

(Q) No. 1. The series of integral multiple of 3 between 6 and 96 is

$$6 + 9 + 12 + 15 + \dots + 96$$

$$\text{Here } a_1 = 6$$

$$d = 9 - 6 = 12 - 9 = 3$$

$$a_n = 96$$

$$\text{Since } a_n = a_1 + (n-1)d$$

$$\Rightarrow 96 = 6 + (n-1)(3)$$

$$\Rightarrow 96 = 6 + 3n - 3$$

$$\Rightarrow 96 - 6 + 3 = 3n$$

$$\Rightarrow 93 = 3n \Rightarrow n = 31$$

Now

$$S_n = \frac{n}{2} (a_1 + a_n) \quad \frac{n}{2}(2a_1 + (n-1)d)$$

$$\Rightarrow S_{31} = \frac{31}{2} (6 + 96)$$

$$= \frac{31}{2} (102)$$

$$= (31)(51) = 1581$$

Answer

(Q) No. 2 i)

$$-3 + (-1) + 1 + 3 + 5 + \dots + a_{16}$$

$$\text{Here } a_1 = -3$$

$$d = -1 - (-3) = -1 + 3 = 2$$

$$n = 16$$

$$\text{Since } S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$\Rightarrow S_{16} = \frac{16}{2} (2(-3) + (16-1)(2))$$

$$= 8(-6 + 30)$$

$$= 8(24) = 192$$

Answer

ii) $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \dots + a_{13}$

$$\text{Here } a_1 = \frac{3}{\sqrt{2}}$$

$$d = 2\sqrt{2} - \frac{3}{\sqrt{2}} = \frac{4 - 3}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$n = 13$$

$$\text{Since } S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$\Rightarrow S_{13} = \frac{13}{2} \left(2 \left(\frac{3}{\sqrt{2}} \right) + (13-1) \frac{1}{\sqrt{2}} \right)$$

$$= \frac{13}{2} \left(\frac{6}{\sqrt{2}} + \frac{12}{\sqrt{2}} \right)$$

$$= \frac{13}{2} \left(\frac{18}{\sqrt{2}} \right) = \frac{117}{\sqrt{2}} \text{ Answer}$$

iii) $1.11 + 1.41 + 1.71 + \dots + a_{10}$

$$\text{Here } a_1 = 1.11$$

$$d = 1.41 - 1.11 = 0.31$$

$$n = 10$$

Do yourself

iv) $-8 - 3\frac{1}{2} + 1 + \dots + a_{11}$

$$-8 - \frac{7}{2} + 1 + \dots + a_{11}$$

$$\text{Here } a_1 = -8$$

$$d = -\frac{7}{2} - (-8) = -\frac{7}{2} + 8$$

$$= \frac{9}{2}, n = 11$$

Now Do yourself as (i)

v) $(x-2) + (x+2) + (x+3a) + \dots$ to n terms

$$\text{Here } a_1 = x - 2$$

$$d = (x+2) - (x-2)$$

$$= x + 2 - x + 2 = 2a$$

$$n = n$$

$$\text{Since } S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

$$\Rightarrow S_n = \frac{n}{2} [2(x-2) + (n-1)(2a)]$$

$$= \frac{n}{2} [2x - 2a + 2an - 2a]$$

$$= \frac{n}{2} [2x + 2an - 4a]$$

$$= \frac{n}{2} \cdot 2(x + an - 2a)$$

$$= n(x + (n-2)a)$$

Answer

vi) $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \dots$ to n term

Here $a_1 = \frac{1}{1-\sqrt{x}}$

$$\begin{aligned} d &= \frac{1}{1-x} - \frac{1}{1-\sqrt{x}} \\ &= \frac{1}{(1-\sqrt{x})(1+\sqrt{x})} - \frac{1}{1-\sqrt{x}} \\ &= \frac{1-(1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})} \\ &= \frac{1-1-\sqrt{x}}{1-x} = \frac{-\sqrt{x}}{1-x} \end{aligned}$$

$\therefore n = n$.

