

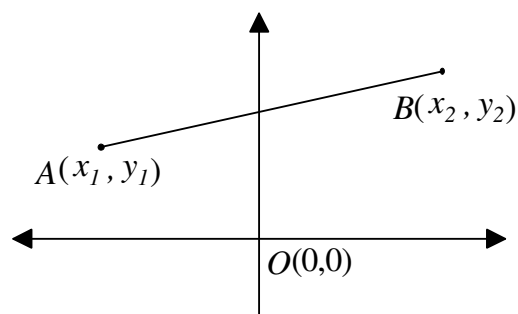
Distance Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane and d be a distance between A and B then

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

See proof on book at page 181

**Ratio Formula**

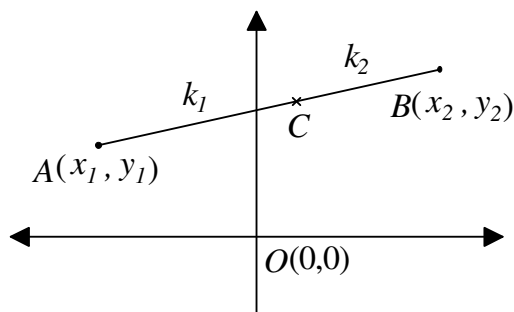
Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane. The coordinates of the point C dividing the line segment AB in the ratio $k_1 : k_2$ are

$$\left(\frac{k_1 x_2 + k_2 x_1}{k_1 + k_2}, \frac{k_1 y_2 + k_2 y_1}{k_1 + k_2} \right)$$

See proof on book at page 182

If C be the midpoint of AB i.e. $k_1 : k_2 = 1 : 1$ then coordinate of C becomes

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Question # 1**

Describe the location in the plane of the point $P(x, y)$ for which

(i) $x > 0$

(ii) $x > 0$ and $y > 0$

(iii) $x = 0$

(iv) $y = 0$

(v) $x < 0$ and $y \geq 0$

(vi) $x = y$

(vii) $|x| = -|y|$

(viii) $|x| \geq 3$

(ix) $x > 2$ and $y = 2$

(x) x and y have opposite signs.

Solution

(i) $x > 0$

Right half plane

(ii) $x > 0$ and $y > 0$

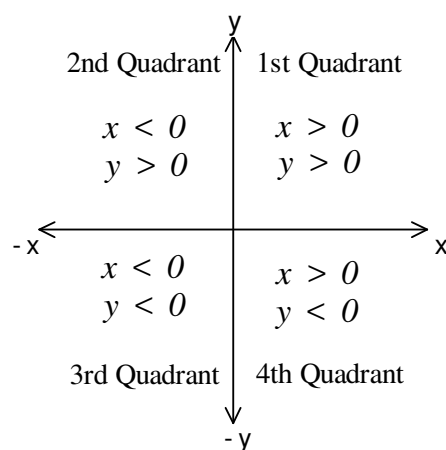
The 1st quadrant.

(iii) $x = 0$

y -axis

(iv) $y = 0$

x -axis



(v) $x < 0$ and $y \geq 0$

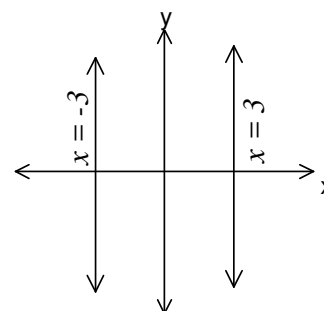
2nd quadrant & negative x-axis

(vi) $x = y$

It is a line bisecting 1st and 3rd quadrant.

(vii) $|x| = -|y|$

A positive value can't equal to a negative value, except number zero, so origin,

(0,0), is the only point which satisfies $|x| = -|y|$ 

(viii) $|x| \geq 3$

$$\Rightarrow \pm x \geq 3 \Rightarrow x \geq 3 \text{ or } -x \geq 3$$

$$\Rightarrow x \geq 3 \text{ or } x \leq -3$$

which is the set of points lying on right side of the line $x = 3$ and the points lying on left side of the line $x = -3$.

(ix) $x > 2$ and $y = 2$

The set of all points on the line $y = 2$ for which $x > 2$.

