Question # 1

A man deposits in a bank $Rs.10$ in the first month; $Rs.15$ in the second month; $Rs.20$ in the third month and so on. Find how much he will have deposited in the bank by the $9th$ month.

Solution

The sequence of the deposits is

10, 15, 20, ……. to 9 terms

Here $a_1 = 10$, $d = 15 - 10 = 5$, $n = 9$

Since $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$\Rightarrow S_9 = \frac{9}{2}\left[2(10) + (9-1)(5)\right] = \frac{9}{2}\left[20 + 40\right] = \frac{9}{2}(60) = 270$$

Hence the total amount he deposits is Rs. 270.

Question # 2

378 trees are planted in rows in the shape of an isosceles triangle, the numbers in successive rows decreasing by one from the base to the top. How many trees are there in the row which forms the base of the triangle?

Solution

The sequence of the trees from top to base row is

1, 2, 3, ……

Let n be the total number of tress in base row then

$a_1 = 1$, $d = 2 - 1 = 1$, $n = n$, $S_n = 378$

Now $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$\Rightarrow 378 = \frac{n}{2}\left[2(1) + (n-1)(1)\right] \quad \Rightarrow 756 = n\left[2 + n - 1\right]$$

$$\Rightarrow 756 = n(n+1) \quad \Rightarrow 756 = n^2 + n$$

$$\Rightarrow \quad n^2 + n - 756 = 0$$

So $n = \frac{-1 \pm \sqrt{(-1)^2 - 4(1)(-756)}}{2(1)}$

$$\Rightarrow n = \frac{-1 \pm \sqrt{1 + 3024}}{2} = \frac{-1 \pm \sqrt{3025}}{2} = \frac{-1 \pm 55}{2}$$

So $n = \frac{-1 + 55}{2} = \frac{54}{2} = 27$ or $n = \frac{-1 - 55}{2} = \frac{-56}{2} = -28$

Since n can never be negative therefore $n = 27$

Now $a_n = a_1 + (n-1)d$

$$\Rightarrow a_{27} = (1) + (27-1)(1) = 1 + 26 = 27$$

Thus the numbers of trees in the base row are 27.

Question # 3

A man borrows $Rs.1100$ and agree to repay with a total interest of $Rs.230$ in 14 installments, each installment being less than the preceding by $Rs.10$. What should be his first installment?

Solution

Let the first installment be x then the sequence of installment will be

$x, x-10, x-20, \ldots\ldots\ldots\ldots$

Here $a_1 = x$, $d = -10$, $n = 14$ and $S_n = 1100 + 230 = 1330$

Now $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow \; 1330 = \frac{14}{2}[2x + (14-1)(-10)] \;\Rightarrow\; 1330 = 7[2x - 130] \;\Rightarrow\; 1330 = 14x - 910$

$\Rightarrow \; 1330 + 910 = 14x \;\Rightarrow\; 2240 = 14x \;\Rightarrow\; x = \frac{2240}{14} = 160$

Hence the first installment is 160.

Question # 4

A clock strikes once when its hour hand is at one, twice when it is at two and so on. How many times does the clock strike in twelve hour?

Solution

The sequence of the strikes is

1, 2, 3, ……., 12

Here $a_1 = 1$, $d = 2 - 1 = 2$, $n = 12$, $a_n = 12$

Now $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow \; S_n = \frac{12}{2}[2(1) + (12-1)(1)] = \frac{12}{2}[2 + 11] = \frac{12}{2}[13] = 78$

Hence clock strikes 78 hours in twelve strikes.

Question # 5

A student saves Rs. 12 at the end to the first week and goes on increasing his saving Rs. 4 weekly. After how many weeks will be able to save Rs. 2100?

Solution

The sequence of the savings is

12, 16, 20, ……

Total Savings = 2100

So here $a_1 = 12$, $d = 16 - 12 = 4$, $S_n = 2100$, $n = ?$

Since $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow \; 2100 = \frac{n}{2}[2(12) + (n-1)(4)] \;\Rightarrow\; 4200 = n[24 + 4n - 4]$

$\Rightarrow \; 4200 = n[4n + 20] \;\Rightarrow\; 4200 = 4n^2 + 20n$

$\Rightarrow \; 4n^2 + 20n - 4200 = 0 \;\Rightarrow\; 4(n^2 + 5n - 1050) = 0$

$\Rightarrow \; n^2 + 5n - 1050 = 0$

$\Rightarrow \; n = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-1050)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 4200}}{2}$

$= \frac{-5 \pm \sqrt{4225}}{2} = \frac{-5 \pm 65}{2}$

So $n = \frac{-5 - 65}{2} = \frac{-70}{2} = -35$ or $n = \frac{-5 + 65}{2} = \frac{60}{2} = 30$

As n can never be negative therefore $n = 30$

Thus student will save Rs. 2100 in 30 weeks.

