

Point of intersection of lines

Let $l_1: a_1x + b_1y + c_1 = 0$

$l_2: a_2x + b_2y + c_2 = 0$ be non-parallel lines.

Let $P(x_1, y_1)$ be the point of intersection of l_1 and l_2 . Then

$$a_1x_1 + b_1y_1 + c_1 = 0 \dots\dots\dots(i)$$

$$a_2x_1 + b_2y_1 + c_2 = 0 \dots\dots\dots(ii)$$

Solving (i) and (ii) simultaneously, we have

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{-y_1}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x_1}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1} \text{ and } \frac{-y_1}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y_1 = -\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

Hence $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, -\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1} \right)$ is the point of intersection of l_1 and l_2 .

Equation of line passing through the point of intersection.

Let $l_1: a_1x + b_1y + c_1 = 0$

$l_2: a_2x + b_2y + c_2 = 0$

Then equation of line passing through the point of intersection of l_1 and l_2 is

$$l_1 + k l_2 = 0, \text{ where } k \text{ is constant.}$$

i.e. $a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$

Question # 1

Find the point of intersection of the lines

(i) $x - 2y + 1 = 0$ $2x - y + 2 = 0$ (ii) $3x + y + 12 = 0$ $x + 2y - 1 = 0$

(iii) $x + 4y - 12 = 0$ $x - 3y + 3 = 0$

Solution

(i) $l_1: x - 2y + 1 = 0$

$l_2: 2x - y + 2 = 0$

Slope of $l_1 = m_1 = -\frac{1}{-2} = \frac{1}{2}$

Slope of $l_2 = m_2 = -\frac{2}{-1} = 2$

$\therefore m_1 \neq m_2$, therefore lines are intersecting.

Now if (x, y) is the point of intersection of l_1 and l_2 then

$$\frac{x}{(-2)(2) - (-1)(1)} = \frac{-y}{(1)(2) - (2)(1)} = \frac{1}{(1)(-1) - (2)(-2)}$$

$$\Rightarrow \frac{x}{-4+1} = \frac{-y}{2-2} = \frac{1}{-1+4}$$

$$\Rightarrow \frac{x}{-3} = \frac{-y}{0} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{-3} = \frac{1}{3} \quad \text{and} \quad \frac{-y}{0} = \frac{1}{3}$$

$$\Rightarrow x = \frac{-3}{3} \quad \text{and} \quad y = -\frac{0}{3}$$

$$\Rightarrow x = -1 \quad \text{and} \quad y = 0$$

Hence $(-1, 0)$ is the point of intersection.

(ii) $l_1: 3x + y + 12 = 0$
 $l_2: x + 2y - 1 = 0$

Slope of $l_1 = m_1 = -\frac{3}{1} = -3$

Slope of $l_2 = m_2 = -\frac{1}{2}$

$\therefore m_1 \neq m_2$, therefore lines are intersecting.

Now if (x, y) is the point of intersection of l_1 and l_2 then

$$\frac{x}{-1-24} = \frac{-y}{-3-12} = \frac{1}{6-1}$$

$$\Rightarrow \frac{x}{-25} = \frac{-y}{-15} = \frac{1}{5}$$

$$\Rightarrow \frac{x}{-25} = \frac{1}{5} \quad \text{and} \quad \frac{-y}{-15} = \frac{1}{5}$$

$$\Rightarrow x = \frac{-25}{5} = -5 \quad \text{and} \quad y = \frac{15}{5} = 3$$

Hence $(-5, 3)$ is the point of intersection.

(iii) *Do yourself as above.*

Question # 2

Find an equation of the line through

- the point $(2, -9)$ and the intersection of the lines $2x + 5y - 8 = 0$ and $3x - 4y - 6 = 0$
- the intersection of the lines $x - y - 4 = 0$, $7x + y + 20 = 0$ and $6x + y - 14 = 0$
- Parallel (ii) Perpendicular to the line $6x + y - 14 = 0$
- through the intersection of the lines $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$

And making equal intercepts on the axes.

Solution

(i) Let $l_1: 2x + 5y - 8 = 0$

$l_2: 3x - 4y - 6 = 0$

Equation of line passing through point of intersection of l_1 and l_2 is

$$2x + 5y - 8 + k(3x - 4y - 6) = 0 \dots (i)$$

Since $(2, -9)$ lies on (i) therefore put $x = 2$ and $y = -9$ in (i)

$$2(2) + 5(-9) - 8 + k(3(2) - 4(-9) - 6) = 0$$

$$\Rightarrow 4 - 45 - 8 + k(6 + 36 - 6) = 0$$

$$\Rightarrow -49 + 36k = 0$$

$$\Rightarrow 36k = 49 \quad \Rightarrow k = \frac{49}{36}$$

Putting value of k in (i)

$$2x + 5y - 8 + \frac{49}{36}(3x - 4y - 6) = 0$$

$$\Rightarrow 72x + 180y - 288 + 49(3x - 4y - 6) = 0 \quad \times \text{ing by } 36$$

$$\Rightarrow 72x + 180y - 288 + 147x - 196y - 294 = 0$$

$$\Rightarrow 219x - 16y - 582 = 0 \quad \text{is the required equation.}$$

(ii) Let $l_1: x - y - 4 = 0$

$l_2: 7x + y + 20 = 0$

$l_3: 6x + y - 14 = 0$

Let l_4 be a line passing through point of intersection of l_1 and l_2 , then

$$l_4: l_1 + k l_2 = 0$$

$$\Rightarrow x - y - 4 + k(7x + y + 20) = 0 \dots (i)$$

$$\Rightarrow (1 + 7k)x + (-1 + k)y + (-4 + 20k) = 0$$

$$\text{Slope of } l_4 = m_1 = -\frac{1 + 7k}{-1 + k}$$

$$\text{Slope of } l_3 = m_2 = -\frac{6}{1} = -6$$

(a) If l_3 and l_4 are parallel then

$$m_1 = m_2$$

$$\Rightarrow -\frac{1 + 7k}{-1 + k} = -6$$

$$\Rightarrow 1 + 7k = 6(-1 + k) \quad \Rightarrow 1 + 7k = -6 + 6k$$

$$\Rightarrow 7k - 6k = -6 - 1 \quad \Rightarrow k = -7$$

Putting value of k in (i)

