Question # 1

How many arrangements of the letters of the following words, taken all together, can be made:

(i) PAKPATTAN
(ii) PAKISTAN
(iii) MATHEMATICS
(iv) ASSASSINATION

Solution

(i) PAKPATTAN
Total number of letters = 9
P is repeated 2 times
A is repeated 3 times
T is repeated 2 times
K and N come only once.

Required number of permutations = $\begin{pmatrix} 9 \\ 2,3,2,1,1 \end{pmatrix}$

$$= \frac{9!}{2!\times 3!\times 2!\times 1!\times 1!}$$

$$= \frac{362880}{(2)(6)(2)} = 15120.$$

(ii) PAKISTAN
Total number of letters = 8
A is repeated 2 times
P, K, I, S, T and N come only once.
Required number of permutations

$$= \begin{pmatrix} 8 \\ 2,1,1,1,1,1,1 \end{pmatrix}$$

$$= \frac{8!}{2!\times 1!\times 1!\times 1!\times 1!\times 1!\times 1!}$$

$$= \frac{40320}{2} = 20160.$$

(iii) MATHEMATICS
Total number of letters = 11
M is repeated 2 times
A is repeated 2 times
T is repeated 2 times
H, E, I, C and S come only once.
Required number of permutations

$$= \begin{pmatrix} 11 \\ 2,2,2,1,1,1,1,1 \end{pmatrix}$$

$$= \frac{11!}{2!\times 2!\times 2!\times 1!\times 1!\times 1!\times 1!\times 1!}$$

$$= \frac{39916800}{8} = 4989600.$$

(iv) ASSASSINATION
Total number of letters $= 13$
A is repeated 3 times
S is repeated 4 times
I is repeated 2 times
N is repeated 2 times
T and O come only once.
Required number of permutations

$$= \begin{pmatrix} 13 \\ 3,4,2,2,1,1 \end{pmatrix}$$

$$= \frac{13!}{3!\times 4!\times 2!\times 2!\times 1!\times 1!}$$

$$= \frac{6227020800}{(6)(24)(2)(2)} = 10810800$$

Question # 2

How many permutations of the letters of the word PANAMA can be made, if P is to be the first letter in each arrangement?

Solution

If P is the first letter then words are of the form $P*****$, where five $*$ can be replace with A,N,A,M,A.
So number of letters $= 5$
A is repeated 3 times
M, N appears only once

So required permutations $= \begin{pmatrix} 5 \\ 3,1,1 \end{pmatrix} = \frac{5!}{3!\times 1!\times 1!}$

$$= \frac{120}{6} = 20.$$

Question # 3

How many arrangements of the letters of the word ATTACKED can be made, if each arrangement begins with C and end with K?

Solution

If C be the first letter and K is the last letter then words are of the form $C******K$. where each $*$ can be replaced with A, T, T, A, E, D.
So number of letters $= 6$
A is repeated 2 times
T is repeated 2 times
E and D come only once.

So required permutations $=\binom{6}{2,\ 2,\ 1,\ 1}$

$$=\frac{6!}{2!\times 2!\times 1!\times 1!}=\frac{720}{4}=180.$$

Question # 4
How many numbers greater than $1000,000$ can be formed from the digits $0,2,2,2,3,4,4$?
Solution
The number greater than 1000000 are of the following forms.
If numbers are of the form $2******$,
where each $*$ can be filled with $0,2,2,3,4,4$
Then number of digits $=6$
2 is repeated 2 times
4 is repeated 2 times
0 and 3 come only once.
So number formed $=\binom{6}{2,\ 2,\ 1,\ 1}$

$$=\frac{6!}{2!\times 2!\times 1!\times 1!}=\frac{720}{4}=180.$$

Now if numbers are of the form $3******$,
where each $*$ can be filled with $0,2,2,2,4,4$
Then number of digits $=6$
2 is repeated 3 times
4 is repeated 2 times
0 comes only once.
So number formed $=\binom{6}{3,\ 2,\ 1}$

$$=\frac{6!}{3!\times 2!\times 1!}=\frac{720}{12}=60.$$

Now if numbers are of the form $4******$,
where each $*$ can be filled with $0,2,2,2,3,4$
Then number of digits $=6$
2 is repeated 3 times
0, 3 and 4 come only once.
So number formed $=\binom{6}{3,\ 1,\ 1}=\frac{6!}{3!\times 1!\times 1!}$

$$=\frac{720}{6}=120.$$

So required numbers greater than 1000000
$= 180+60+120$
$= 360.$

➢ **Alternative**
(Submitted by **Waqas Ahmad** - FAZMIC Sargodha – 2004-06)
No. of digits = 7
No. of 2's = 3
No. of 4's = 2
0 and 3 come only once.