(x) x and y have opposite signs.

It is the set of points lying in 2nd and 4th quadrant.**Question # 2**

Find each of the following

(i) the distance between the two given points

(ii) Midpoint of the line segment joining the two points

(a) $A(3,1); B(-2,-4)$ (b) $A(-8,3); B(2,-1)$ (c) $A\left(-\sqrt{5}, -\frac{1}{3}\right); B(-3\sqrt{5}, 5)$

Solution

(a) $A(3,1)$; $B(-2,-4)$

$$(i) \quad |AB| = \sqrt{(-2-3)^2 + (-4-1)^2} = \sqrt{(-5)^2 + (-5)^2} \\ = \sqrt{25+25} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$(ii) \quad \text{Midpoint of } AB = \left(\frac{3-2}{2}, \frac{1-4}{2} \right) = \left(\frac{1}{2}, \frac{-3}{2} \right)$$

(b) $A(-8,3)$; $B(2,-1)$

*Do yourself as above.***Review:**

The midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$

is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

(c) $A\left(-\sqrt{5}, -\frac{1}{3}\right)$; $B(-3\sqrt{5}, 5)$

(i) $|AB| = \sqrt{(-3\sqrt{5} + \sqrt{5})^2 + \left(5 + \frac{1}{3}\right)^2} = \sqrt{(2\sqrt{5})^2 + \left(\frac{16}{3}\right)^2}$

$$= \sqrt{20 + \frac{256}{9}} = \sqrt{\frac{436}{9}} = \sqrt{\frac{4 \times 109}{9}} = \frac{2\sqrt{109}}{3}$$

(ii) Midpoint of $AB = \left(\frac{-\sqrt{5} - 3\sqrt{5}}{2}, \frac{-\frac{1}{3} + 5}{2}\right) = \left(\frac{-4\sqrt{5}}{2}, \frac{\frac{14}{3}}{2}\right) = \left(-2\sqrt{5}, \frac{7}{3}\right)$

Question # 3

Which of the following points are at a distance of 15 units from the origin?

(a) $(\sqrt{176}, 7)$ (b) $(10, -10)$ (c) $(1, 15)$ (d) $\left(\frac{15}{2}, \frac{15}{2}\right)$

Solution

(a) Distance of $(\sqrt{176}, 7)$ from origin $= \sqrt{(\sqrt{176} - 0)^2 + (7 - 0)^2}$

$$= \sqrt{(176) + (49)}$$

$$= \sqrt{(176) + (49)} = \sqrt{225} = 15$$

\Rightarrow the point $(\sqrt{176}, 7)$ is at 15 unit away from origin.

(b) Distance of $(10, -10)$ from origin $= \sqrt{(10 - 0)^2 + (-10 - 0)^2}$

$$= \sqrt{100 + 100} = \sqrt{200}$$

$$= \sqrt{100 \times 2} = 10\sqrt{2} \neq 15$$

\Rightarrow the point $(10, -10)$ is not at distance of 15 unit from origin.

(c) *Do yourself as above*

(d) Distance of $\left(\frac{15}{2}, \frac{15}{2}\right)$ from origin $= \sqrt{\left(\frac{15}{2} - 0\right)^2 + \left(\frac{15}{2} - 0\right)^2}$

$$= \sqrt{\frac{225}{4} + \frac{225}{4}} = \sqrt{\frac{225}{2}} = \frac{15}{\sqrt{2}} \neq 15$$

Hence the point $\left(\frac{15}{2}, \frac{15}{2}\right)$ is not at distance of 15 unit from origin.

Question # 4

Show that

(i) the point $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.

- (ii) the point $A(3,1)$, $B(-2,-3)$ and $C(2,2)$ are vertices of an isosceles triangle.
- (iii) the point $A(3,1)$, $B(-2,-3)$ and $C(2,2)$ and $D(4,-5)$ are vertices of a parallelogram. Is the parallelogram a square?

