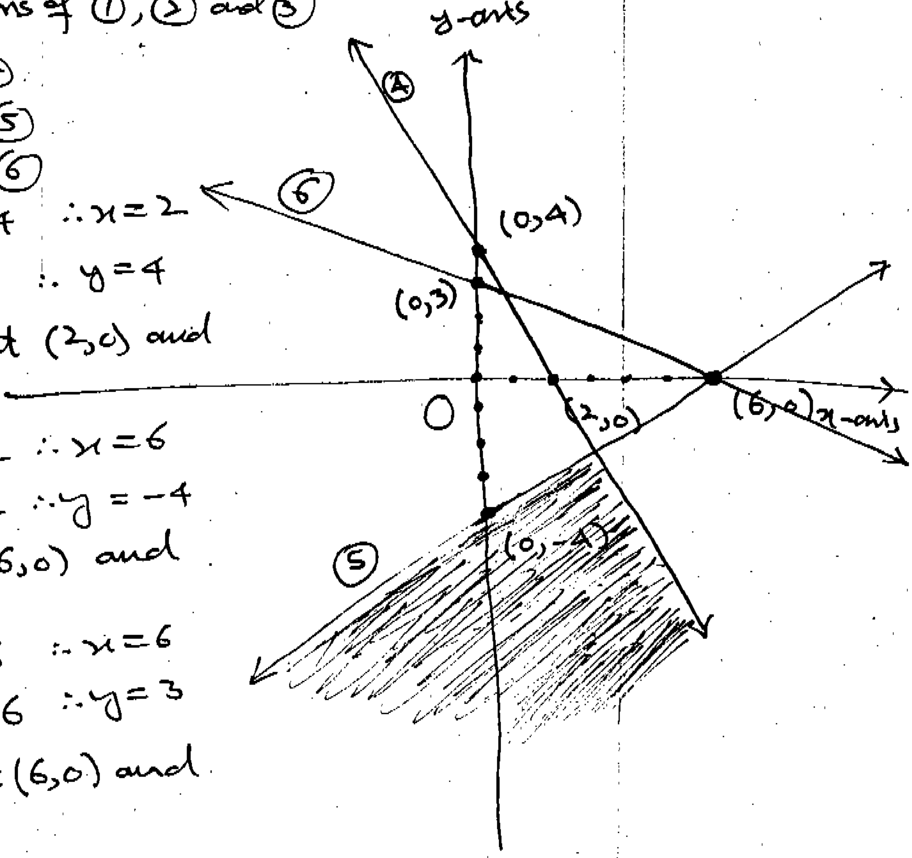
$x+2y \le 6 \rightarrow (3)$

The associated equations of (1), (2) and (3)

are $2x+y=4 \rightarrow (4)$

$2x-3y=12 \rightarrow (5)$

$x+2y=6 \rightarrow (6)$



For $y=0$, (4) $\Rightarrow 2x+0=4$ $\therefore x=2$

for $x=0$, (4) $\Rightarrow 0+y=4$ $\therefore y=4$

$\therefore$ The line (4) cuts x-axis at (2,0) and y-axis at (0,4)

For $y=0$, (5) $\Rightarrow 2x-0=12$ $\therefore x=6$

for $x=0$, (5) $\Rightarrow 0-3y=12$ $\therefore y=-4$

$\therefore$ Line (5) cuts x-axis at (6,0) and y-axis at (0,-4).

for $y=0$, (6) $\Rightarrow x+0=6$ $\therefore x=6$

for $x=0$, (6) $\Rightarrow 0+2y=6$ $\therefore y=3$

$\therefore$ Line (6) cuts x-axis at (6,0) and y-axis at (0,3)

We take (0,0) as test point.

Putting (0,0) in L.H.S of (1)

$2(0)+0=0+0=0<4$

$\therefore$ (0,0) satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin O.

Putting (0,0) in L.H.S of (2)

$2(0)-3(0)=0-0=0<12$

$\therefore$ (0,0) does not satisfy (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) not on the side of origin O.

