Question # 1
The two points P and O' are given in $xy-coordinates$ system. Find the $XY-coordinates$ of P referred to the translated axes $O'X$ and $O'Y$

(i) $P(3,2);O'(1,3)$

(ii) $P(-2,6);O'(-3,2)$

(iii) $P(-6,-8);O'(-4,-6)$

(iv) $P\left(\frac{3}{2},\frac{5}{2}\right);O'\left(-\frac{1}{2},\frac{7}{2}\right)$

Solution

(i) Since $P(x,y) = P(3,2)$

i.e. $x=3$ and $y=2$

$O'(h,k) = O'(1,3)$

i.e. $h=1$ and $k=3$

$\because\ X = x-h$
$= 3-1 = 2$

Also $Y = y-k$
$= 2-3 = -1$

Hence $(2,-1)$ is point P in $XY-$coordinates.

(ii) *Do yourself*

(iii) *Do yourself*

(iv) Since $P(x,y) = P\left(\frac{3}{2},\frac{5}{2}\right)$

i.e. $x = \frac{3}{2}$ and $y = \frac{5}{2}$

$O'(h,k) = O'\left(-\frac{1}{2},\frac{7}{2}\right)$

i.e. $h=-\frac{1}{2}$ and $k=\frac{7}{2}$

$\because\ X = x-h$
$= \frac{3}{2}-\left(-\frac{1}{2}\right) = 2$

And $Y = y-k$
$= \frac{5}{2}-\frac{7}{2} = -1$

Hence $(2,-1)$ are coordinates of P in XY-axes.

Question # 2
The $xy-coordinates$ axes are translated through the point O' whose coordinates are given in $xy-coordinates$. The coordinates of P are given in the $XY-coordinates$ system. Find the coordinates of P in $xy-coordinates$ system.

(i) $P(8,10);O'(3,4)$

(ii) $P(-5,-3);O'(-2,-6)$

(iii) $P\left(-\frac{3}{4},-\frac{7}{6}\right);O'\left(\frac{1}{4},-\frac{1}{6}\right)$

(iv) $P(4,-3);O'(-2,3)$

Solution

(i) $\because\ P(X,Y) = P(8,10)$

$\Rightarrow X=8$ and $Y=10$

$O'(h,k) = O'(3,4)$

$\Rightarrow h=3$ and $k=4$

$\because\ X = x-h$

$\Rightarrow 8 = x-3$

$\Rightarrow x = 8+3 \quad \Rightarrow \quad x = 11$

Also $Y = y-k$

$\Rightarrow 10 = y-4$

$\Rightarrow y = 10+4 \quad \Rightarrow \quad y = 14$

Hence $(11,14)$ are coordinates of P in xy-axes.

(ii) *Do yourself*

(iii) $\because\ P(X,Y) = P\left(-\frac{3}{4},-\frac{7}{6}\right)$

$\Rightarrow X=-\frac{3}{4}$ and $Y=-\frac{7}{6}$

$O'(h,k) = O'\left(\frac{1}{4},-\frac{1}{6}\right)$

$\Rightarrow h=\frac{1}{4}$ and $k=-\frac{1}{6}$

$\because\ X = x-h$

$\Rightarrow -\frac{3}{4} = x-\frac{1}{4}$

$\Rightarrow x = -\frac{3}{4}+\frac{1}{4} \quad \Rightarrow \quad x = -\frac{1}{2}$

Also $Y = y-k$

$\Rightarrow\ -\dfrac{7}{6} = y+\dfrac{1}{6}$

$\Rightarrow\ y = -\dfrac{7}{6}-\dfrac{1}{6} \quad \Rightarrow\ y = -\dfrac{4}{3}$

Hence $\left(-\dfrac{1}{2},-\dfrac{4}{3}\right)$ is the required point.