Solution

- (i) Given: $A(0,2)$, $B(\sqrt{3},-1)$ and $C(0,-2)$

$$|AB| = \sqrt{(\sqrt{3}-0)^2 + (-1-2)^2} = \sqrt{(\sqrt{3})^2 + (-3)^2}$$

$$= \sqrt{3+9} = \sqrt{12} \quad \Rightarrow |AB|^2 = 12$$

$$|BC| = \sqrt{(0-\sqrt{3})^2 + (-2+1)^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2}$$

$$= \sqrt{3+1} = \sqrt{4} = 2 \quad \Rightarrow |BC|^2 = 4$$

$$|CA| = \sqrt{(0-0)^2 + (2+2)^2} = \sqrt{0+(4)^2}$$

$$= \sqrt{16} = 4 \quad \Rightarrow |CA|^2 = 16$$

$$\therefore |AB|^2 + |BC|^2 = 12 + 4 = 16 = |CA|^2$$

\therefore by Pythagoras theorem A, B & C are vertices of a right triangle.

- (ii) Given: $A(3,1)$, $B(-2,-3)$ and $C(2,2)$

$$|AB| = \sqrt{(-2-3)^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

$$|BC| = \sqrt{(2-(-2))^2 + (2-(-3))^2} = \sqrt{(4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$$

$$|CA| = \sqrt{(3-2)^2 + (1-2)^2} = \sqrt{(1)^2 + (-1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$\therefore |AB| = |BC| \Rightarrow A, B$ & C are vertices of an isosceles triangle.

- (iii) Given: $A(5,2)$, $B(-2,3)$ & $C(-3,-4)$ and $D(4,-5)$

$$|AB| = \sqrt{(-2-5)^2 + (3-2)^2} = \sqrt{(-7)^2 + (1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$|BC| = \sqrt{(-3+2)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$$

$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

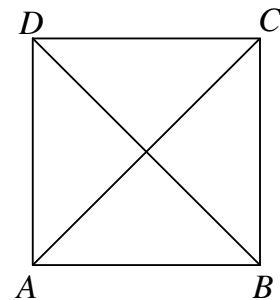
$$|CD| = \sqrt{(4+3)^2 + (-5+4)^2} = \sqrt{(7)^2 + (-1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$|DA| = \sqrt{(5-4)^2 + (2+5)^2} = \sqrt{(1)^2 + (7)^2}$$

$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$\therefore |AB| = |CD|$ and $|BC| = |DA| \Rightarrow A, B, C$ and D are vertices of parallelogram.



$$\begin{aligned}
 \text{Now } |AC| &= \sqrt{(-3-5)^2 + (-4-2)^2} = \sqrt{(-8)^2 + (-6)^2} \\
 &= \sqrt{64 + 36} = \sqrt{100} = 10 \\
 |BD| &= \sqrt{(4+2)^2 + (-5-3)^2} = \sqrt{(6)^2 + (-8)^2} \\
 &= \sqrt{36 + 64} = \sqrt{100} = 10
 \end{aligned}$$

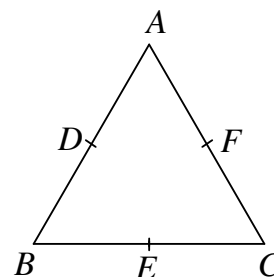
Since all sides are equal and also both diagonals are equal therefore A, B, C, D are vertices of a square.

Question # 5

The midpoints of the sides of a triangle are $(1, -1)$, $(-4, -3)$ and $(-1, 1)$. Find coordinates of the vertices of the triangle.

Solution

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle ABC , and let $D(1, -1)$, $E(-4, -3)$ and $F(-1, 1)$ are midpoints of sides AB , BC and CA respectively. Then



$$\begin{aligned}
 \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= (1, -1) \\
 \Rightarrow x_1 + x_2 &= 2 \dots \text{(i)} \quad \text{and} \quad y_1 + y_2 = -2 \dots \text{(ii)} \\
 \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right) &= (-4, -3) \\
 \Rightarrow x_2 + x_3 &= -8 \dots \text{(iii)} \quad \text{and} \quad y_2 + y_3 = -6 \dots \text{(iv)} \\
 \left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2} \right) &= (-1, 1) \\
 \Rightarrow x_1 + x_3 &= -2 \dots \text{(v)}, \quad \text{and} \quad y_1 + y_3 = 2 \dots \text{(vi)}
 \end{aligned}$$