$$x - y - 4 - 7(7x + y + 20) = 0$$

$$\Rightarrow x - y - 4 - 49x - 7y - 140 = 0$$

$$\Rightarrow -48x - 8y - 144 = 0$$

$$\Rightarrow 6x + y + 18 = 0$$

is the required equation

(b) If l_3 and l_4 are \perp then

$$m_1 m_2 = -1$$

$$\Rightarrow \left(-\frac{1+7k}{-1+k} \right) (-6) = -1$$

$$\Rightarrow 6(1+7k) = -(-1+k) \Rightarrow 6+42k = 1-k$$

$$\Rightarrow 42k + k = 1-6 \Rightarrow 43k = -5 \Rightarrow k = -\frac{5}{43}$$

Putting in (i) we have

$$x - y - 4 - \frac{5}{43}(7x + y + 20) = 0$$

$$\Rightarrow 43x - 43y - 172 - 5(7x + y + 20) = 0$$

$$\Rightarrow 43x - 43y - 172 - 35x - 5y - 100 = 0$$

$$\Rightarrow 8x - 48y - 272 = 0$$

$$\Rightarrow x - 6y - 34 = 0 \text{ is the required equation.}$$

(iii) Suppose $l_1: x + 2y + 3 = 0$

$$l_2: 3x + 4y + 7 = 0$$

Equation of line passing through the intersection of l_1 and l_2 is given by:

$$x + 2y + 3 + k(3x + 4y + 7) = 0 \dots\dots\dots (i)$$

$$\Rightarrow (1+3k)x + (2+4k)y + (3+7k) = 0$$

$$\Rightarrow (1+3k)x + (2+4k)y = -(3+7k)$$

$$\Rightarrow \frac{(1+3k)x}{-(3+7k)} + \frac{(2+4k)y}{-(3+7k)} = 1$$

$$\Rightarrow \frac{x}{\cancel{-(3+7k)} / (1+3k)} + \frac{y}{\cancel{-(3+7k)} / (2+4k)} = 1$$

Which is two-intercept form of equation of line with

$$x\text{-intercept} = \frac{-(3+7k)}{(1+3k)} \quad \text{and} \quad y\text{-intercept} = \frac{-(3+7k)}{(2+4k)}$$

We have given

$$x\text{-intercept} = y\text{-intercept}$$

$$\Rightarrow \frac{-(3+7k)}{(1+3k)} = \frac{-(3+7k)}{(2+4k)}$$

$$\Rightarrow \frac{1}{(1+3k)} = \frac{1}{(2+4k)} \Rightarrow (2+4k) = (1+3k)$$

$$\Rightarrow 4k - 3k = 1 - 2 \Rightarrow k = -1$$

Putting value of k in (i)

$$\begin{aligned}
 x + 2y + 3 - 1(3x + 4y + 7) &= 0 \\
 \Rightarrow x + 2y + 3 - 3x - 4y - 7 &= 0 \Rightarrow -2x - 2y - 4 = 0 \\
 \Rightarrow x + y + 2 &= 0
 \end{aligned}$$

is the required equation.

Question # 3

Find an equation of the line through the intersection of
 $16x - 10y - 33 = 0$; $12x + 14y + 29 = 0$ and the intersection of
 $x - y + 4 = 0$; $x - 7y + 2 = 0$

Solution

Let $l_1: 16x - 10y - 33 = 0$

$l_2: 12x + 14y + 29 = 0$

$l_3: x - y + 4 = 0$

$l_4: x - 7y + 2 = 0$

For point of intersection of l_1 and l_2

$$\frac{x}{-290 + 462} = \frac{-y}{464 + 396} = \frac{1}{224 + 120}$$

$$\Rightarrow \frac{x}{172} = \frac{-y}{860} = \frac{1}{334}$$

$$\Rightarrow \frac{x}{172} = \frac{1}{334} \quad \text{and} \quad \frac{-y}{860} = \frac{1}{334}$$

$$\Rightarrow x = \frac{172}{334} = \frac{1}{2} \quad \text{and} \quad y = -\frac{860}{334} = -\frac{5}{2}$$

$$\Rightarrow \left(\frac{1}{2}, -\frac{5}{2}\right) \text{ is a point of intersection of } l_1 \text{ and } l_2.$$

For point of intersection of l_3 and l_4 .

$$\frac{x}{-2 + 28} = \frac{-y}{2 - 4} = \frac{1}{-7 + 1}$$

$$\Rightarrow \frac{x}{26} = \frac{-y}{-2} = \frac{1}{-6}$$

$$\Rightarrow \frac{x}{26} = \frac{1}{-6} \quad \text{and} \quad \frac{-y}{-2} = \frac{1}{-6}$$

$$\Rightarrow x = \frac{26}{-6} = -\frac{13}{3} \quad \text{and} \quad y = \frac{2}{-6} = -\frac{1}{3}$$

$$\Rightarrow \left(-\frac{13}{3}, -\frac{1}{3}\right) \text{ is a point of intersection of } l_3 \text{ and } l_4.$$

Now equation of line passing through $\left(\frac{1}{2}, -\frac{5}{2}\right)$ and $\left(-\frac{13}{3}, -\frac{1}{3}\right)$

$$y + \frac{5}{2} = \frac{-\frac{1}{3} + \frac{5}{2}}{-\frac{13}{3} - \frac{1}{2}} \left(x - \frac{1}{2}\right)$$

$$\begin{aligned} \Rightarrow y + \frac{5}{2} &= \frac{13/6}{-29/6} \left(x - \frac{1}{2} \right) \Rightarrow y + \frac{5}{2} = -\frac{13}{29} \left(x - \frac{1}{2} \right) \\ \Rightarrow 29y + \frac{145}{2} &= -13x + \frac{13}{2} \Rightarrow 13x - \frac{13}{2} + 29y + \frac{145}{2} = 0 \\ \Rightarrow 13x + 29y + 66 &= 0 \end{aligned}$$

is the required equation.

Three Concurrent Lines

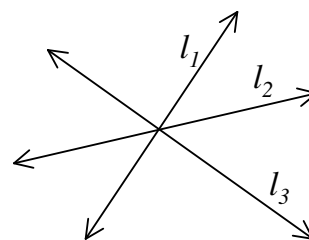
Suppose $l_1: a_1x + b_1y + c_1 = 0$

$l_2: a_2x + b_2y + c_2 = 0$

$l_3: a_3x + b_3y + c_3 = 0$

If l_1, l_2 and l_3 are concurrent (intersect at one point) then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$



See proof on book at page 208

Question # 4

Find the condition that the lines $y = m_1x + c_1$; $y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent.

