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Definitions: Mathematics HSSC-I
Textbook of Algebra and Trigonometry for Class XI
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Chapter # 1 (Number system)

Rational number: A number which can be written in the form of $\frac{p}{q}$, where $p,q\in\mathbb{Z}$, $q\neq 0$ is called a rational number.

Irrational number: A real number which cannot be written in the form of $\frac{p}{q}$, where $p,q\in\mathbb{Z}$, $q\neq 0$ is called an irrational number.

Real number: The field of all rational and irrational numbers is called the real numbers, or simply the "reals," and denoted $\mathbb{R}$.

Terminating decimal: A decimal which has only a finite number of digits in its decimal part, is called terminating decimal.
e.g. $202.04,\ 0.25,\ 0.5$ example of terminating decimal.

Recurring decimal: A decimal in which one or more digits repeats indefinitely is called recurring decimal or periodic decimal.
e.g. $0.33333\ldots,\ 21.134134\ldots$
Note: Every terminating and recurring decimal is a rational number because it can be converted into common fraction.

Non-terminating, non-recurring decimal: Decimal which neither terminates nor it is recurring. it is not possible to convert it into a common fraction. Thus non-terminating, non-recurring decimals represent irrational number.
e.g. $\pi=3.1415\ldots$, we don't have exact decimal representation of this number.

Binary operations: A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A to another element of A.
e.g. two important binary operations are addition and multiplication in a set of real numbers.

Complex number: The number of the form of $z=x+iy$, where $x,y\in\mathbb{R},\ i=\sqrt{-1},$ is called complex number. Here x is called real part and y is called imaginary part of z.
e.g. $2,\ 3+\sqrt{2}i,\ \frac{1}{2}-i$.

Real plane or coordinate plane: The geometrical plane on which coordinate system has been specified is called the real plane or the coordinate plane.

Argand diagram: The figure representing one or more complex numbers on the complex plane is called argand diagram.
Modulus of complex number: The modulus of a complex number is the distance from the origin of the point representing the number. It is denoted by $|x+yi|$ or $|(x,y)|$.

Chapter # 2 (Sets, Functions and Groups)

Set: A set is generally described as a well-defined collection of distinct objects or a well-defined object collection of distinct object is called set.

There are three ways to describe a set,
Descriptive method: A method by which a set is described in words.
Example; N = the set of all nature numbers.

Tabular method: A set may be described by listing its elements within brackets.
e.g. $N = \{1,2,3,4,\ldots\}$

Set-builder method: In this form, we use a latter or symbol for an arbitrary element of a set and also stating the property that is common to all members.
Example; $\{x \mid x$ is any nature number$\}$

Order of a set: Number of elements in a set is called its order.
e.g. $A = \{2,4\}$ then order of A is 2.

Equal set: Two sets A and B are said to be equal sets if each element of set A is an element of set B both entries are same so A=B.
Example $A = \{2,4,6,8\}$, $B = \{2,8,4,6\}$

Equivalent set: Two sets are said to be equivalent if one to one correspondence can be established between them.
Example $A = \{2,4,6,8\}$, $B = \{a,b,c,d\}$

Singleton set: A set having one element is called singleton set.
Example $A = \{2\}$

Null set: A set having no element is called null set.
Example $A = \{\}$ *or* Φ

Finite set: A set having finite number of elements.
Example $A = \{2,4,6,8\ldots100\}$

Infinite set: A set having infinite number of elements.
Example $A = \{2,4,6,8\ldots\}$

Subset: If each element of set A is also an element set B. Then A is called sub set of B written as
$A \subseteq B$ and in case of B is called B super set of A.

(i) Empty set is a sub set of every set.
(ii) Every set is subset of itself.

Proper subset: if A is a subset of B and contains at least one element which is not in A then A is called proper subset of B denoted by $A \subseteq B$.

Improper subset: If a set of B and A = B then A is improper subset of B its follow that every set is improper subset of itself.

Power set: The set of all subset of set A is called power set of A, denoted by $P(A)$. Power set of empty set is not empty.
Example $A = \{2,4\}$, then $P(A) = \{\Phi, \{2\}, \{4\} \{2,4\}\}$ $P(A) = 2^n$.

Universal set: Universal set is the set that contains all the elements and objects involved in the problem under consideration or the set containing all objects or elements and of which all other sets are subsets.

Compliment of a set: The compliment of a set A, denoted by A' *or* A^c relative to the universal set U is the set of all elements of U, which do not belong to A.
e.g. $U = N$ then $E' = O$

Deduction: To draw general conclusion from well knows facts is called deduction.