Subtracting (i) and (iii)

$$\begin{array}{rcl}
 x_1 + x_2 & = & 2 \\
 - x_2 + x_3 & = & -8 \dots \text{(vii)} \\
 \hline
 x_1 - x_3 & = & 10
 \end{array}$$

Adding (v) and (vii)

$$\begin{array}{rcl}
 x_1 + x_3 & = & -2 \\
 x_1 - x_3 & = & 10 \\
 \hline
 2x_1 & = & 8 \Rightarrow \boxed{x_1 = 4}
 \end{array}$$

Putting value of x_1 in (i)

$$\begin{array}{l}
 4 + x_2 = 2 \\
 \Rightarrow x_2 = 2 - 4 \Rightarrow \boxed{x_2 = -2}
 \end{array}$$

Putting value of x_1 in (v)

$$\begin{array}{l}
 4 + x_3 = -2 \\
 \Rightarrow x_3 = -2 - 4 \Rightarrow \boxed{x_3 = -6}
 \end{array}$$

Subtracting (ii) and (iv)

$$\begin{array}{rcl}
 y_1 + y_2 & = & -2 \\
 - y_2 + y_3 & = & -6 \dots \text{(viii)} \\
 \hline
 y_1 - y_3 & = & 4
 \end{array}$$

Adding (vi) and (viii)

$$\begin{array}{rcl}
 y_1 + y_3 & = & 2 \\
 y_1 - y_3 & = & 4 \\
 \hline
 2y_1 & = & 6 \Rightarrow \boxed{y_1 = 3}
 \end{array}$$

Putting value of y_1 in (ii)

$$\begin{array}{l}
 3 + y_2 = -2 \\
 \Rightarrow y_2 = -2 - 3 \Rightarrow \boxed{y_2 = -5}
 \end{array}$$

Putting value of y_1 in (v)

$$\begin{array}{l}
 3 + y_3 = 2 \\
 \Rightarrow y_3 = 2 - 3 \Rightarrow \boxed{y_3 = -1}
 \end{array}$$

Hence vertices of triangle are $(4,3), (-2,-5)$ & $(-6,-1)$.

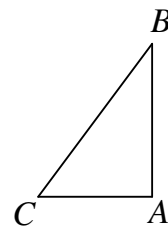
Question # 6

Find h such that the point $A(\sqrt{3}, -1)$, $B(0, 2)$ and $C(h, -2)$ are vertices of a right angle with right angle at the vertex A .

Solution

Since ABC is a right triangle therefore by Pythagoras theorem

$$\begin{aligned}
 |AB|^2 + |CA|^2 &= |BC|^2 \\
 \Rightarrow \left[(0 - \sqrt{3})^2 + (2 + 1)^2 \right] + \left[(\sqrt{3} - h)^2 + (-1 + 2)^2 \right] &= (h - 0)^2 + (-2 - 2)^2 \\
 \Rightarrow [3 + 9] + [3 - 2\sqrt{3}h + h^2 + 1] &= h^2 + 16 \\
 \Rightarrow 12 + 4 - 2\sqrt{3}h + h^2 &= h^2 + 16 \\
 \Rightarrow -2\sqrt{3}h = h^2 + 16 - 12 - 4 - h^2 &\Rightarrow -2\sqrt{3}h = 0 \Rightarrow \boxed{h = 0}.
 \end{aligned}$$



Question # 7

Find h such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.

Solution

Points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are collinear if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Since given points are collinear therefore

$$\begin{aligned}
 &\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0 \\
 \Rightarrow -1(2 - 3) - h(3 - 7) + 1(9 - 14) &= 0 \Rightarrow -1(-1) - h(-4) + 1(-5) = 0 \\
 \Rightarrow 1 + 4h - 5 = 0 &\Rightarrow 4h - 4 = 0 \Rightarrow 4h = 4 \Rightarrow \boxed{h = 1}
 \end{aligned}$$

Question # 8

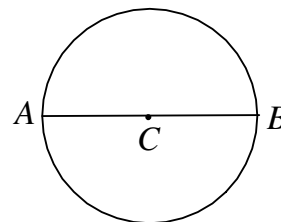
The points $A(-5, -2)$ and $B(5, -4)$ are end of a diameter of a circle. Find the centre and radius of the circle.