Solution

Assume that

$$l_1: y = m_1x + c_1$$

$$\Rightarrow m_1x - y + c_1 = 0$$

$$l_2: y = m_2x + c_2$$

$$\Rightarrow m_2x - y + c_2 = 0$$

$$l_3: y = m_3x + c_3$$

$$\Rightarrow m_3x - y + c_3 = 0$$

If l_1, l_2 and l_3 are concurrent then

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 - m_1 & 0 & c_2 - c_1 \\ m_3 - m_1 & 0 & c_3 - c_1 \end{vmatrix} = 0 \quad \text{by } \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

Expanding by C_2

$$-(-1)[(m_2 - m_1)(c_3 - c_1) - (m_3 - m_1)(c_2 - c_1)] + 0 - 0 = 0$$

$$\Rightarrow [(m_2 - m_1)(c_3 - c_1) - (m_3 - m_1)(c_2 - c_1)] = 0$$

$$\Rightarrow (m_2 - m_1)(c_3 - c_1) = (m_3 - m_1)(c_2 - c_1)$$

is the required condition.

Question # 5

Determine the value of P such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.

Solution

$$\text{Let } l_1: 2x - 3y - 1 = 0$$

$$l_2: 3x - y - 5 = 0$$

$$l_3: 3x + py + 8 = 0$$

Since l_1 , l_2 and l_3 meet at a point i.e. concurrent therefore

$$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & p & 8 \end{vmatrix} = 0$$

$$\Rightarrow 2(-8 + 5p) + 3(24 + 15) - 1(3p + 3) = 0$$

$$\Rightarrow -16 + 10p + 72 + 45 - 3p - 3 = 0$$

$$\Rightarrow 7p + 98 = 0 \Rightarrow 7p = -98$$

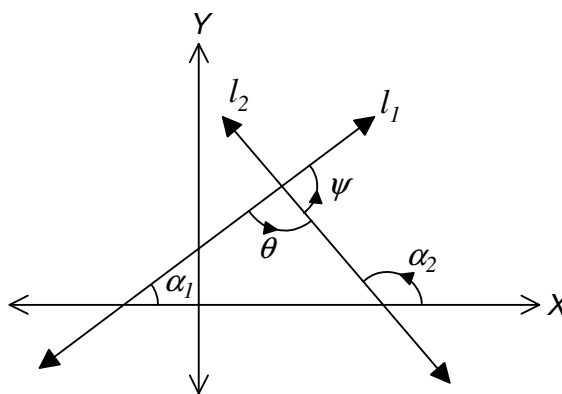
$$\Rightarrow p = -\frac{98}{7} \Rightarrow \boxed{p = -14}$$

Angle between lines

Let l_1 and l_2 be two lines. If α_1 and α_2 be inclinations and m_1 and m_2 be slopes of lines l_1 and l_2 respectively, Let θ be an angle from line l_1 to l_2 then θ is given by

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

See proof on book at page 219

**Question # 6**

Show that the lines $4x - 3y - 8 = 0$, $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent and the third-line bisects the angle formed by the first two lines.

Solution

$$\text{Let } l_1: 4x - 3y - 8 = 0$$

$$l_2: 3x - 4y - 6 = 0$$

$$l_3: x - y - 2 = 0$$

To check l_1 , l_2 and l_3 are concurrent, let

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= 4(8 - 6) + 3(-6 + 6) - 8(-3 + 4)$$

$$= 4(2) + 3(0) - 8(1)$$

$$= 8 + 0 - 8 = 0$$

Hence l_1 , l_2 and l_3 are concurrent.

$$\text{Slope of } l_1 = m_1 = -\frac{4}{-3} = \frac{4}{3}$$

$$\text{Slope of } l_2 = m_2 = -\frac{3}{-4} = \frac{3}{4}$$

$$\text{Slope of } l_3 = m_3 = -\frac{1}{-1} = 1$$

Now let θ_1 be angle from l_1 to l_3 and θ_2 be a angle from l_3 to l_2 . Then

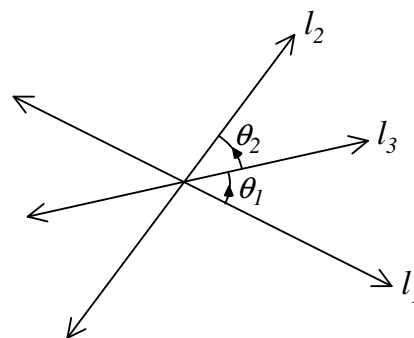
$$\tan \theta_1 = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{1 - \frac{4}{3}}{1 + (1)\left(\frac{4}{3}\right)} = \frac{-\frac{1}{3}}{\frac{7}{3}} = -\frac{1}{7} \dots\dots\dots (i)$$

$$\text{And } \tan \theta_2 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{3}{4} - 1}{1 + \left(\frac{3}{4}\right)(1)} = \frac{-\frac{1}{4}}{\frac{7}{4}} = -\frac{1}{7} \dots\dots\dots (ii)$$

From (i) and (ii)

$$\tan \theta_1 = \tan \theta_2 \Rightarrow \theta_1 = \theta_2$$

$\Rightarrow l_3$ bisect the angle formed by the first two lines.



Question # 7

The vertices of a triangle are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$.

Find coordinates of the

- (i) centroid (ii) orthocentre (iii) circumcentre of the triangle
Are these three points are collinear?

Solution

Given vertices of triangles are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$.

- (i) Centroid of triangle is the intersection of medians and is given by

$$\begin{aligned} & \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \\ &= \left(\frac{-2 - 4 + 3}{3}, \frac{3 + 1 + 5}{3} \right) = \left(\frac{-3}{3}, \frac{9}{3} \right) = (-1, 3) \end{aligned}$$

Hence $(-1, 3)$ is the centroid of the triangle.