Since $S_n = \frac{n}{2} [2a_1 + (n-1)d]$

$$\begin{aligned} \Rightarrow S_n &= \frac{n}{2} \left[2 \cdot \frac{1}{1-\sqrt{x}} + (n-1) \cdot \left(\frac{-\sqrt{x}}{1-x} \right) \right] \\ &= \frac{n}{2} \left[\frac{2}{1-\sqrt{x}} - \frac{\sqrt{x}(n-1)}{1-x} \right] \\ &= \frac{n}{2} \left[\frac{2}{1-\sqrt{x}} - \frac{\sqrt{x}(n-1)}{(1-\sqrt{x})(1+\sqrt{x})} \right] \\ &= \frac{n}{2} \left[\frac{2(1+\sqrt{x}) - \sqrt{x}(n-1)}{(1-\sqrt{x})(1+\sqrt{x})} \right] \\ &= \frac{n}{2} \left[\frac{2+2\sqrt{x} - \sqrt{x}(n-1)}{1-x} \right] \\ &= \frac{n}{2} \left[\frac{2+(2-n+1)\sqrt{x}}{1-x} \right] \\ &= \frac{n}{2} \left[\frac{2+(3-n)\sqrt{x}}{1-x} \right] \end{aligned}$$

Answer

vii)

Do yourself as above

(Q No. 3)

$-7 + (-5) + (-3) + \dots$ amount to 65

Here $a_1 = -7$

$d = (-5) - (-7) = -5 + 7 = 2$

$S_n = 65$, $n = ?$

Since $S_n = \frac{n}{2} [2a_1 + (n-1)d]$

$$\Rightarrow 65 = \frac{n}{2} [2(-7) + (n-1)(2)]$$

$$\Rightarrow 130 = n(-14 + 2n - 2)$$

$$\Rightarrow 130 = n(2n - 16)$$

$$\Rightarrow 130 = 2n^2 - 16n$$

$$\Rightarrow 2n^2 - 16n - 130 = 0$$

$$\Rightarrow n^2 - 8n - 65 = 0 \text{ dividing by 2.}$$

$$\Rightarrow n^2 - 13n + 5n - 65 = 0$$

$$\Rightarrow n(n-13) + 5(n-13) = 0$$

$$\Rightarrow (n-13)(n+5) = 0$$

$$\Rightarrow n-13 = 0 \text{ or } n+5 = 0$$

$$n = 13 \text{ or } n = -5$$

As n can not be -ive.

So $n = 13$ Answer

ii) Do yourself

Hint: you will get equation

$$3n^2 - 17n - 288 = 0$$

You may use quadratic

formula to find value of n.

(Q No. 4 i)

$$3+5-7+9+11-13+15+17-19+\dots$$

----- to 3n terms

$$(3+5-7)+(9+11-13)+(15+17-19)+\dots$$

----- to n terms

$1+7+13+\dots$ to n terms

Here $a_1 = 1$, $d = 7-1 = 6$, $n = n$

Since $S_n = \frac{n}{2} [2a_1 + (n-1)d]$

$$\Rightarrow S_n = \frac{n}{2} [2(1) + (n-1)(6)]$$

$$= \frac{n}{2} (2 + 6n - 6) = \frac{n}{2} (6n - 4)$$

$$= \frac{n}{2} \cdot 2(3n - 2)$$

$$= n(3n - 2) \text{ Answer}$$

ii) Do yourself as above

Q No. 5. Since $a_1 = 3r + 1$

$$\text{put } r=1, a_1 = 3(1) + 1 = 4$$

$$\text{put } r=2, a_2 = 3(2) + 1 = 7$$

$$\text{So } d = a_2 - a_1 = 7 - 4 = 3$$

$$\text{also } n = 20, S_n = ?$$

$$\text{Since } S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$\Rightarrow S_{20} = \frac{20}{2} (2(4) + (20-1)(3))$$