Putting (0,0) in L.H.S of (3)

$0+2(0)=0+0=0<6$

$\therefore$ (0,0) satisfies (3).

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) on the side of (0,0).

The intersection of graphs of (1), (2) and (3) is the solution region of system(I) as shown partly in the figure by shading the region.

(iv) Given inequalities are

$2x+y \le 10 \longrightarrow (1)$

$x+y \le 7 \longrightarrow (2)$

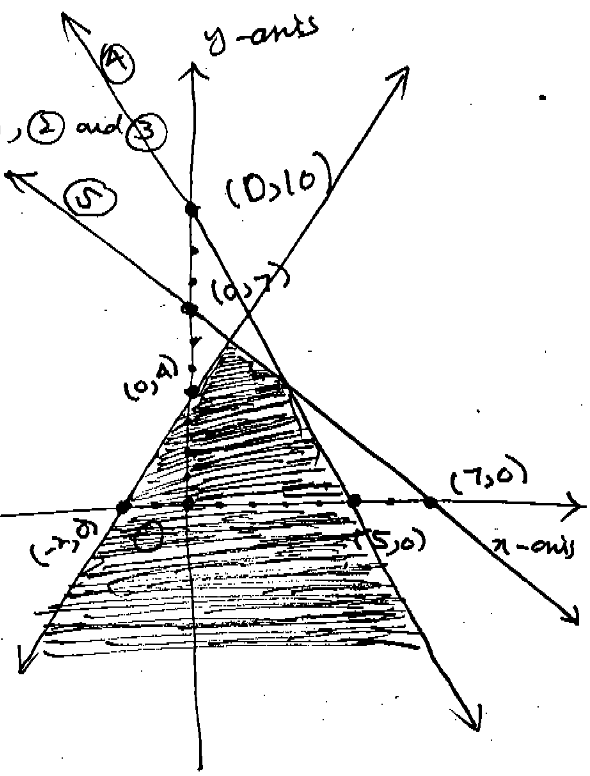
$-2x+y \le 4 \longrightarrow (3)$

The associated equations of (1), (2) and (3) are

$2x+y = 10 \longrightarrow (4)$

$x+y = 7 \longrightarrow (5)$

$-2x+y = 4 \longrightarrow (6)$



For $y=0$, (4) $\Rightarrow 2x+0=10 \quad \therefore x=5$

for $x=0$, (4) $\Rightarrow 0+y=10 \quad \therefore y=10$

$\therefore$ Line (4) cuts x-axis at $(5,0)$ and y-axis at $(0,10)$.

For $y=0$, (5) $\Rightarrow x+0=7 \quad \therefore x=7$

for $x=0$, (5) $\Rightarrow 0+y=7 \quad \therefore y=7$

$\therefore$ Line (5) cuts x-axis at $(7,0)$ and y-axis at $(0,7)$.

For $y=0$, (6) $\Rightarrow -2x+0=4 \quad \therefore x=-2$

for $x=0$, (6) $\Rightarrow -0+y=4 \quad \therefore y=4$

$\therefore$ Lines (6) cuts x-axis at $(-2,0)$ and y-axis at $(0,4)$

We take $(0,0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1)

$2(0)+0 = 0+0 = 0 < 10$

$\therefore (0,0)$ satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (2)

$0+0 = 0 < 7$

$\therefore (0,0)$ satisfies (2)

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$

Putting $(0,0)$ in L.H.S of (3).

$-2(0)+0 = -0+0 = 0 < 4$

$\therefore (0,0)$ satisfies (3)

$\therefore$ The graph of (3) is the closed half plane (formed by line (6)) on the side of $(0,0)$.