(iv) *Do yourself*

Rotation of Axes

Let (x, y) be the coordinates of point P in *xy*-coordinate system. If the axes are rotated through at angle of θ and (X,Y) are coordinate of P in new *XY*-coordinate system then

$$X = x\cos\theta + y\sin\theta$$
$$Y = y\cos\theta - x\sin\theta$$

Question # 3

The $xy-coordinates$ axes are rotated about the origin through the indicated angle. The new axes are OX and OY. Find the $XY-coordinates$ of the point P with the given $xy-coordinates$.

(i) $P(5,3);\theta=45^\circ$

(ii) $P(3,-7);\theta=30^\circ$

(iii) $P(11,-15);\theta=60^\circ$

(iv) $P(15,10);\theta=\arctan\dfrac{1}{3}$

Solution

(i) $\because\ P(x,y) = P(5,3)$

$\Rightarrow\ x=5\ \ \&\ \ y=3\ \ ,\ \theta=45^\circ$

Since $X = x\cos\theta + y\sin\theta$

$= 5\cos45^\circ + 3\sin45^\circ$

$= 5\left(\dfrac{1}{\sqrt{2}}\right)+3\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{1}{\sqrt{2}}(5+3)$

$= \dfrac{8}{\sqrt{2}} = \dfrac{4\times2}{\sqrt{2}} = 4\sqrt{2}$

Now $Y = y\cos\theta - x\sin\theta$

$= 3\cos45^\circ - 5\sin45^\circ$

$= 3\left(\dfrac{1}{\sqrt{2}}\right)-5\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{1}{\sqrt{2}}(3-5)$

$= -2\left(\dfrac{1}{\sqrt{2}}\right) = -\sqrt{2}$

Hence the required point is $\left(4\sqrt{2},-\sqrt{2}\right)$.

(ii) $\because\ P(x,y) = P(3,-7)$

$\Rightarrow\ x=3\ \ \&\ \ y=-7\ \ ,\ \theta=30^\circ$

Since $X = x\cos\theta + y\sin\theta$

$= 3\cos30^\circ - 7\sin30^\circ$

$\Rightarrow\ X = 3\left(\dfrac{\sqrt{3}}{2}\right)-7\left(\dfrac{1}{2}\right)$

$= \dfrac{3\sqrt{3}-7}{2}$

Now $Y = y\cos\theta - x\sin\theta$

$= -7\cos30^\circ - 3\sin30^\circ$

$= -7\left(\dfrac{\sqrt{3}}{2}\right)-3\left(\dfrac{1}{2}\right) = \dfrac{-7\sqrt{3}-3}{2}$

Hence the required point is

$\left(\dfrac{3\sqrt{3}-7}{2},\dfrac{-7\sqrt{3}-3}{2}\right)$.

(iv) *Do yourself*

(iv) $\because\ P(x,y) = P(15,10)$

$\Rightarrow x=15\ \ \&\ \ y=10$

Also $\theta=\tan^{-1}\left(\dfrac{1}{3}\right)$

$\Rightarrow\ \tan\theta=\dfrac{1}{3}$

$\Rightarrow\ \dfrac{\sin\theta}{\cos\theta}=\dfrac{1/\sqrt{10}}{3/\sqrt{10}}$

$\tan\theta=\dfrac{y}{x}=\dfrac{1}{3}$

$\Rightarrow\ x=3\ ,\ y=1$

$r=\sqrt{x^2+y^2}$

$=\sqrt{3^2+1^2}$

$=\sqrt{10}$

$\Rightarrow\ \sin\theta=\dfrac{1}{\sqrt{10}}\ \ ,\ \ \cos\theta=\dfrac{3}{\sqrt{10}}$

Now $X = x\cos\theta + y\sin\theta$

$= 15\left(\dfrac{3}{\sqrt{10}}\right)+10\left(\dfrac{1}{\sqrt{10}}\right)$

$= \dfrac{1}{\sqrt{10}}(45+10) = \dfrac{55}{\sqrt{10}}$

$Y = y\cos\theta - x\sin\theta$

$=10\left(\dfrac{3}{\sqrt{10}}\right)-15\left(\dfrac{1}{\sqrt{10}}\right)$

$= \dfrac{1}{\sqrt{10}}(30-15) = \dfrac{15}{\sqrt{10}}$

Hence the required point is $\left(\dfrac{55}{\sqrt{10}},\dfrac{15}{\sqrt{10}}\right)$.