Solution

The centre of the circle is mid point of AB

$$\text{i.e. centre 'C'} = \left(\frac{-5 + 5}{2}, \frac{-2 - 4}{2} \right) = \left(\frac{0}{2}, \frac{-6}{2} \right) = (0, -3)$$

$$\begin{aligned}
 \text{Now radius} &= |AC| \\
 &= \sqrt{(0 + 5)^2 + (-3 + 2)^2} \\
 &= \sqrt{25 + 1} = \sqrt{26}
 \end{aligned}$$



Question # 9

Find h such that the points $A(h,1)$, $B(2,7)$ and $C(-6,-7)$ are vertices of a right triangle with right angle at the vertex A

Solution

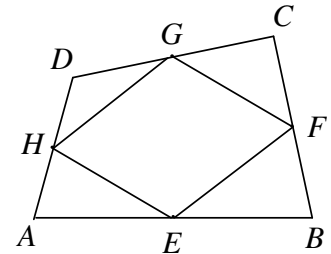
Do yourself as Question # 6

Hint: you will get a equation $h^2 + 4h - 60 = 0$

Solve this quadratic equation to get two values of h .

Question # 10

A quadrilateral has the points $A(9,3)$, $B(-7,7)$, $C(-3,-7)$ and $D(-5,5)$ as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.

**Solution**

Given: $A(9,3)$, $B(-7,7)$, $C(-3,-7)$ and $D(-5,5)$

Let E , F , G and H be the mid-points of sides of quadrilateral

$$\text{Coordinate of } E = \left(\frac{9-7}{2}, \frac{3+7}{2} \right) = \left(\frac{2}{2}, \frac{10}{2} \right) = (1,5)$$

$$\text{Coordinate of } F = \left(\frac{-7-3}{2}, \frac{7-7}{2} \right) = \left(\frac{-10}{2}, \frac{0}{2} \right) = (-5,0)$$

$$\text{Coordinate of } G = \left(\frac{-3+5}{2}, \frac{-7-5}{2} \right) = \left(\frac{2}{2}, \frac{-12}{2} \right) = (1,-6)$$

$$\text{Coordinate of } H = \left(\frac{9+5}{2}, \frac{3-5}{2} \right) = \left(\frac{14}{2}, \frac{-2}{2} \right) = (7,-1)$$

$$\text{Now } |EF| = \sqrt{(-5-1)^2 + (0-5)^2} = \sqrt{36+25} = \sqrt{61}$$

$$|FG| = \sqrt{(1+5)^2 + (-6-0)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$|GH| = \sqrt{(7-1)^2 + (-1+6)^2} = \sqrt{36+25} = \sqrt{61}$$

$$|HE| = \sqrt{(1-7)^2 + (5+1)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

Since $|EF| = |GH|$ and $|FG| = |HE|$

Therefore $EFGH$ is a parallelogram.

Question # 11

Find h such that the quadrilateral with vertices $A(-3,0)$, $B(1,-2)$, $C(5,0)$ and $D(1,h)$ is parallelogram. Is it a square?

Solution

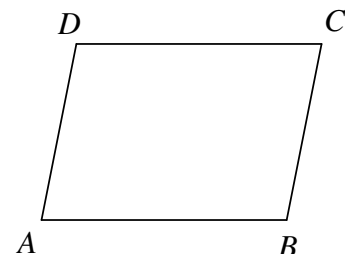
Given: $A(-3,0)$, $B(1,-2)$, $C(5,0)$, $D(1,h)$

Quadrilateral $ABCD$ is a parallelogram if

$$|AB| = |CD| \quad \& \quad |BC| = |AD|$$

when $|AB| = |CD|$

$$\Rightarrow \sqrt{(1+3)^2 + (-2-0)^2} = \sqrt{(1-5)^2 + (h-0)^2}$$



$$\Rightarrow \sqrt{16+4} = \sqrt{16+h^2} \Rightarrow \sqrt{20} = \sqrt{16+h^2}$$