Permutations of 7 digits number $=\binom{7}{3,\ 2,\ 1,\ 1}$

$$=\frac{7!}{3!\times 2!\times 1!\times 1!}=\frac{5040}{12}=420.$$

Number less than 1,000,000 are of the form $0******$, where each $*$ can be replaced with 2, 2, 3, 4, 4.
No. of digits = 6
No. of 2's = 3
No. of 4's = 2
3 comes only once
So permutations $=\binom{6}{3,\ 2,\ 1}=\frac{6!}{3!\times 2!\times 1!}$

$$=\frac{720}{12}=60$$

Hence number greater than $1000000=420-60$
$= 360.$

Question # 5
How many 6-digits numbers can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them will lie between 400,000 and 430,000?
Solution
Total number of digits $=6$
Number of 2's $=2$
Number of 3's $=2$
Number of 4's $=2$
So number formed by these 6 digits

$$=\binom{6}{2,\ 2,\ 2}=\frac{6!}{(2!)(2!)(2!)}$$

$$=\frac{720}{(2)(2)(2)}=90.$$

The numbers lie between 400,000 and 430,000 are only of the form 42****, where each * can be filled by 2, 3, 3, 4.
Here number of digits $=4$.
Number of 2's $=1$
Number of 3's $=2$
Number of 4's $=1$
So number formed $=\binom{4}{1,\ 2,\ 1}=\frac{4!}{(1!)(2!)(1!)}$

$$=\frac{24}{2}=12.$$

Question # 6
11 members of a club form 4 committees of 3, 4, 2, 2 members so that no member is a member is a member of more than one committee. Find the number of committees.
Solution
Total members $=11$

Members in first committee $=3$
Members in second committee $=4$
Members in third committee $=2$
Members in fourth committee $=2$
So required number of committees

$$=\binom{11}{3,\,4,\,2,\,2} = \frac{11!}{3!\cdot 4!\cdot 2!\cdot 2!}$$

$$=\frac{39916800}{(6)(24)(2)(2)} = 69300.$$

Question # 7
The D.C.Os of 11 districts meet to discuss the law and order situation in their districts. In how many ways can they be seated at a round table, when two particular D.C.Os insist on sitting together?
Solution
Number of D.C.O's $=9$
Let D_1 and D_2 be the two D.C.O's insisting to sit together so consider them one.
If D_1D_2 sit together then permutations

$$= {}^9P_9 = 362880$$

If D_2D_1 sit together then permutations

$$= {}^9P_9 = 362880$$

So total permutations $=362880+362880$
$=725760$

Question # 8
The Governor of the Punjab calls a meeting of 12 officers. In how many ways can they be seated at a round table?
Solution
Fixing one officer on a particular seat,
we have permutations of remaining 11 officers

$$= {}^{11}P_{11} = 39916800.$$

Question # 9
Fatima invites 14 people to a dinner. There are 9 males and 5 females who are seated at two different tables so that guests of one sex seat at one round table and the guests of other sex at the second table. Find the number of ways in which all guests are seated.
Solution
9 males can be seated on a round table

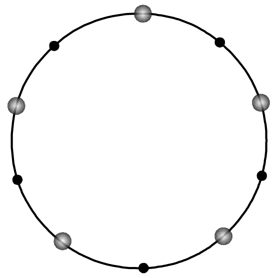
$$= {}^8P_8 = 40320$$

And 5 females can be seated on a round table

$$= {}^4P_4 = 24$$

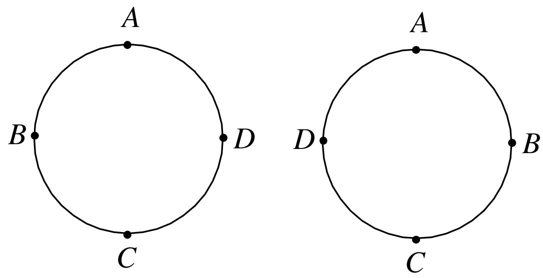
So permutations of both $= 40320\times 24$
$= 967680.$

Question # 10
Find the numbers of ways in which 5 men and 5 women can be seated at a round table in such a way that no two persons of same sex sit together.
Solution



If we fix one man round a table
then their permutations $= {}^4P_4 = 24$
Now if women sit between the two men
then their permutations $= {}^5P_5 = 120$
So total permutations $=24\times 120 = 2880$

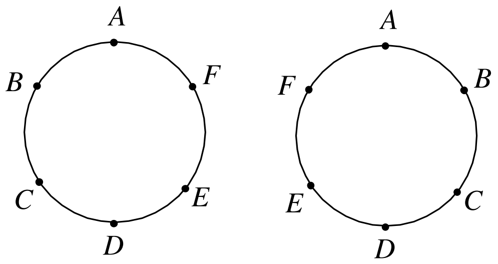
Question # 11
In how many ways can 4 keys be arranged on a circular key ring?
Solution
Number of keys $=4$
Fixing one key we have permutation $= {}^3P_3 = 6$



Since above figures of arrangement are reflections of each other

Therefore permutations $=\frac{1}{2}\times 6 = 3$

Question # 12
How many necklaces can be made from 6 beads of different colours?
Solution
Number of beads $=6$
Fixing one bead, we have permutation $= {}^5P_5$
$= 120$



Since above figures of arrangement are reflections of each other

Therefore permutations $=\frac{1}{2}\times 120=60$

Book: Exercise 7.3 (Page 238)
Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore – PAKSITAN.

Available online at
http://www.MathCity.org *in PDF Format*
(Picture format to view online).
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