(iv)

$\because P(x,y) = P(15,10)$

$\Rightarrow x=15 \quad \& \quad y=10$

Also $\theta = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/2}{\sqrt{3}/2}$

$\Rightarrow \sin\theta = \dfrac{1}{2}\ , \qquad \cos\theta = \dfrac{\sqrt{3}}{2}$

$\tan\theta = \dfrac{y}{x} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow x = \sqrt{3}\ ,\ y = 1$

$r = \sqrt{x^2+y^2}$

$= \sqrt{\left(\sqrt{3}\right)^2 + 1^2}$

$= \sqrt{4} = 2$

Now $X = x\cos\theta + y\sin\theta$

$= 15\left(\dfrac{\sqrt{3}}{2}\right) + 10\left(\dfrac{1}{2}\right)$

$= \dfrac{15\sqrt{3}+10}{2}$

$Y = y\cos\theta - x\sin\theta$

$= 10\left(\dfrac{\sqrt{3}}{2}\right) - 15\left(\dfrac{1}{2}\right)$

$= \dfrac{10\sqrt{3}-15}{2}$

Hence the required point is

$\left(= \dfrac{15\sqrt{3}+10}{2}, \dfrac{10\sqrt{3}-15}{2}\right).$

Question # 4

The $xy-coordinates$ axes are rotated about the origin through the indicated angle and the new axes are OX and OY , Find the $xy-coordinates$ of P with the given $XY-coordinates$.

(i) $P(-5,3); \theta = 30^\circ$

(ii) $P\left(-7\sqrt{2}, 5\sqrt{2}\right); \theta = 45^\circ$

Solution

(i) $\because P(X,Y) = P(-5,3)$

$\Rightarrow X = -5 \quad \& \quad Y = 3$

Also $\theta = 30^\circ$

Therefore $\sin\theta = \dfrac{1}{2} \quad \& \quad \cos\theta = \dfrac{\sqrt{3}}{2}$

Now $X = x\cos\theta + y\sin\theta$

$\Rightarrow -5 = x\left(\dfrac{\sqrt{3}}{2}\right) + y\left(\dfrac{1}{2}\right)$

$\Rightarrow \sqrt{3}x + y = -10$ .... (i)

Also $Y = y\cos\theta - x\sin\theta$

$\Rightarrow 3 = y\left(\dfrac{\sqrt{3}}{2}\right) - x\left(\dfrac{1}{2}\right)$

$\Rightarrow 6 = \sqrt{3}y - x$

$\Rightarrow x = \sqrt{3}y - 6$ .... (ii)

Putting value of x in (i)

$\sqrt{3}\left(\sqrt{3}y-6\right) + y = -10$

$\Rightarrow 3y - 6\sqrt{3} + y = -10$

$\Rightarrow 4y = -10 + 6\sqrt{3}$

$\Rightarrow y = \dfrac{-10+6\sqrt{3}}{4}$

$= \dfrac{-5+3\sqrt{3}}{2}$

Putting value of y in (ii)

$x = \sqrt{3}\left(\dfrac{-5+3\sqrt{3}}{2}\right) - 6$

$= \dfrac{-5\sqrt{3}+9}{2} - 6 = \dfrac{-5\sqrt{3}+9-12}{2}$

$= \dfrac{-5\sqrt{3}-3}{2}$

Hence $\left(\dfrac{-5\sqrt{3}-3}{2}, \dfrac{-5+3\sqrt{3}}{2}\right)$ is required point.

(ii) *Do yourself*

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Exercise 4.1 *(Page 189)*
Calculus and Analytic Geometry
Mathematative 12
Punjab Textbook Board, Lahore.

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