On squaring

$$20 = 16 + h^2 \Rightarrow h^2 = 20 - 16 \Rightarrow h^2 = 4 \Rightarrow h = \pm 2$$

When $h = 2$, then $D(1, h) = D(1, 2)$

$$\text{Then } |AB| = \sqrt{(1+3)^2 + (-2-0)^2} = \sqrt{16+4} = \sqrt{20}$$

$$|BC| = \sqrt{(5-1)^2 + (0+2)^2} = \sqrt{16+4} = \sqrt{20}$$

$$|CA| = \sqrt{(1-5)^2 + (2-0)^2} = \sqrt{16+4} = \sqrt{20}$$

$$|DA| = \sqrt{(-3-1)^2 + (-0-2)^2} = \sqrt{16+4} = \sqrt{20}$$

Now for diagonals

$$|AC| = \sqrt{(5+3)^2 + (0-0)^2} = \sqrt{64+0} = 8$$

$$|BD| = \sqrt{(1-1)^2 + (2+2)^2} = \sqrt{0+16} = 4$$

Since all sides are equal but diagonals $|AC| \neq |BD|$

Therefore $ABCD$ is not a square.

Now when $h = -2$, then $D(1, h) = D(1, -2)$ but we also have $B(1, -2)$

i.e. B and D represents the same point, which can not happened in quadrilateral so we can not take $h = -2$.

Question # 12

If two vertices of an equilateral triangle are $A(-3, 0)$ and $B(3, 0)$, find the third vertex. How many of these triangles are possible?

Solution

Given: $A(-3, 0)$, $B(3, 0)$

Let $C(x, y)$ be a third vertex of an equilateral triangle ABC .

$$\text{Then } |AB| = |BC| = |CA|$$

$$\Rightarrow \sqrt{(3+3)^2 + (0-0)^2} = \sqrt{(x-3)^2 + (y-0)^2} = \sqrt{(x+3)^2 + (y-0)^2}$$

$$\Rightarrow \sqrt{36+0} = \sqrt{x^2 - 6x + 9 + y^2} = \sqrt{x^2 + 6x + 9 + y^2}$$

On squaring

$$36 = x^2 + y^2 - 6x + 9 = x^2 + y^2 + 6x + 9 \dots\dots\dots(i)$$

From equation (i)

$$x^2 + y^2 - 6x + 9 = x^2 + y^2 + 6x + 9$$

$$\Rightarrow x^2 + y^2 - 6x + 9 - x^2 - y^2 - 6x - 9 = 0$$

$$\Rightarrow -12x = 0 \Rightarrow x = 0$$

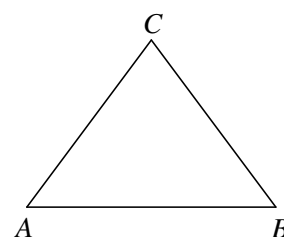
Again from equation (i)

$$36 = x^2 + y^2 - 6x + 9$$

$$\Rightarrow 36 = (0)^2 + y^2 - 6(0) + 9 \quad \because x = 0$$

$$\Rightarrow 36 = y^2 + 9 \Rightarrow y^2 = 36 - 9 = 27 \Rightarrow y = \pm 3\sqrt{3}$$

so coordinate of C is $(0, 3\sqrt{3})$ or $(0, -3\sqrt{3})$.



And hence two triangle can be formed with vertices $A(-3,0), B(3,0), C(0,3\sqrt{3})$ and $A(-3,0), B(3,0), C(0,-3\sqrt{3})$.

Question # 13

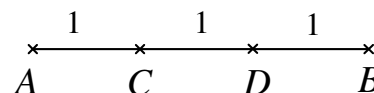
Find the points trisecting the join of $A(-1,4)$ and $B(6,2)$.