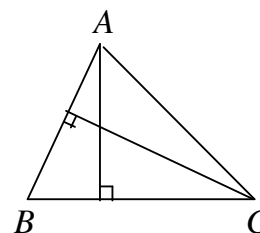
- (ii) Orthocentre is the point of intersection of altitudes.

$$\text{Slope of } \overline{AB} = m_1 = \frac{1-3}{-4+2} = \frac{-2}{-2} = 1$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{5-1}{3+4} = \frac{4}{7}$$

Since altitudes are \perp to sides therefore

$$\text{Slope of altitude on } \overline{AB} = -\frac{1}{m_1} = -\frac{1}{1} = -1$$



$$\text{Slope of altitude on } \overline{BC} = -\frac{1}{m_2} = -\frac{1}{\frac{4}{7}} = -\frac{7}{4}$$

Equation of altitude on \overline{AB} with slope -1 from $C(3,5)$

$$\begin{aligned} y-5 &= -1(x-3) \\ \Rightarrow y-5 &= -x+3 \quad \Rightarrow x-3+y-5=0 \\ \Rightarrow x+y-8 &= 0 \dots\dots\dots (i) \end{aligned}$$

Now equation of altitude on \overline{BC} with slope $-\frac{7}{4}$ from $A(-2,3)$

$$\begin{aligned} y-3 &= -\frac{7}{4}(x+2) \\ \Rightarrow 4y-12 &= -7x-14 \quad \Rightarrow 7x+14+4y-12=0 \\ \Rightarrow 7x+4y+2 &= 0 \dots\dots\dots (ii) \end{aligned}$$

For point of intersection of (i) and (ii)

$$\begin{aligned} \frac{x}{2+32} &= \frac{-y}{2+56} = \frac{1}{4-7} \\ \Rightarrow \frac{x}{34} &= \frac{-y}{58} = \frac{1}{-3} \\ \Rightarrow \frac{x}{34} &= \frac{1}{-3} \quad \text{and} \quad \frac{-y}{58} = \frac{1}{-3} \\ \Rightarrow x &= -\frac{34}{3} \quad \text{and} \quad y = -\frac{58}{-3} = \frac{58}{3} \end{aligned}$$

Hence $\left(-\frac{34}{3}, \frac{58}{3}\right)$ is orthocentre of triangle ABC .

(iii) Circumcentre of the triangle is the point of intersection of perpendicular bisector.

Let D and E are midpoints of side \overline{AB} and \overline{BC} respectively.

$$\text{Then coordinate of } D = \left(\frac{-4-2}{2}, \frac{1+3}{2}\right) = \left(\frac{-6}{2}, \frac{4}{2}\right) = (-3, 2)$$

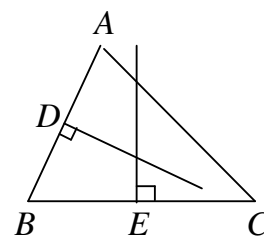
$$\text{Coordinate of } E = \left(\frac{-4+3}{2}, \frac{1+5}{2}\right) = \left(\frac{-1}{2}, \frac{6}{2}\right) = \left(-\frac{1}{2}, 3\right)$$

$$\text{Slope of } \overline{AB} = m_1 = \frac{1-3}{-4+2} = \frac{-2}{-2} = 1$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{5-1}{3+4} = \frac{4}{7}$$

$$\text{Slope of } \perp \text{ bisector on } \overline{AB} = -\frac{1}{m_1} = -\frac{1}{1} = -1$$

$$\text{Slope of } \perp \text{ bisector on } \overline{BC} = -\frac{1}{m_2} = -\frac{1}{\frac{4}{7}} = -\frac{7}{4}$$



Now equation of \perp bisector having slope -1 through $D(-3,2)$

$$\begin{aligned}y - 2 &= -1(x + 3) \\ \Rightarrow y - 2 &= -x - 3 \quad \Rightarrow x + 3 + y - 2 = 0 \\ \Rightarrow x + y + 1 &= 0 \dots\dots\dots (iii)\end{aligned}$$

Now equation of \perp bisector having slope $-\frac{7}{4}$ through $E\left(-\frac{1}{2}, 3\right)$

$$\begin{aligned}y - 3 &= -\frac{7}{4}\left(x + \frac{1}{2}\right) \Rightarrow 4y - 12 = -7x - \frac{7}{2} \\ \Rightarrow 7x + \frac{7}{2} + 4y - 12 &= 0 \Rightarrow 7x + 4y - \frac{17}{2} = 0 \\ \Rightarrow 14x + 8y - 17 &= 0 \dots\dots\dots (iv)\end{aligned}$$

For point of intersection of (iii) and (iv)

$$\begin{aligned}\frac{x}{-17-8} &= \frac{-y}{-17-14} = \frac{1}{8-14} \\ \Rightarrow \frac{x}{-25} &= \frac{-y}{-31} = \frac{1}{-6} \\ \Rightarrow \frac{x}{-25} &= \frac{1}{-6} \quad \text{and} \quad \frac{-y}{-31} = \frac{1}{-6} \\ \Rightarrow x &= \frac{-25}{-6} = \frac{25}{6} \quad \text{and} \quad y = -\frac{31}{6}\end{aligned}$$

Hence $\left(\frac{25}{6}, -\frac{31}{6}\right)$ is the circumcentre of the triangle.

Now to check $(-1, 3), \left(-\frac{34}{3}, \frac{58}{3}\right)$ and $\left(\frac{25}{6}, -\frac{31}{6}\right)$ are collinear, let

$$\begin{aligned}&\begin{vmatrix} -1 & 3 & 1 \\ -\frac{34}{3} & \frac{58}{3} & 1 \\ \frac{25}{6} & -\frac{31}{6} & 1 \end{vmatrix} \\ &= -1\left(\frac{58}{3} + \frac{31}{6}\right) - 3\left(-\frac{34}{3} - \frac{25}{6}\right) + 1\left(\frac{1054}{18} - \frac{1450}{18}\right) \\ &= -1\left(\frac{49}{2}\right) - 3\left(-\frac{31}{2}\right) + 1(-22) \\ &= -\frac{49}{2} + \frac{93}{2} - 22 = 0\end{aligned}$$

Hence centroid, orthocentre and circumcentre of triangle are collinear.

Question # 8

Check whether the lines $4x - 3y - 8 = 0$; $3x - 4y - 6 = 0$; $x - y - 2 = 0$ are concurrent. If so, find the point where they meet.

Solution

$$\text{Let } l_1: 4x - 3y - 8 = 0$$

$$l_2: 3x - 4y - 6 = 0$$

$$l_3: x - y - 2 = 0$$

To check lines are concurrent, let

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= 4(8 - 6) + 3(-6 + 6) - 8(-3 + 4)$$

$$= 4(2) + 3(0) - 8(1) = 8 + 0 - 8 = 0$$

Hence l_1 , l_2 and l_3 are concurrent.