$$= 10(8 + 57) = 10(65)$$

$$= 650 \quad \text{Answer}$$

Q No. 6. $S_n = n(2n-1)$

$$\text{put } n=1, 2, 3, 4$$

$$S_1 = 1(2(1)-1) = 1(2-1) = 1$$

$$S_2 = 2(2(2)-1) = 2(4-1) = 6$$

$$S_3 = 3(2(3)-1) = 3(6-1) = 15$$

$$S_4 = 4(2(4)-1) = 4(8-1) = 28$$

Now

$$a_1 = S_1 = 1$$

$$a_2 = S_2 - S_1 = 6 - 1 = 5$$

$$a_3 = S_3 - S_2 = 15 - 6 = 9$$

$$a_4 = S_4 - S_3 = 28 - 15 = 13$$

hence required series is

$$1 + 5 + 9 + 13 + \dots$$

Q No. 7.

Consider a_1, a'_1 are the first terms and d, d' are the common differences of two series in A.P.

Now we have given

$$S_n : S'_n = 3n+2 : n+1$$

$$\Rightarrow \frac{S_n}{S'_n} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{\frac{n}{2}(2a_1 + (n-1)d)}{\frac{n}{2}(2a'_1 + (n-1)d')} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{2a_1 + (n-1)d}{2a'_1 + (n-1)d'} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{2\left[a_1 + \left(\frac{n-1}{2}\right)d\right]}{2\left[a'_1 + \left(\frac{n-1}{2}\right)d'\right]} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d}{a'_1 + \left(\frac{n-1}{2}\right)d'} = \frac{3n+2}{n+1} \quad \text{(i)}$$

For 8th term

$$\text{Consider } \frac{n-1}{2} = 7$$

$$\Rightarrow n-1 = 14$$

$$\Rightarrow n = 14 + 1 = 15$$

Putting in (i),

$$\frac{a_1 + 7d}{a'_1 + 7d'} = \frac{3(15)+2}{15+1}$$

$$\Rightarrow \frac{a_8}{a'_8} = \frac{47}{16} \quad \because a_8 = a_1 + 7d$$

$$\Rightarrow a_8 : a'_8 = 47 : 16 \quad \text{or } \frac{47}{16}$$

Answer

Q No. 8.

Since

$$S_2 = S_{2n} = \frac{2n}{2} (2a_1 + (2n-1)d) \quad \text{(i)}$$

$$S_3 = S_{3n} = \frac{3n}{2} (2a_1 + (3n-1)d) \quad \text{(ii)}$$

$$S_5 = S_{5n} = \frac{5n}{2} (2a_1 + (5n-1)d) \quad \text{(iii)}$$

Now

$$R.H.S = 5(S_3 - S_2)$$

~~$$= 5 \left[\frac{3n}{2} (2a_1 + (3n-1)d) - \frac{2n}{2} (2a_1 + (2n-1)d) \right]$$~~

$$= 5 \left[\frac{3n}{2} (2a_1 + (3n-1)d) - 2(2a_1 + (2n-1)d) \right]$$

$$= 5 \left[6a_1 + 3(3n-1)d - 4a_1 - 2(2n-1)d \right]$$

$$= 5 \left[2a_1 + [3(3n-1) - 2(2n-1)]d \right]$$

$$= 5 \left[2a_1 + (9n-3 - 4n+2)d \right]$$

$$\Rightarrow \text{R.H.S} = \frac{5n}{2}(2a_1 + (5n-1)d)$$

$$= S_5 = \text{L.H.S from (iii)}$$

Hence $S_5 = 5(S_3 - S_2)$ proved

(b) QNo 9 The series of integers which are neither divisible by 5 nor by 2 are

$$1+3+7+9+11+13+17+19+21+$$

$$23+27+29+\dots+991+993+997+999 \\ (\text{400 terms})$$

$$(1+3+7+9)+(11+13+17+19)+(21+23+27+29)+\dots+(991+993+997+999) \\ (\text{100 terms})$$

