

Question # 1

Find from first principles, the derivatives of the following expansions w.r.t. their respective independent variables:

$$\begin{array}{lll} \text{(i)} & (ax+b)^3 & \text{(ii)} & (2x+3)^5 & \text{(iii)} & (3t+2)^{-2} \\ \text{(iv)} & (ax+b)^{-5} & \text{(v)} & \frac{1}{(az-b)^7} \end{array}$$

Solution

$$\text{(i) Let } y = (ax+b)^3$$

$$\begin{aligned} \Rightarrow y + \delta y &= (a(x+\delta x)+b)^3 \\ \Rightarrow \delta y &= (ax+b+a\delta x)^3 - y \\ &= ((ax+b)+a\delta x)^3 - (ax+b)^3 \\ &= [(ax+b)^3 + 3(ax+b)^2(a\delta x) + 3(ax+b)(a\delta x)^2 + (a\delta x)^3] - (ax+b)^3 \\ &= 3a(ax+b)^2\delta x + 3a^2(ax+b)\delta x^2 + a^3\delta x^3 \\ &= \delta x(3a(ax+b)^2 + 3a^2(ax+b)\delta x + a^3\delta x^2) \end{aligned}$$

Dividing by δx

$$\frac{\delta y}{\delta x} = 3a(ax+b)^2 + 3a^2(ax+b)\delta x + a^3\delta x^2$$

Taking limit as $\delta x \rightarrow 0$

$$\begin{aligned} \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= \lim_{\delta x \rightarrow 0} [3a(ax+b)^2 + 3a^2(ax+b)\delta x + a^3\delta x^2] \\ \Rightarrow \frac{dy}{dx} &= 3a(ax+b)^2 + 3a^2(ax+b)(0) + a^3(0)^2 \\ \Rightarrow \frac{dy}{dx} &= 3a(ax+b)^2 + 0 + 0 \quad \Rightarrow \boxed{\frac{dy}{dx} = 3a(ax+b)^2} \end{aligned}$$

$$\text{(ii) Let } y = (2x+3)^5$$

$$\begin{aligned} \Rightarrow y + \delta y &= (2(x+\delta x)+3)^5 \\ \Rightarrow \delta y &= (2x+2\delta x+3)^5 - y \\ &= ((2x+3)+2\delta x)^5 - (2x+3)^5 \\ &= \left[\binom{5}{0}(2x+3)^5 + \binom{5}{1}(2x+3)^4(2\delta x) + \binom{5}{2}(2x+3)^3(2\delta x)^2 + \dots \right. \\ &\quad \left. \dots + \binom{5}{5}(2\delta x)^5 \right] - (2x+3)^5 \end{aligned}$$

$$\begin{aligned}
&= \left[(1)(2x+3)^5 + 2\binom{5}{1}(2x+3)^4 \delta x + 4\binom{5}{2}(2x+3)^3 \delta x^2 + \dots \right. \\
&\quad \left. \dots + 32\binom{5}{5} \delta x^5 \right] - (2x+3)^5 \\
&= 2\binom{5}{1}(2x+3)^4 \delta x + 4\binom{5}{2}(2x+3)^3 \delta x^2 + \dots + 32\binom{5}{5} \delta x^5
\end{aligned}$$

Dividing by δx

$$\frac{\delta y}{\delta x} = 2\binom{5}{1}(2x+3)^4 + 4\binom{5}{2}(2x+3)^3 \delta x + \dots + 32\binom{5}{5} \delta x^4$$

Taking limit as $\delta x \rightarrow 0$

$$\begin{aligned}
\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= \lim_{\delta x \rightarrow 0} \left[2\binom{5}{1}(2x+3)^4 + 4\binom{5}{2}(2x+3)^3 \delta x + \dots + 32\binom{5}{5} \delta x^4 \right] \\
&\Rightarrow \frac{dy}{dx} = \left[2\binom{5}{1}(2x+3)^4 + 0 + 0 + \dots + 0 \right] \\
&\Rightarrow \frac{dy}{dx} = 2(5)(2x+3)^4 \quad \text{or} \quad \boxed{\frac{dy}{dx} = 10(2x+3)^4}
\end{aligned}$$

(iii) Let $y = (3t+2)^{-2}$

$$\begin{aligned}
&\Rightarrow y + \delta y = (3(t+\delta t)+2)^{-2} \\
&\Rightarrow \delta y = (3t+3\delta t+2)^{-2} - y \\
&\Rightarrow \delta y = ((3t+2)+3\delta t)^{-2} - (3t+2)^{-2} \\
&\quad = (3t+2)^{-2} \left(1 + \frac{3\delta t}{3t+2} \right)^{-2} - (3t+2)^{-2} = (3t+2)^{-2} \left[\left(1 + \frac{3\delta t}{3t+2} \right)^{-2} - 1 \right] \\
&\quad = (3t+2)^{-2} \left[\left(1 + (-2) \frac{3\delta t}{3t+2} + \frac{-2(-2-1)}{2!} \left(\frac{3\delta t}{3t+2} \right)^2 + \dots \right) - 1 \right] \\
&\Rightarrow \delta y = (3t+2)^{-2} \left[1 - \frac{6\delta t}{3t+2} + \frac{-2(-3)}{2} \left(\frac{\delta t}{3t+2} \right)^2 + \dots - 1 \right] \\
&\quad = (3t+2)^{-2} \left[-\frac{6\delta t}{3t+2} + 3 \left(\frac{3\delta t}{3t+2} \right)^2 + \dots \right] \\
&\quad = (3t+2)^{-1} \cdot \frac{3\delta t}{3t+2} \left[-2 + 3 \left(\frac{3\delta t}{3t+2} \right) + \dots \right]
\end{aligned}$$

Dividing by δt

$$\frac{\delta y}{\delta t} = 3(3t+2)^{-2-1} \left[-2 + \left(\frac{3\delta t}{3t+2} \right) + \dots \right]$$

Taking limit when $\delta t \rightarrow 0$, we have

$$\lim_{\delta t \rightarrow 0} \frac{\delta y}{\delta t} = \lim_{\delta t \rightarrow 0} 3(3t+2)^{-3} \left[-2 + \left(\frac{3\delta t}{3t+2} \right) + \dots \right]$$

$$\Rightarrow \frac{dy}{dx} = 3(3t+2)^{-3} [-2 + 0 - 0 + \dots] \Rightarrow \boxed{\frac{dy}{dx} = -6(3t+2)^{-3}}$$

(iv) Let $y = (ax+b)^{-5}$ *Do yourself*

(v) Let $y = \frac{1}{(az-b)^7} = (az-b)^{-7}$

$\Rightarrow y + \delta y = (a(z+\delta z)-b)^{-7}$

$\Rightarrow \delta y = ((az-b)+a\delta z)^{-7} - (az-b)^{-7}$

$\Rightarrow \delta y = (az-b)^{-7} \left[\left(1 + \frac{a\delta z}{(az-b)} \right)^{-7} - 1 \right]$

*Now do yourself***Error Analyst****Irfan Mehmood 2015-16 Fazaia Degree College Risalpur**Please report us error at www.mathcity.org/error**Book:** *Exercise 2.2 , page 53**Calculus and Analytic Geometry Mathematic 12
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