For point of concurrency, we find intersection of l_1 and l_2 (You may choose any two lines)

$$\frac{x}{18-32} = \frac{-y}{-24+24} = \frac{1}{-16+9}$$

$$\Rightarrow \frac{x}{-14} = \frac{-y}{0} = \frac{1}{-7}$$

$$\Rightarrow \frac{x}{-14} = \frac{1}{-7} \quad \text{and} \quad \frac{-y}{0} = \frac{1}{-7}$$

$$\Rightarrow x = \frac{-14}{-7} = 2 \quad \text{and} \quad y = -\frac{0}{-7} = 0$$

Hence $(2,0)$ is the point of concurrency.

Question # 9

Find the coordinates of the vertices of the triangle formed by the lines

$x - 2y - 6 = 0$; $3x - y + 3 = 0$; $2x + y - 4 = 0$. Also find measures of the angles of the triangle.

Solution

$$\text{Let } l_1: x - 2y - 6 = 0$$

$$l_2: 3x - y + 3 = 0$$

$$l_3: 2x + y - 4 = 0$$

For point of intersection of l_1 and l_2

$$\frac{x}{-6-6} = \frac{-y}{3+18} = \frac{1}{-1+6}$$

$$\Rightarrow \frac{x}{-12} = \frac{-y}{21} = \frac{1}{5}$$

$$\Rightarrow \frac{x}{-12} = \frac{1}{5} \quad \text{and} \quad \frac{-y}{21} = \frac{1}{5}$$

$$\Rightarrow x = -\frac{12}{5} \quad \text{and} \quad y = -\frac{21}{5}$$

$\Rightarrow \left(-\frac{12}{5}, -\frac{21}{5}\right)$ is the point of intersection of l_1 and l_2 .

For point of intersection of l_2 and l_3 .

$$\begin{aligned}\frac{x}{4-3} &= \frac{-y}{-12-6} = \frac{1}{3+2} \\ \Rightarrow \frac{x}{1} &= \frac{-y}{-18} = \frac{1}{5} \\ \Rightarrow \frac{x}{1} &= \frac{1}{5} \quad \text{and} \quad \frac{-y}{-18} = \frac{1}{5} \\ \Rightarrow x &= \frac{1}{5} \quad \text{and} \quad y = \frac{18}{5}\end{aligned}$$

$\Rightarrow \left(\frac{1}{5}, \frac{18}{5}\right)$ is the point of intersection of l_2 and l_3 .

Now for point of intersection of l_1 and l_3

$$\begin{aligned}\frac{x}{8+6} &= \frac{-y}{-4+12} = \frac{1}{1+4} \\ \Rightarrow \frac{x}{14} &= \frac{-y}{8} = \frac{1}{5} \\ \Rightarrow \frac{x}{14} &= \frac{1}{5} \quad \text{and} \quad \frac{-y}{8} = \frac{1}{5} \\ \Rightarrow x &= \frac{14}{5} \quad \text{and} \quad y = -\frac{8}{5}\end{aligned}$$

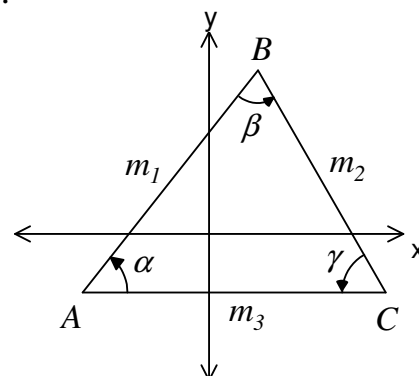
$\Rightarrow \left(\frac{14}{5}, -\frac{8}{5}\right)$ is the point of intersection of l_1 and l_3 .

Hence $\left(-\frac{12}{5}, -\frac{21}{5}\right)$, $\left(\frac{1}{5}, \frac{18}{5}\right)$ and $\left(\frac{14}{5}, -\frac{8}{5}\right)$ are vertices of triangle made by l_1 , l_2 and l_3 . We say these vertices as A, B and C respectively.

$$\text{Slope of side } AB = m_1 = \frac{\frac{18}{5} - \left(-\frac{21}{5}\right)}{\frac{1}{5} - \left(-\frac{12}{5}\right)} = \frac{\frac{39}{5}}{\frac{13}{5}} = \frac{39}{13} = 3$$

$$\text{Slope of side } BC = m_2 = \frac{-\frac{8}{5} - \frac{18}{5}}{\frac{14}{5} - \frac{1}{5}} = \frac{-\frac{26}{5}}{\frac{13}{5}} = -\frac{26}{13} = -2$$

$$\text{Slope of side } CA = m_3 = \frac{-\frac{21}{5} - \frac{8}{5}}{-\frac{12}{5} - \frac{14}{5}} = \frac{-\frac{29}{5}}{-\frac{26}{5}} = \frac{29}{26} = \frac{1}{2}$$



Let α, β and γ denotes angles of triangle at vertices A, B and C respectively.
Then

$$\tan \alpha = \frac{m_1 - m_3}{1 + m_1 m_3} = \frac{3 - \frac{1}{2}}{1 + (3)\left(\frac{1}{2}\right)} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$$

$$\Rightarrow \alpha = \tan^{-1}(1) \Rightarrow \boxed{\alpha = 45^\circ}$$

Now $\tan \beta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{-2 - 3}{1 + (-3)(3)} = \frac{-5}{-5} = 1$

$$\Rightarrow \beta = \tan^{-1}(1) \Rightarrow \boxed{\beta = 45^\circ}$$

Now $\tan \gamma = \frac{m_3 - m_2}{1 + m_3 m_2} = \frac{\frac{1}{2} + 2}{1 + \left(\frac{1}{2}\right)(-2)} = \frac{\frac{5}{2}}{0} = \infty$

$$\Rightarrow \gamma = \tan^{-1}(\infty) \Rightarrow \boxed{\gamma = 90^\circ}.$$

Question # 10

Find the angle measured from the line l_1 to the line l_2 where

(a) l_1 : joining (2,7) and (7,10)

l_2 : joining (1,1) and (-5,3)

(b) l_1 : joining (3,-1) and (5,7)

l_2 : joining (2,4) and (-8,2)

(c) l_1 : joining (1,-7) and (6,-4)

l_2 : joining (-1,2) and (-6,-1)

(d) l_1 : joining (-9,-1) and (3,-5)

l_2 : joining (2,7) and (-6,-7)

Solution

(a) Since l_1 : joining (2,7) and (7,10)

Therefore slope of $l_1 = m_1 = \frac{10-7}{7-2} = \frac{3}{5}$

Also l_2 : joining (1,1) and (-5,3)