Question # 5

An object falling from rest, falls 9 meters during the first second, 27 meters during the next second, 45 meters during the third second and so on.

(i) How far will it fall during the fifth second?

(ii) How far will it fall up to the fifth second?

Solution

The sequence of the falls is

$$9, 27, 45, \ldots\ldots\ldots\ldots\ldots$$

(i) $a_1 = 9,\ d = 27 - 9 = 18,\ a_5 = ?$

Since $a_5 = a_1 + 4d = 9 + 4(18) = 9 + 72 = 81$

Hence in fifth second the object will fall 81 meters.

(ii) Here $a_1 = 9,\ d = 27 - 9 = 18,\ \text{n} = 5,\ S_5 = ?$

Since $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$\Rightarrow\ S_5 = \frac{5}{2}\left[2(9) + (5-1)(18)\right] = \frac{5}{2}\left[18 + 72\right] = \frac{5}{2}(90) = 225$$

Thus up to 5^{th} second the object will fall 225 meters.

Question # 7

An investor earned Rs. 6000 for year 1980 and Rs. 12000 for year 1990 on the same investment. If his earning have increased by the same amount each year, how much income he has received from the investment over the past eleven years?

Solution

Here $a_1 = 6000$, $a_{11} = 12000$, $n = 11$

Now $S_n = \frac{n}{2}\left[a_1 + a_n\right]$

$$\Rightarrow\ S_{11} = \frac{11}{2}\left[6000 + 12000\right] = \frac{11}{2}(18000) = 99000$$

Hence he will receive Rs. 99000 in past eleven years.

Question # 8

The sum of interior angles of polygons having sides $3, 4, 5, \ldots\ldots\ldots$ *etc.* from an A.P. Find the sum of the interior angles for a 16 sided polygon.

Solution

Since the sum of angels of 3 sided polygon (triangle) = $a_1 = \pi$

Sum of angels of 4 sided polygon (quadrilateral) = $a_2 = 2\pi$

Sum of the angels of 5 sided polygon (pentagon) = $a_3 = 3\pi$

So

The sum of interior angels of 16 side polygon = $a_{14} = ?$

Here $a_1 = \pi$, $d = a_2 - a_1 = 2\pi - \pi = \pi$, $n = 14$

Since $a_n = a_1 + (n-1)\,d$

$$\Rightarrow\ a_{14} = \pi + (14-1)(\pi) = \pi + 13\pi = 14\pi$$

Hence sum of interior angels of 16 side polygon is 14π.

Question # 9

The prize money $Rs60,000$ will be distributed among the eight teams according to their positions determined in the match-series. The award increases by the same amount for each higher position. If the last place tem is given $Rs.4000$, how much will be awarded to the first place team?

Solution

Let a_1 denotes the prize money for the last position

Then $a_1 = 4000$, $S_n = 60000$, $n = 8$, $a_n = ?$

Since $S_n = \frac{n}{2}\left(a_1 + a_n\right)$

$$\Rightarrow\ 60000 = \frac{8}{2}\left(4000 + a_n\right)\quad \Rightarrow\ 60000 = 4\left(4000 + a_n\right)$$

$$\Rightarrow\ 60000 = 16000 + 4a_n \quad \Rightarrow\ 60000 - 16000 = 4a_n \quad \Rightarrow\ 44000 = 4a_n$$

$$\Rightarrow\ a_n = \frac{44000}{4} = 11000$$

Hence the team at 1st place will get 11000 Rs.

Question # 10

An equilateral triangular base is filled by placing eight balls in the first row, 7 balls in the second row and so on with one ball in the last row, After this base layer, second layer is formed by placing 7 balls in the first row, 6 balls in the second row and so on with one ball in its last row. Continuing this process, a pyramid of balls is formed with one ball on top. How many balls are there in the pyramid?

Solution

Balls in the first layer = 8 + 7 + 6 +…… + 2 + 1

$$= \frac{8}{2}\left[2(8) + (8-1)(-1)\right] = 4(16-7) = 36$$

Balls in the second layer = 7 + 6 + 5 + ……+ 2 + 1

$$= \frac{7}{2}\left[2(7) + (7-1)(-1)\right] = \frac{7}{2}\left[14-6\right] = \frac{7}{2}\left[8\right] = 28$$

Balls in the third layer = 6 + 5 + 4 +3 + 2 + 1 = 21

Balls in the fourth layer = 5 + 4 + 3 + 2 +1 = 15

Balls in the fifth layer = 4 + 3 + 2 + 1 = 10

Balls in the sixth layer = 3 + 2 + 1 = 6

Balls in the seventh layer = 2 + 1 = 3

Balls in the eighth layer = 1

Hence the number of balls in pyramid

= 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 120

Book: ***Exercise 6.5***
Text Book of Algebra and Trigonometry Class XI

Punjab Textbook Board, Lahore.

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