Solution

Given: $A(-1,4), B(6,2)$

Let C and D be points trisecting A and B

Then $AC:CB = 1:2$



$$\begin{aligned}\text{So coordinate of } C &= \left(\frac{1(6) + 2(-1)}{1+2}, \frac{1(2) + 2(4)}{1+2} \right) \\ &= \left(\frac{6-2}{3}, \frac{2+8}{3} \right) = \left(\frac{4}{3}, \frac{10}{3} \right)\end{aligned}$$

Also $AD:DB = 2:1$

$$\begin{aligned}\text{So coordinate of } D &= \left(\frac{2(6) + 1(-1)}{2+1}, \frac{2(2) + 1(4)}{2+1} \right) \\ &= \left(\frac{12-1}{3}, \frac{4+4}{3} \right) = \left(\frac{11}{3}, \frac{8}{3} \right)\end{aligned}$$

Hence $\left(\frac{4}{3}, \frac{10}{3} \right)$ and $\left(\frac{11}{3}, \frac{8}{3} \right)$ are points trisecting A and B .

Question # 14

Find the point three-fifth of the way along the line segment from $A(-5,8)$ to $B(5,3)$.

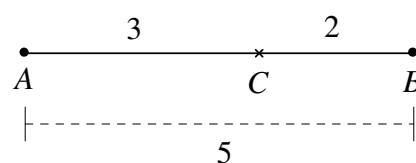
Solution

Given: $A(-5,8), B(5,3)$

Let $C(x, y)$ be a required point

$\therefore AC:CB = 3:2$

$$\begin{aligned}\therefore \text{Co-ordinate of } C &= \left(\frac{3(5) + 2(-5)}{3+2}, \frac{3(3) + 2(8)}{3+2} \right) \\ &= \left(\frac{15-10}{5}, \frac{9+16}{5} \right) = \left(\frac{5}{5}, \frac{25}{5} \right) = (1, 5)\end{aligned}$$



Question # 15

Find the point P on the joint of $A(1,4)$ and $B(5,6)$ that is twice as far from A as B is from A and lies

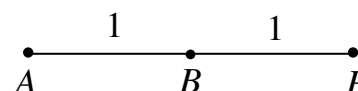
(i) on the same side of A as B does.

(ii) on the opposite side of A as B does.

Solution

Given: $A(1,4), B(5,6)$

(i) Let $P(x, y)$ be required point, then



$$AB:AP = 1:2$$

$$\Rightarrow AB:BP = 1:1 \quad \text{i.e. } B \text{ is midpoint of } AP$$

$$\text{Then } B(5,6) = \left(\frac{1+x}{2}, \frac{4+y}{2} \right)$$

$$\Rightarrow 5 = \frac{1+x}{2} \quad \text{and} \quad 6 = \frac{4+y}{2}$$

$$\Rightarrow 10 = 1+x \quad \text{and} \quad 12 = 4+y$$

$$\Rightarrow x = 10-1, \quad y = 12-4$$

$$= 9, \quad = 8$$

Hence $P(9,8)$ is required point.

(ii) Since $PA:AB = 2:1$

$$\Rightarrow A(1,4) = \left(\frac{2(5)+1(x)}{2+1}, \frac{2(6)+1(y)}{2+1} \right)$$

$$= \left(\frac{10+x}{3}, \frac{12+y}{3} \right)$$

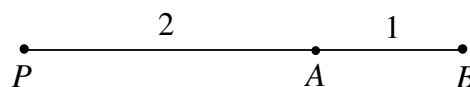
$$\Rightarrow 1 = \frac{10+x}{3} \quad \text{and} \quad 4 = \frac{12+y}{3}$$

$$\Rightarrow 3 = 10+x \quad \text{and} \quad 12 = 12+y$$

$$\Rightarrow x = 3-10 \quad \text{and} \quad y = 12-12$$

$$= -7, \quad = 0$$

Hence $P(-7,0)$ is required point.



Question # 16

Find the point which is equidistant from the points $A(5,3)$, $B(2,-2)$ and $C(4,2)$.

What is the radius of the circumcircle of the ΔABC ?