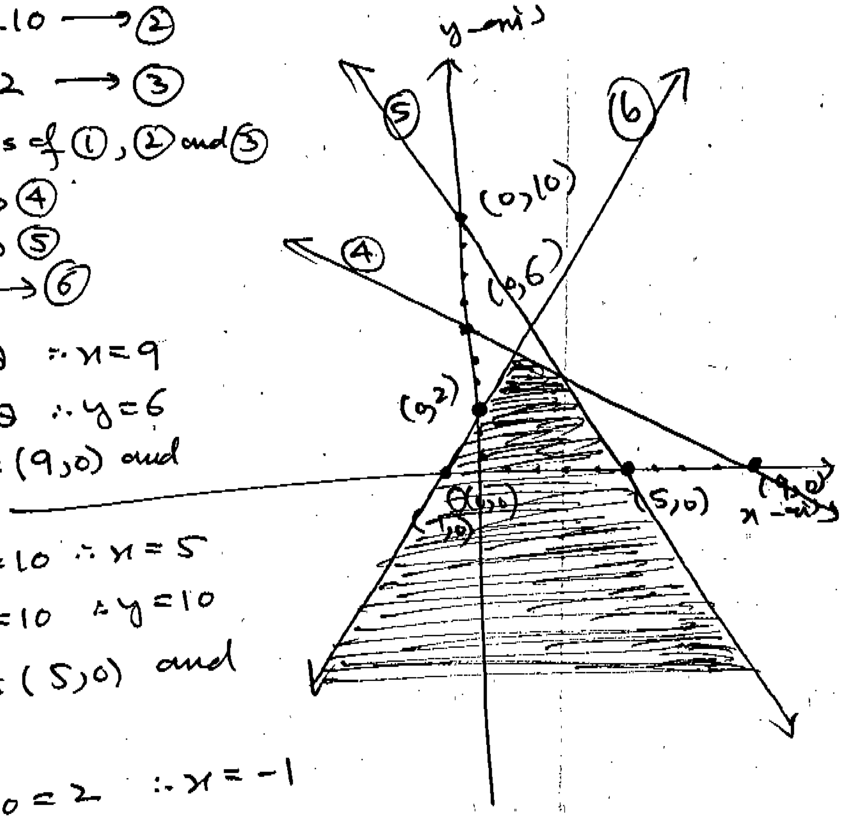
The intersection of graphs of (1), (2) and (3) is the solution region of system (I), the part of which is as shown in the figure by shading the region.

(v) Given inequalities are

$$2x+3y \le 18 \to (1)$$
$$2x+y \le 10 \to (2)$$
$$-2x+y \le 2 \to (3)$$

The associated equations of (1), (2) and (3)

are
$$2x+3y=18 \to (4)$$
$$2x+y=10 \to (5)$$
$$-2x+y=2 \to (6)$$



For $y=0$, (4) $\Rightarrow 2x+0=18 \quad \therefore x=9$

for $x=0$, (4) $\Rightarrow 0+3y=18 \quad \therefore y=6$

$\therefore$ The line (4) cuts x-axis at $(9,0)$ and y-axis at $(0,6)$

For $y=0$, (5) $\Rightarrow 2x+0=10 \quad \therefore x=5$

for $x=0$, (5) $\Rightarrow 0+y=10 \quad \therefore y=10$

$\therefore$ Line (5) cuts x-axis at $(5,0)$ and y-axis at $(0,10)$

For $y=0$, (6) $\Rightarrow -2x+0=2 \quad \therefore x=-1$

for $x=0$, (6) $\Rightarrow -0+y=2 \quad \therefore y=2$

$\therefore$ Line (6) cuts x-axis at $(-1,0)$ and y-axis at $(0,2)$.

We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$$2(0)+3(0)=0+0=0<18.$$

$\therefore (0,0)$ satisfies (1).

$\therefore$ The graph of (1) is the closed half plane (formed by line (4)) on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (2)

$$2(0)+0=0+0=0<10$$

$\therefore (0,0)$ also satisfies (2)

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (3)

$$-2(0)+0=-0+0=0<2$$

$\therefore (0,0)$ also satisfies (3)

$\therefore$ The graph of (3) is the closed half planed (made by line (6)) on the side of $(0,0)$.