Therefore slope of $l_2 = m_2 = \frac{3-1}{-5-1} = \frac{2}{-6} = -\frac{1}{3}$

Let θ be an angle from l_1 to l_2 then

$$\begin{aligned} \tan \theta &= \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{-\frac{1}{3} - \frac{3}{5}}{1 + \left(\frac{3}{5}\right)\left(-\frac{1}{3}\right)} \\ &= \frac{-\frac{14}{15}}{1 - \frac{1}{5}} = \frac{-\frac{14}{15}}{\frac{4}{5}} = -\frac{14}{15} \times \frac{5}{4} = -\frac{7}{6} \end{aligned}$$

$$\Rightarrow -\tan \theta = \frac{7}{6} \Rightarrow \tan(180 - \theta) = \frac{7}{6}$$

$$\therefore \tan(180 - \theta) = -\tan \theta$$

$$\Rightarrow 180 - \theta = \tan^{-1}\left(\frac{7}{6}\right) = 49.4$$

$$\Rightarrow \theta = 180 - 49.4 \Rightarrow \boxed{\theta = 130.6^\circ}$$

Now acute angle between lines = $180 - 130.6 = 49.4^\circ$

(b) *Do yourself as above.*

(c) Since l_1 : joining $(1, -7)$ and $(6, -4)$

$$\text{Therefore slope of } l_1 = m_1 = \frac{-4 + 7}{6 - 1} = \frac{3}{5}$$

Also l_2 : joining $(-1, 2)$ and $(-6, -1)$

$$\text{Therefore slope of } l_2 = m_2 = \frac{-1 - 2}{-6 + 1} = \frac{-3}{-5} = \frac{3}{5}$$

Let θ be a angle from l_1 to l_2 then

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{3}{5} - \frac{3}{5}}{1 + \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)} = \frac{0}{1 + \frac{9}{25}} = 0$$

$$\Rightarrow \theta = \tan^{-1}(0) \Rightarrow \boxed{\theta = 0^\circ}$$

Also acute angle between lines = 0°

(d) Since l_1 : joining $(-9, -1)$ and $(3, -5)$

$$\text{Therefore slope of } l_1 = m_1 = \frac{-5 + 1}{3 + 9} = \frac{-4}{12} = -\frac{1}{3}$$

Also l_2 : joining $(2, 7)$ and $(-6, -7)$

$$\text{Therefore slope of } l_2 = m_2 = \frac{-7 - 7}{-6 - 2} = \frac{-14}{-8} = \frac{7}{4}$$

Let θ be a angle from l_1 to l_2 then

$$\begin{aligned} \tan \theta &= \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{7}{4} - \left(-\frac{1}{3}\right)}{1 + \left(\frac{7}{4}\right)\left(-\frac{1}{3}\right)} \\ &= \frac{\frac{7}{4} + \frac{1}{3}}{1 - \frac{7}{12}} = \frac{\frac{25}{12}}{\frac{5}{12}} = \frac{25}{12} \times \frac{12}{5} = 5 \end{aligned}$$

$$\Rightarrow \theta = \tan^{-1}(5) \Rightarrow \boxed{\theta = 78.69^\circ}$$

Also acute angle between lines = 78.69°

Question # 11

Find the interior angle of the triangle whose vertices are

(a) $A(-2, 11)$, $B(-6, -3)$, $C(4, -9)$

(b) $A(6,1)$, $B(2,7)$, $C(-6,-7)$

(c) $A(2,-5)$, $B(-4,-3)$, $C(-1,5)$

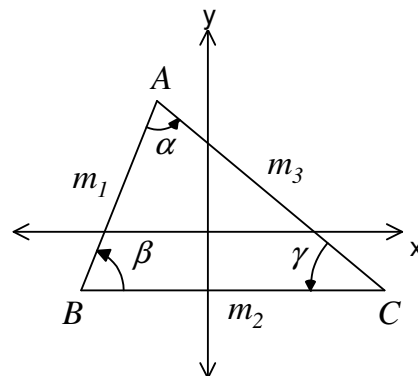
(d) $A(2,8)$, $B(-5,4)$ and $C(4,-9)$

Solution(a) Given vertices $A(-2,11)$, $B(-6,-3)$ and $C(4,-9)$ Let m_1, m_2 and m_3 denotes the slopes of side AB , BC and CA respectively. Then

$$m_1 = \frac{-3-11}{-6+2} = \frac{-14}{-4} = \frac{7}{2}$$

$$m_2 = \frac{-9+3}{4+6} = \frac{-6}{10} = -\frac{3}{5}$$

$$m_3 = \frac{11+9}{-2-4} = \frac{20}{-6} = -\frac{10}{3}$$

Let α, β and γ denotes angles of triangle at vertex A, B and C respectively. Then

$$\tan \alpha = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{-10/3 - 7/2}{1 + (-10/3)(7/2)}$$

$$= \frac{-41/6}{1 - 35/3} = \frac{-41/6}{-32/3} = \frac{41}{6} \times \frac{3}{32} = \frac{41}{64}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{41}{64}\right) \Rightarrow \boxed{\alpha = 32.64^\circ}$$

$$\tan \beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{7/2 - (-3/5)}{1 + (7/2)(-3/5)}$$

$$= \frac{7/2 + 3/5}{1 - 21/10} = \frac{41/10}{-11/10} = -\frac{41}{10} \times \frac{10}{11} = -\frac{41}{11}$$

$$\Rightarrow -\tan \beta = \frac{41}{11} \Rightarrow \tan(180 - \beta) = \frac{41}{11}$$

$$\because \tan(180 - \theta) = -\tan \theta$$

$$\Rightarrow 180 - \beta = \tan^{-1}\left(\frac{41}{11}\right) = 74.98$$

$$\Rightarrow \beta = 180 - 74.98 \Rightarrow \boxed{\beta = 105.02}$$

$$\tan \gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-3/5 - (-10/3)}{1 + (-3/5)(-10/3)}$$

$$= \frac{-3/5 + 10/3}{1 + 2} = \frac{41/15}{3} = \frac{41}{15 \times 3} = \frac{41}{45}$$

$$\Rightarrow \gamma = \tan^{-1}\left(\frac{41}{45}\right) \Rightarrow \boxed{\gamma = 42.34^\circ}$$