Induction: To draw general conclusion from limited number of observation or experience is called induction.

Aristotelian logic: Deductive logic in which every statement is regarded as true or false is called Aristotelian logic.

Non Aristotelian: Deductive logic in which every statement is regarded scope of third or fourth is called non-Aristotelian logic.

Truth Table: A table to drives truth values of a given compound statement in terms of its component parts is called truth table.

Tautology: A statement which is true for all possible values of variable involved in it is called tautology.
e.g. $p \to q \leftrightarrow (\sim q \to \sim p)$ is a tautology.

Contradiction: A statement which is always false is called Contradiction or absurdity.
e.g. $p \to \sim p$

Contingency: A statement which can be true or false depending upon the truth values of variable.
e.g. $(p \to q) \wedge (p \vee q)$ is the contingency.

Function: Let A and B be two non-empty set sets. If
(1) F is a relation from A to B i.e. F is a subset of $A \times B$.
(2) Domain of F = A (3) No two ordered pairs of F have same 1st elements .Then F is called a function from A to B and is written as $F : A \to B$ denoted by $y = f(x)$.

Bijective function: (Range $f = \text{B}$ and 1-1) A function f which is both one to one and onto is called bijective function.

Injective function : (Rage $f \neq \text{B}$ and 1-1) A function f which is both one to one and into is called injective function.

Groupoid: A non-empty set which is closed under given Binary Operation '*' is called Groupoid.

Binary operation: Any mapping of $G \times G$ into G, where G is non empty set, is called binary operation.

Semi group: A non-empty set is called semi group if
(i) it is closed under given Binary operation.
(ii) The Binary operation is associative.

Monoid: A non-empty set is called Monoid.
(i) it is closed under given Binary operation.
(ii) The Binary operation is associative.
(iii) The set have identity element w.r.t. Binary operation

Group: A non-empty set G id called a group w.r.t Binary operation '*'.
(i) it is closed under given Binary operation.
(ii) The Binary operation is associative.
(iii) The set have identity element w.r.t. Binary operation.
(iv) Every element of G w.r.t Binary operation i.e. $a * a' = a' * a = e$.

Abelian group: A group G under Binary operation '*' is called Abelian group if Binary operation is commutative i.e. $a * b = b * a$. if $a * b \neq b * a$ then this is a Non Abelian group under Binary operation.

Linear function: The function $\{(x, y) \mid y = mx + c\}$ is called a linear function. Geometrical representation of linear function is a straight line.

Quadratic function: The function $\{(x, y) \mid y = ax^2 + bx + c\}$ is called a quadratic function, because it is defined by second degree equation in x, y.

Unary Operation: A mathematical producer that changes one number into another. Or it is an operation which is applied on a single number to give another single number .e.g $\sqrt{\ }$ *is a unary*.

Chapter # 3 (Matrices and Determinates)

Matrix: An arrangement of different elements in the rows and columns, within square brackets is called Matrix.

e.g. $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 8 & 4\\ 2 & 1 & 1\end{bmatrix}$.

Order: Order of Matrix tells us about no of rows and columns order of a matrix = no. of rows × no. of column.

Example $A=\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}_{m\times n}$ order of A = 2 x 3.

Row matrix: A matrix having single row is called Row Matrix.
e.g. $A=\begin{bmatrix}1 & 4 & 6\end{bmatrix}$

Colum matrix: A matrix having single column is called column Matrix.

e. g $A=\begin{bmatrix}3\\ 1\\ 5\end{bmatrix}$

Square matrix: A matrix in which no of rows and columns are equal is called square matrix.

e.g. $A=\begin{bmatrix}2 & 5\\ 4 & 8\end{bmatrix}$

Rectangular matrix: A matrix in which no of rows and columns are not equal is called square matrix. e.g. $A=\begin{bmatrix}2 & 1 & 3\\ 1 & 0 & 6\end{bmatrix}$

Diagonal matrix: A square matrix having each of its elements excepts principle diagonal equal to zero and at least one elements in its principle diagonal matrix.

e.g. $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$

Scalar matrix: A square matrix having same elements in principle diagonal except 1 is called scalar matrix.

e.g. $A=\begin{bmatrix}a & 0\\ 0 & a\end{bmatrix}$, $B=\begin{bmatrix}7 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 7\end{bmatrix}$.