The intersection of graphs of (1), (2), (3) is the solution region of system (I), partly shown in the figure by shading the region.

(vi) Given inequalities are

$$\left.\begin{aligned} 3x-2y &\geq 3 \rightarrow (1) \\ x+4y &\leq 12 \rightarrow (2) \\ 3x+y &\leq 12 \rightarrow (3) \end{aligned}\right] \rightarrow (I)$$

The associated equations of (1), (2) and (3) are

$3x-2y=3 \rightarrow (4)$

$x+4y=12 \rightarrow (5)$

$3x+y=12 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow 3x-0=3 \quad \therefore x=1$

for $x=0$, (4) $\Rightarrow 0-2y=3 \quad \therefore y=-\frac{3}{2}$

$\therefore$ Line (4) cuts x-axis at $(1,0)$ and y-axis at $(0,-\frac{3}{2})$

For $y=0$, (5) $\Rightarrow x+0=12 \quad \therefore x=12$

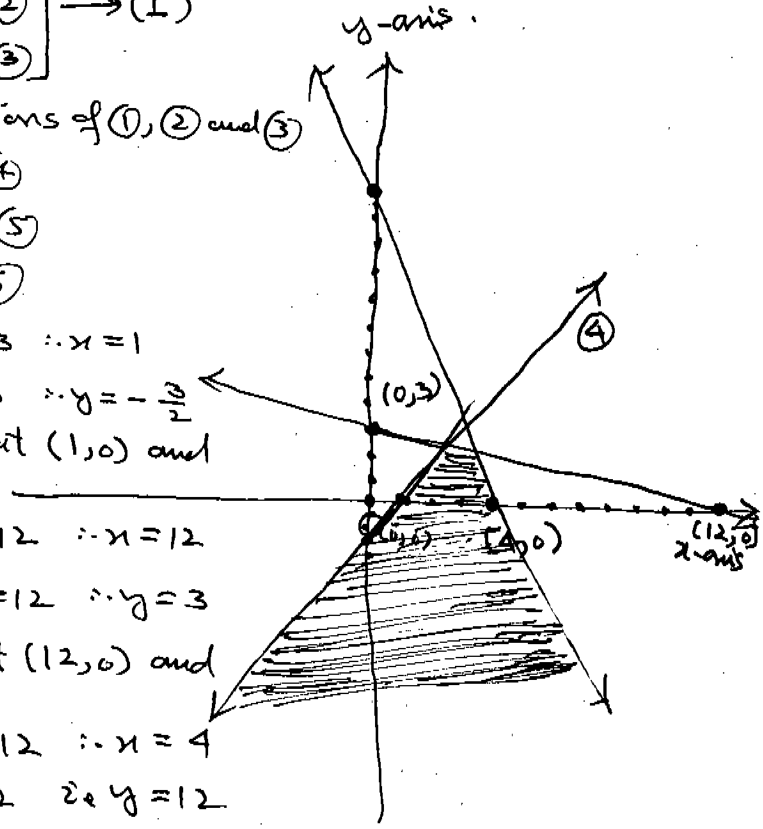
for $x=0$, (5) $\Rightarrow 0+4y=12 \quad \therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(12,0)$ and y-axis at $(0,3)$.

For $y=0$, (6) $\Rightarrow 3x+0=12 \quad \therefore x=4$

for $x=0$, (6) $\Rightarrow 0+y=12$ i.e. $y=12$

$\therefore$ Line (6) cuts x-axis at $(4,0)$ and y-axis at $(0,12)$.



We take $(0,0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1)

$3(0)-2(0)=0-0=0<3$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) not on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (2)

$0+4(0)=0+0=0<12$

$\therefore (0,0)$ satisfies (2).

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (3)

$3(0)+0=0+0=0<12$

$\therefore (0,0)$ also satisfies (3)

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) on the side of $(0,0)$.

The intersection of graphs of (1), (2), (3) is the solution region of system (I) is partly shown in the figure by shading the region.