Exercise 3.1 (Solutions) Page 123 MathCity.org **Calculus and Analytic Geometry, MATHEMATICS 12** Available online @ http://www.mathcity.org, Version: 3.0 Merging man and maths **Question #1** $=\frac{1}{2u^{\frac{1}{2}}}dx$ Find δy and dy in the following cases: (i) $y = x^2 - 1$ when x changes from 3 to 3.02 Put x = 4 & $dx = \delta x = 0.41$ $dy = \frac{1}{2(4)^{\frac{1}{2}}} (0.41)$ (ii) $y = x^2 + 2x$ when x changes from 2 to 1.8 $=\frac{0.41}{4}$ (iii) $y = \sqrt{x}$ when x changes from 4 to 4.41 Solution $\Rightarrow dy = 0.1025$ $y = x^2 - 1$ (i) (i) x = 3 & $\delta x = 3.02 - 3 = 0.02$ **Ouestion # 2** Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the $y + \delta y = (x + \delta x)^2 - 1$ $\Rightarrow \delta y = (x + \delta x)^2 - 1 - x^2 + 1$ following equations. (ii) $x^2 + 2y^2 = 16$ $=(x+\delta x)^2-x^2$ (i) xy + x = 4(iii) $x^4 + y^2 = xy^2$ (iv) $xy - \ln x = c$ Put $x = 3 \& \delta x = 0.02$ $\delta y = (3+0.02)^2 - (3)^2$ **Solution** xy + x = 4(i) $\Rightarrow \delta y = 0.1204$ Taking differential on both sides Taking differential of (i) d(xy) + dx = d(4) $dy = d\left(x^2 - 1\right)$ $\Rightarrow xdy + ydx + dx = 0$ $\Rightarrow dv = 2x dx$ $\Rightarrow xdy + (y+1)dx = 0$ Put $x = 3 \& dx = \delta x = 0.02$ $\Rightarrow xdy = -(y+1)dx$ dy = 2(3)(0.02) $\Rightarrow \frac{dy}{dx} = -\frac{y+1}{x}$ dy = 0.12 \Rightarrow & $\frac{dx}{dy} = -\frac{x}{y+1}$ Do yourself as above. (ii) Do yourself as above (ii) (iii) $v = \sqrt{x} = x^{\frac{1}{2}}$ (i) x = 4 & $\delta x = 4.41 - 4 = 0.41$ $x^4 + y^2 = xy^2$ (iii) $y + \delta y = (x + \delta x)^{\frac{1}{2}}$ Taking differential $d(x^4) + d(y^2) = d(xy^2)$ $\Rightarrow \delta v = (x + \delta x)^{\frac{1}{2}} - x^{\frac{1}{2}}$ $\Rightarrow 4x^3dx + 2ydy = x \cdot 2ydy + y^2dx$ Put x = 4 & $\delta x = 0.41$ $\Rightarrow 2ydy - 2xydy = y^2dx - 4x^3dx$ $\delta v = (4+0.41)^{\frac{1}{2}} - (4)^{\frac{1}{2}}$ $\Rightarrow 2y(1-x)dy = (y^2 - 4x^3)dx$ =2.1-2 $\Rightarrow | \delta y = 0.1$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - 4x^3}{2y(1-x)}$ Taking differential of (i) $dy = \frac{d}{dx} \left(x^{\frac{1}{2}} \right) dx$ $\& \frac{dx}{dy} = \frac{2y(1-x)}{y^2 - 4x^3}$ $=\frac{1}{2}x^{-\frac{1}{2}}dx$ (iv) $xy - \ln x = c$

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Taking differential

$$d(xy) - d(\ln x) = d(c)$$

$$\Rightarrow xdy + ydx - \frac{1}{x}dx = 0$$

$$\Rightarrow xdy = \frac{1}{x}dx - ydx$$

$$= \left(\frac{1}{x} - y\right)dx$$

$$\Rightarrow xdy = \left(\frac{1 - xy}{x}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - xy}{x^{2}}$$

$$\frac{dx}{dy} = \frac{x^{2}}{1 - xy}$$

Question # 3

(i)

Using differentials to approximate the values of

(i) ⁴ √17	$(ii)(31)^{\frac{1}{5}}$
(iii) $\cos 29^{\circ}$	$(iv) \sin 61^{\circ}$
Solution	_

Let $y = f(x) = \sqrt[4]{x}$ where x = 16 and $\delta x = dx = 1$ Taking differential of above

$$dy = d(\sqrt[4]{x})$$

$$= d(x)^{\frac{1}{4}}$$

$$= \frac{1}{4}x^{\frac{1}{4}-1}dx$$

$$= \frac{1}{4}x^{-\frac{3}{4}}dx$$

$$= \frac{1}{4x^{\frac{3}{4}}}dx$$
Put $x = 16$ and $dx = 1$
 $dy = \frac{1}{4(16)^{\frac{3}{4}}}(1)$

$$= \frac{1}{4(2^4)^{\frac{3}{4}}}$$

$$= \frac{1}{4(8)} = 0.03125$$
Now $f(x+dx) \approx y+dy$
 $= f(x)+dy$
 $\therefore y = f(x)$
 $\Rightarrow \sqrt[4]{16+1} \approx \sqrt[4]{16}+0.03125$
 $\Rightarrow \sqrt[4]{17} \approx (2^4)^{\frac{1}{4}}+0.03125$

= 2 + 0.03125= 2.03125(ii) Let $y = f(x) = x^{\frac{1}{5}}$ Where x = 32 & $\delta x = dx = -1$ Try yourself as above. (iii) Let $y = f(x) = \cos x$ Where $x = 30^{\circ}$ & $\delta x = -1^{\circ} = -\frac{\pi}{180}$ rad = -0.01745 rad Now $dy = d(\cos x)$ $= -\sin x \, dx$ Put $x = 30^{\circ}$ and $dx = \delta x = -0.01745$ $dy = -\sin 30^{\circ} (-0.01745)$ = -(0.5)(-0.01745) = 0.008725Now $f(x+\delta x) \approx y+dy$ = f(x) + dy $\Rightarrow \cos(30-1) = \cos 30^\circ + 0.008725$ $\Rightarrow \cos 29^{\circ} = 0.866 + 0.008725$ = 0.8747(iv) Let $y = f(x) = \sin x$ Where $x = 60^{\circ}$ & $\delta x = 1^{\circ} = \frac{\pi}{180}$ rad = 0.01745 rad Now $dy = d(\sin x)$ $= \cos x \, dx$ Put $x = 60^{\circ}$ and $dx = \delta x = 0.01745$ $dy = \cos 60^{\circ} (0.01745)$ = (0.5)(0.01745) = 0.008725Now $f(x+\delta x) \approx y+dy$ = f(x) + dy \Rightarrow sin(60+1) = sin 60° + 0.008725 $\Rightarrow \sin 61^{\circ} = 0.866 + 0.008725$ = 0.8747

Question # 4

Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02... Solution Let x be the length of side of cube where $x=5 \& \delta x = 5.02 - 5 = 0.02$ Assume V denotes the volume of the cube. Then $V = x \cdot x \cdot x$ = x^3 Taking differential $dV = 3x^2 dx$ Put x = 5 & $dx = \delta x = 0.02$ $dV = 3(5)^{2} (0.02)$ = 1.5 Hence increase in volume is 1.5 cubic unit.

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