Solution

Given: $A(5,3)$, $B(-2,2)$ and $C(4,2)$

Let $D(x,y)$ be a point equidistance from A , B and C then

$$|\overline{DA}| = |\overline{DB}| = |\overline{DC}|$$

$$\Rightarrow |\overline{DA}|^2 = |\overline{DB}|^2 = |\overline{DC}|^2$$

$$\Rightarrow (x-5)^2 + (y-3)^2 = (x+2)^2 + (y-2)^2 = (x-4)^2 + (y-2)^2 \dots\dots\dots (i)$$

From eq. (i)

$$(x-5)^2 + (y-3)^2 = (x+2)^2 + (y-2)^2$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 6y + 9 = x^2 + 4x + 4 + y^2 - 4y + 4$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 6y + 9 - x^2 - 4x - 4 - y^2 + 4y - 4 = 0$$

$$\Rightarrow -14x - 2y + 26 = 0 \quad \Rightarrow 7x + y - 13 = 0 \dots\dots\dots (ii)$$

Again from equation (i)

$$(x+2)^2 + (y-2)^2 = (x-4)^2 + (y-2)^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2 - 4y + 4$$

$$\Rightarrow 12x - 12 = 0 \Rightarrow 12x = 12 \Rightarrow x = 1$$

Put $x = 1$ in eq. (ii)

$$7(1) + y - 13 = 0 \Rightarrow y - 6 = 0 \Rightarrow y = 6$$

Hence $(1, 6)$ is required point.

$$\begin{aligned} \text{Now radius of circumcircle} &= |\overline{DA}| \\ &= \sqrt{(5-1)^2 + (3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units} \end{aligned}$$

Intersection of Median

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle.

Intersection of median is called centroid of triangle and can be determined as

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \quad \text{See proof at page 184}$$

Centre of In-Circle (In-Centre)

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle.

And $|AB| = c$, $|BC| = a$, $|CA| = b$

$$\text{Then in-centre of triangle} = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) \quad \text{See proof at page 184}$$

Question # 17

The points $(4, -2)$, $(-2, 4)$ and $(5, 5)$ are the vertices of a triangle. Find in-centre of the triangle.

Solution

Let $A(4, -2)$, $B(-2, 4)$, $C(5, 5)$ are vertices of triangle then

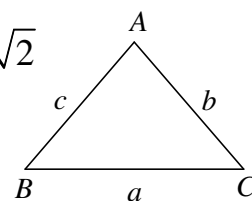
$$a = |BC| = \sqrt{(5+2)^2 + (5-4)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$b = |CA| = \sqrt{(4-5)^2 + (-2-5)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$c = |AB| = \sqrt{(-2-4)^2 + (4+2)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

Now

$$\begin{aligned} \text{In-centre} &= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) \\ &= \left(\frac{5\sqrt{2}(4) + 5\sqrt{2}(-2) + 6\sqrt{2}(5)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}, \frac{5\sqrt{2}(-2) + 5\sqrt{2}(4) + 6\sqrt{2}(5)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} \right) \\ &= \left(\frac{20\sqrt{2} - 10\sqrt{2} + 30\sqrt{2}}{16\sqrt{2}}, \frac{-10\sqrt{2} + 20\sqrt{2} + 30\sqrt{2}}{16\sqrt{2}} \right) \\ &= \left(\frac{40\sqrt{2}}{16\sqrt{2}}, \frac{40\sqrt{2}}{16\sqrt{2}} \right) = \left(\frac{5}{2}, \frac{5}{2} \right) \end{aligned}$$



Question # 18

Find the points that divide the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ into four equal parts.

Solution

Given: $A(x_1, y_1)$, $B(x_2, y_2)$

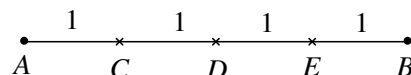
Let C , D and E are points dividing AB into four equal parts.

$$\therefore AC:CB = 1:3$$

$$\Rightarrow \text{Co-ordinates of } C = \left(\frac{1(x_2) + 3(x_1)}{1+3}, \frac{1(y_2) + 3(y_1)}{1+3} \right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4} \right)$$

Now $AD:DB = 2:2$

$= 1:1$ i.e. D is midpoint of AB .



$$\Rightarrow \text{Co-ordinates of } D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Now $AE:EB = 3:1$

$$\Rightarrow \text{Co-ordinates of } E = \left(\frac{3(x_2) + 1(x_1)}{3+1}, \frac{3(y_2) + 1(y_1)}{3+1} \right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4} \right)$$

Hence $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4} \right)$, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ and $\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4} \right)$ are the points dividing AB into four equal parts.

Error Analyst

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Book:**Exercise 4.1 (Page 185)**

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