(b) Given vertices $A(6,1)$, $B(2,7)$ and $C(-6,-7)$

Let m_1, m_2 and m_3 denotes the slopes of side AB , BC and CA respectively. Then

$$m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$$

$$m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

$$m_3 = \frac{1+7}{6+6} = \frac{8}{12} = \frac{2}{3}$$

Let α, β and γ denotes angles of triangle at vertex A, B and C respectively. Then

$$\begin{aligned} \tan \alpha &= \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{\frac{2}{3} - \left(-\frac{3}{2}\right)}{1 + \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right)} \\ &= \frac{\frac{2}{3} + \frac{3}{2}}{1 - 1} = \frac{\frac{13}{6}}{0} = \infty \end{aligned}$$

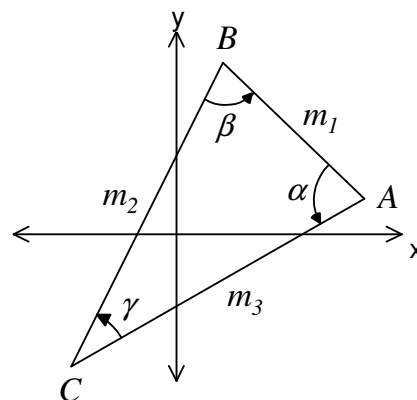
$$\Rightarrow \alpha = \tan^{-1}(\infty) \Rightarrow \boxed{\alpha = 90^\circ}$$

$$\begin{aligned} \tan \beta &= \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-\frac{3}{2} - \frac{7}{4}}{1 + \left(-\frac{3}{2}\right)\left(\frac{7}{4}\right)} \\ &= \frac{-\frac{13}{4}}{1 - \frac{21}{8}} = \frac{-\frac{13}{4}}{-\frac{13}{8}} = \frac{13}{4} \times \frac{8}{13} = 2 \end{aligned}$$

$$\Rightarrow \beta = \tan^{-1}(2) \Rightarrow \boxed{\beta = 63.43^\circ}$$

$$\begin{aligned} \tan \gamma &= \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{7}{4} - \frac{2}{3}}{1 + \left(\frac{7}{4}\right)\left(\frac{2}{3}\right)} \\ &= \frac{\frac{13}{12}}{1 + \frac{7}{6}} = \frac{\frac{13}{12}}{\frac{13}{6}} = \frac{13}{12} \times \frac{6}{13} = \frac{1}{2} \end{aligned}$$

$$\Rightarrow \gamma = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\gamma = 26.57^\circ}$$



(c) *Do yourself as above.*

(d) *Do yourself as above.*

Question # 12

Find the interior angles of the quadrilateral whose vertices are $A(5,2)$, $B(-2,3)$, $C(-3,-4)$ and $D(4,-5)$

Solution Given vertices are $A(5,2)$, $B(-2,3)$, $C(-3,-4)$ and $D(4,-5)$

Let m_1 , m_2 , m_3 and m_4 be slopes of side AB, BC, CD and DA . Then

$$m_1 = \frac{3-2}{-2-5} = \frac{1}{-7}$$

$$m_2 = \frac{-4-3}{-3+2} = \frac{-7}{-1} = 7$$

$$m_3 = \frac{-5+4}{4+3} = \frac{-1}{7}$$

$$m_4 = \frac{2+5}{5-4} = \frac{7}{1} = 7$$

Now suppose α, β, γ and δ are angles of quadrilateral at vertices A, B, C and D respectively. Then

$$\tan \alpha = \frac{m_4 - m_1}{1 + m_4 m_1} = \frac{7 - (-1/7)}{1 + (7)(-1/7)} = \frac{7 + 1/7}{1 - 1} = \frac{50/7}{0} = \infty$$

$$\Rightarrow \alpha = \tan^{-1}(\infty) \Rightarrow \boxed{\alpha = 90^\circ}$$

$$\tan \beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-1/7 - 7}{1 + (-1/7)(7)} = \frac{-50/7}{0} = \infty$$

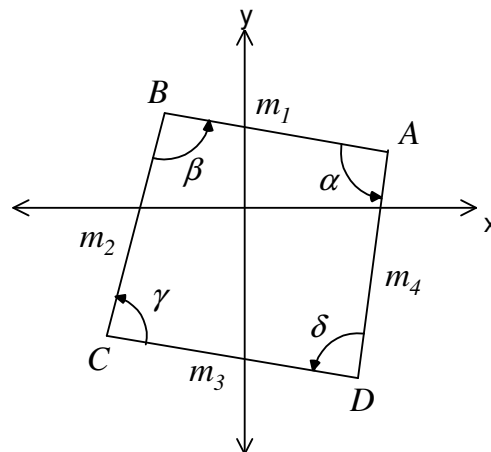
$$\Rightarrow \beta = \tan^{-1}(\infty) \Rightarrow \boxed{\beta = 90^\circ}$$

$$\tan \gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{7 - (-1/7)}{1 + (7)(-1/7)} = \frac{7 + 1/7}{1 - 1} = \frac{50/7}{0} = \infty$$

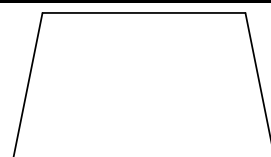
$$\Rightarrow \gamma = \tan^{-1}(\infty) \Rightarrow \boxed{\gamma = 90^\circ}$$

$$\tan \delta = \frac{m_3 - m_4}{1 + m_3 m_4} = \frac{-1/7 - 7}{1 + (-1/7)(7)} = \frac{-50/7}{0} = \infty$$

$$\Rightarrow \delta = \tan^{-1}(\infty) \Rightarrow \boxed{\delta = 90^\circ}$$

**Trapezium**

If any two opposite sides of the quadrilateral are parallel then it is called *trapezium*.



Question # 13

Show that the points $A(-1,-1)$, $B(-3,0)$, $C(3,7)$ and $D(1,8)$ are the vertices of the rhombus and find its interior angle.