Unit matrix or identity matrix: Let $A=[a_{ij}]$ be a square matrix of order n. If $a_{ij}=0$ for all $i\neq j$ and $a_{ij}=1$ for all $i=j$, then the matrix A is called a unit matrix or identity matrix of order n. It is denoted by I_n.

e.g. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Null matrix or zero matrix: A square or rectangular matrix whose each element is zero, is called a null or zero matrix. It is denoted by $O_{m\times n}$.

$O_{2\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equal matrix: Two matrix are said to be equal if they are of same order with the same correspondence elements.

e.g. $\mathrm{A} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $\mathrm{B} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

Upper triangular matrix: If all elements below the principle diagonal of square matrix are zero then it is called upper triangular matrix.

e.g. $\mathrm{A} = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$

Lower triangular matrix: If all elements above the principle diagonal of square matrix are zero then it is called lower triangular matrix.

e.g. $\mathrm{A} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix}$.

Singular matrix: A square matrix A is called singular if $|\mathrm{A}| = 0$

Non-Singular matrix: : A square matrix A is called non-singular if $|\mathrm{A}| \neq 0$

Adjoint of a 2×2 **matrix**: The adjoint of a matrix $\mathrm{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted by $adj\,\mathrm{A}$ and is defined as $adj\mathrm{A} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Symmetric matrix: Let 'A' be the square matrix if $\mathrm{A}^t = \mathrm{A}$ then 'A' is called symmetric matrix.

Skew symmetric matrix: Let 'A' be the square matrix if $\mathrm{A}^t = -\mathrm{A}$ then 'A' is called skew symmetric matrix.

Hermitian matrix :Let 'A' be the square matrix if $\overline{\mathrm{A}}^t = \mathrm{A}$ then 'A' is called Hermitian matrix .

Skew hermitian matrix :Let 'A' be the square matrix if $\bar{A}^t = -A$ then 'A' is called skew Hermitian matrix .

Rank: Non zero row in a matrix is called rank of the matrix.

Chapter # 4 (Quadratic Equations)

Quadratic Equation: An equation of second degree polynomial in a certain variable is called Quadratic Equation.
e.g. $x^2 - 4 = 0, \quad 5x^2 - 7x = 0, \quad x^2 + x + 5 = 0.$
Equation of type $ax^2 + bx + c = 0$ where $a = b = c \neq 0$ is called standard form of Quadratic Equation.

Solution of Quadratic Equation: (i) Factorization (ii) Quadratic Formula (iii) Completing Square.

Exponential Equation: Equations in which variable occur in exponents.
e.g. 2^x and 5^x

Reciprocal Equation: An equation which remains unchanged when x is replaced by $\frac{1}{x}$, is called a reciprocal equation.

Radical Equation: Equation involving radical expression of the variable is called radical equation.
Example $\sqrt{x+2} + \sqrt{x-3} = 7$.

Remainder Theorem: If a polynomial $f(x)$ of degree $n \geq 1$ is divided by $(x-a)$ till no x term exits in the remainder then $f(a)$ is remainder .

Polynomial function: A polynomial in x is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + ... + a_{1n} x + a_0, \quad a_n \neq 0$ where n is a non-negative integer and the coefficients $a_n, a_{n-1}, ..., a_0$ are real numbers. It can be considered as a polynomial function of x.

Factor Theorem: The polynomial $(x-a)$ is a factor of the polynomial $f(x)$ if and only if $f(x) = 0$.

Chapter # 5 (Partial Fractions)

Partial fraction: Partial fraction is an expression of a single rational function as a sum of two or more single rational fraction.

Identity: It is an equation which holds good for all values of the variable.

Rational Fraction: The Quotient of two polynomials $\frac{P(x)}{Q(x)}$ where $Q(x) \neq 0$, with no common factor is called Rational Fraction.

Proper Rational Fraction: A rational Fraction $\frac{P(x)}{Q(x)}$ is called. if the degree of polynomial $P(x)$ is less degree of polynomial $Q(x)$.

e.g. $\frac{3}{x+2}, \frac{2x+5}{x^2+9}$.

Improper Rational Fraction: A Improper rational Fraction $\frac{P(x)}{Q(x)}$ is called. if the degree of polynomial $P(x)$ is greater than or equal to the degree of polynomial.

e.g. $Q(x)$. $\frac{3x-9}{x+2}, \frac{2x^3-5x+1}{x^2-9}$.

Conditional Equation: It is an equation which is true for particular values of variable.

e.g. $2x=3 \quad if \quad x=\frac{2}{3}$.

Chapter # 6 (Sequences and Series)

Sequence: Sequence is a function whose domain is subset of the set of natural numbers.

Real sequence: If all members of a sequence are real numbers, then it is called a real sequence.