Solution Given vertices are $A(-1,-1)$, $B(-3,0)$, $C(3,7)$ and $D(1,8)$

Let m_1 , m_2 , m_3 and m_4 be slopes of side \overline{AB} , \overline{BD} , \overline{DC} and \overline{CA} . Then

$$m_1 = \frac{0+1}{-3+1} = \frac{1}{-2}$$

$$m_2 = \frac{8-0}{1+3} = \frac{8}{4} = 2$$

$$m_3 = \frac{7-8}{3-1} = \frac{-1}{2}$$

$$m_4 = \frac{-1-7}{-1-3} = \frac{-8}{-4} = 2$$

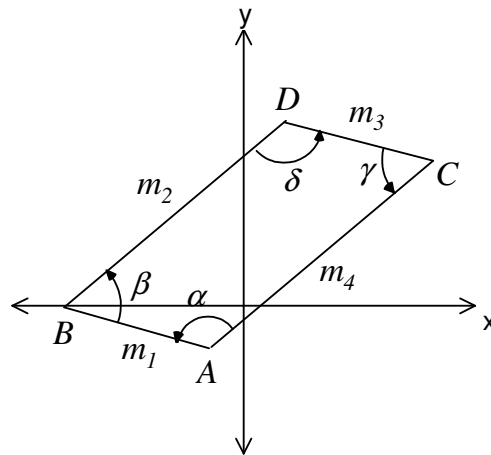
Since $m_2 = m_4$ or $m_1 = m_3$

Hence A, B, C and D are vertices of trapezium.

Now suppose α, β, γ and δ are angles of

quadrilateral at vertices A, B, C and D respectively. Then

Now do yourself as above in Question # 12

**Question # 14**

Find the area of the region bounded by the triangle, whose sides are

$$7x - y - 10 = 0; 10x + y - 41 = 0; 3x + 2y + 3 = 0$$

Solution

Let $l_1: 7x - y - 10 = 0$

$l_2: 10x + y - 41 = 0$

$l_3: 3x + 2y + 3 = 0$

For intersection of l_1 and l_2

$$\frac{x}{41+10} = \frac{-y}{-287+100} = \frac{1}{7+10}$$

$$\Rightarrow \frac{x}{51} = \frac{-y}{-187} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{51} = \frac{1}{17} \quad \text{and} \quad \frac{y}{187} = \frac{1}{17}$$

$$\Rightarrow x = \frac{51}{17} = 3 \quad \text{and} \quad y = \frac{187}{17} = 11$$

$\Rightarrow (3, 11)$ is the point of intersection of l_1 and l_2 .

Now for point of intersection of l_2 and l_3

$$\frac{x}{3+82} = \frac{-y}{30+123} = \frac{1}{20-3}$$

$$\Rightarrow \frac{x}{85} = \frac{-y}{153} = \frac{1}{17}$$

$$\begin{aligned}\Rightarrow \frac{x}{85} &= \frac{-y}{153} = \frac{1}{17} \\ \Rightarrow \frac{x}{85} &= \frac{1}{17} \quad \text{and} \quad \frac{-y}{153} = \frac{1}{17} \\ \Rightarrow x &= \frac{85}{17} = 5 \quad \text{and} \quad y = -\frac{153}{17} = -9\end{aligned}$$

$\Rightarrow (5, -9)$ is the point of intersection of l_2 and l_3 .

For point of intersection of l_1 and l_3

$$\begin{aligned}\frac{x}{-3+20} &= \frac{-y}{21+30} = \frac{1}{14+3} \\ \Rightarrow \frac{x}{17} &= \frac{-y}{51} = \frac{1}{17} \\ \Rightarrow \frac{x}{17} &= \frac{1}{17} \quad \text{and} \quad \frac{-y}{51} = \frac{1}{17} \\ \Rightarrow x &= \frac{17}{17} = 1 \quad \text{and} \quad y = -\frac{51}{17} = 3\end{aligned}$$

$\Rightarrow (1, -3)$ is the point of intersection of l_1 and l_3 .

Now area of triangle having vertices $(3, 11)$, $(5, -9)$ and $(1, -3)$ is given by:

$$\begin{aligned}& \frac{1}{2} \begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix} \\ &= \frac{1}{2} |3(-9+3) - 11(5-1) + 1(-15+9)| \\ &= \frac{1}{2} |3(-6) - 11(4) + 1(-6)| = \frac{1}{2} |-18 - 44 - 6| \\ &= \frac{1}{2} |-68| = \frac{1}{2} (68) = 34 \text{ sq. unit}\end{aligned}$$

Question # 15

The vertices of a triangle are $A(-2, 3)$, $B(-4, 1)$ and $C(3, 5)$. Find the centre of the circum centre of the triangle?

Solution Same Question # 7(c)

Question # 16

Express the given system of equations in matrix form. Find in each case whether in lines are concurrent.

- (a) $x + 3y - 2 = 0$; $2x - y + 4 = 0$; $x - 11y + 14 = 0$
 (b) $2x + 3y + 4 = 0$; $x - 2y - 3 = 0$; $3x + y - 8 = 0$
 (c) $3x - 4y - 2 = 0$; $x + 2y - 4 = 0$; $3x - 2y + 5 = 0$

Solution

$$\begin{aligned}
 \text{(a)} \quad & x + 3y - 2 = 0 \\
 & 2x - y + 4 = 0 \\
 & x - 11y + 14 = 0
 \end{aligned}$$

In matrix form

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

$$\begin{aligned}
 \Rightarrow |A| &= 1(-14 + 44) - 3(28 - 4) - 2(-22 + 1) \\
 &= 1(30) - 3(24) - 2(-21) \\
 &= 30 - 72 + 42 = 0
 \end{aligned}$$

Hence given lines are concurrent.

$$\begin{aligned}
 \text{(b)} \quad & 2x + 3y + 4 = 0 \\
 & x - 2y - 3 = 0 \\
 & 3x + y - 8 = 0
 \end{aligned}$$

In matrix form

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}$$

$$\begin{aligned}
 \Rightarrow |A| &= 2(16 + 3) - 3(-8 + 9) + 4(1 + 6) \\
 &= 2(19) - 3(1) + 4(7) = 38 - 3 + 28 = 63 \neq 0
 \end{aligned}$$

Hence given lines are not concurrent.

(c) *Do yourself as above*

Question # 17

Find a system of linear equations corresponding to the given matrix form. Check whether the lines responded by the system are concurrent.

$$\text{(a)} \quad \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{(b)} \quad \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

(a)

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+0-1 \\ 2x+0+1 \\ 0-y+2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x-1 \\ 2x+1 \\ -y+2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Equating the elements

$$\begin{aligned} x-1 &= 0 \\ 2x+1 &= 0 \\ -y+2 &= 0 \end{aligned}$$

are the required equation of lines.

Coefficients matrix of the system

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \det A = 1(0+1) - 0 - 1(-2-0)$$

$$= 1 + 2 = 3 \neq 0$$

Hence system is not concurrent.

(b)

*Do yourself as above.*Please report us error at www.mathcity.org/error**Book:****Exercise 4.4 (Page 223)**

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