Finite Sequence: If the domain of a sequence is a finite set, then the sequence is called finite sequence.

Infinite Sequence: If the domain of a sequence is an infinite set, then the sequence is called infinite sequence.

Series: The sum of an indicated number of terms in a sequence is called series.
e.g. $1+4+9+16+25$

Arithmetic Sequence: A sequence $\{a_n\}$ is an Arithmetic Sequence or Arithmetic progression if $a_n - a_{n-1}$ is the same number for all $n \in$ N and $n>1$. $a_n = a_1 + (n-1)d$.

Arithmetic Mean: A number A is said to be the A.M. between the two numbers a and b. If a, A, b are in A.P. If d is the common difference of this A.P., then $A-a=d$ and $b-A=d$.

Thus $A-a=b-A \Rightarrow A=\frac{a+b}{2}$

Geometric Progression: A sequence $\{a_n\}$ is geometric sequence or geometric progression if a_n/a_{n-1} is the same non zero number of all $n \in$ N & $n>1$.

Geometric Mean: A number is said to be geometric means between two numbers a and b. If a, G, b are in $G.P.$ Therefore $\frac{G}{a}=\frac{b}{G} \Rightarrow G^2=ab \Rightarrow G=\pm\sqrt{ab}$

Harmonic Progression: A sequence of numbers is called harmonic progression or harmonic sequence if the reciprocal of its terms are in arithmetic progression. The sequence $1,\frac{1}{3},\frac{1}{5},\frac{1}{7}$ are in harmonic sequence since there reciprocals $1,3,5,7$ are in $A.P.$

Harmonic means: A number H is said to be the harmonic means ($H.M$) between two numbers a and b, if a, H, b are in H.P.

Chapter # 7 (Permutation, Combination, Probability)

Permutation: An ordering arrangement of n objects is called permutation.

Circular Permutation: The permutation of things which can be represents by the points on a circle.

Probability: Probability is the numerical evaluation of a chance that a particular event would occur.

Sample Space: The set S consisting of all possible outcome of a given experiment is called sample space.

Combination: When a selection of objects is the made without paying regard to the order of selection.

Event: An event is a subset of sample space. $P(\mathrm{A})=n(\mathrm{A})/n(S)$

Equally Likely: Two events A and B are said to be Equally Likely if one event is as likely to occur as other.

Mutually exclusive: A and B are said to be mutually if and only if they cannot both occur at the same time.

Chapter # 8 (Mathematical Induction and Binomial Theorem)

Binomial Theorem: An algebraic expression consisting of two terms is called binomial expression. e.g. $a+x,\ x-2y, ax+b$

Chapter # 9,10,11,12,13,14 (Trigonometry)

Trigonometry: The word trigonometry has been derived from three Greek words Trei (three) Goni (angles) and Metron (measurement). its mean measurement of triangle.

Angle: Two rays with common starting point from an angle is called an angle.

Degree: If the circumference of circle is divided into 360 equal parts in length, the angle subtended by one part at the centre of the circle is called a degree.

Allied angles: The angles associated with basic angles of measure θ to the right angle or its multiple are called allied angles. The angles of measure $90^\circ \pm \theta$, $180^\circ \pm \theta$, $270^\circ \pm \theta$, $360^\circ \pm \theta$ are known as allied angles.

Period: period is the smallest positive number which, when added to the original circular measure of the angle, gives the same value of the function.

Circular system (Radians): A radian is the measure of the central angle of an arc of a circle whose length is equal to the radius of the circle.

Sexagesimal system: The system of measurement in which the angle is measured in degree, and its sub-units, minutes and seconds is called the Sexagesimal system.
Example $16^\circ 40' 34''$

Period of Trigonometric Function: The smallest positive number which when added to the original circular measurement of the angle gives same value of function is called period.
Example 2π is period of sine function as $sin(\alpha + 2\pi) = sin\alpha$.

Trigonometry equation: The equation, containing at least one trigonometry function are called Trigonometry equation.
Example. $sinx = 2/7$, $cosx - tanx = 0$

Oblique Triangles: A triangle, which is not right, is called an oblique triangle.

Circum-Circle: The circle passes through the three vertices of a triangle is called circum-circle.

In-Circle: A circle drawn inside a triangle touching its three sides is called its inscribed circle or in-circle.

Escribed Circles: A circle, which touches one side of the triangle externally and the other two produced sides, is called an escribed circle or ex-circle or e-circle.

Trigonometric function: The equation, containing at least one trigonometric function, are called trigonometric function.

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