$$20+60+100+\dots+3980 \quad (\text{100 terms})$$

$$\text{here } a_1 = 20, d = 60 - 20 = 40$$

$$n = 100$$

Since

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$\Rightarrow S_{100} = \frac{100}{2}(2(20) + (100-1)(40))$$

$$= 50(40 + 3960)$$

$$= 50(4000) = 200000$$

Answer

(QNo 10) $50S_9 = 63S_8 \rightarrow a_1 = 2$

$$\Rightarrow 50\left(\frac{9}{2}(2a_1 + (9-1)d)\right) = 63\left[\frac{8}{2}(2a_1 + (8-1)d)\right]$$

$$\Rightarrow 50\left(\frac{9}{2}(2a_1 + 8d)\right) = 63(4(2a_1 + 7d))$$

$$\Rightarrow 225(2a_1 + 8d) = 252(2a_1 + 7d)$$

$$\therefore a_1 = 2$$

$$\Rightarrow 225(2(2) + 8d) = 252(2(2) + 7d)$$

$$\Rightarrow 225(4 + 8d) = 252(4 + 7d)$$

$$\Rightarrow 900 + 1800d = 1008 + 1764d$$

$$\Rightarrow 1800d - 1764d = 1008 - 900$$

$$\Rightarrow 36d = 108 \Rightarrow d = \frac{108}{36} = 3$$

Now $S_9 = \frac{9}{2}(2a_1 + (9-1)d)$

$$\Rightarrow S_9 = \frac{9}{2}(2(2) + 8(3))$$

$$= \frac{9}{2}(4 + 24) = \frac{9}{2}(28)$$

= 126 Answer

(QNo 11)

Since $S_9 = 171$

$$\Rightarrow \frac{9}{2}(2a_1 + (9-1)d) = 171$$

$$\Rightarrow \frac{9}{2}(2a_1 + 8d) = 171$$

$$\Rightarrow \frac{9}{2} \cdot 2(a_1 + 4d) = 171$$

$$\Rightarrow 9a_1 + 36d = 171 \quad \text{(i)}$$

Now $a_8 = 31$

$$\Rightarrow a_1 + 7d = 31 \quad \text{(ii)}$$

Multiplying eq. (ii) by 9 and subtracting from (i)

$$9a_1 + 36d = 171$$

$$\underline{9a_1 + 63d = 279}$$

$$- 27d = -108$$

$$\Rightarrow d = \frac{-108}{-27} \Rightarrow d = 4$$

put $d = 4$ in eq. (ii)

$$a_1 + 7(4) = 31$$

$$\Rightarrow a_1 + 28 = 31$$

$$\Rightarrow a_1 = 31 - 28 \Rightarrow a_1 = 3$$

Now

$$a_2 = a_1 + d = 3 + 4 = 7$$

$$a_3 = a_1 + 2d = 3 + 2(4) = 11$$

$$a_4 = a_1 + 3d = 3 + 3(4) = 15$$

Hence the required series is

$$3+7+11+15+\dots$$

Q No 12. Since

$$S_9 + S_7 = 203 \quad \text{--- (i)}$$

$$\text{also } S_9 - S_7 = 49 \quad \text{--- (ii)}$$

adding (i) and (ii)

$$S_9 + S_7 = 203$$

$$S_9 - S_7 = 49$$

$$\frac{2S_9}{2} = 252$$

$$\Rightarrow S_9 = 126$$

If a_1 be the first term and d be the common difference then

$$\frac{9}{2} [2a_1 + (9-1)d] = 126$$

$$\Rightarrow 9(2a_1 + 8d) = 252$$

$$\Rightarrow 18a_1 + 72d = 252$$

$$\Rightarrow 18(a_1 + 4d) = 252$$

$$\Rightarrow a_1 + 4d = 14 \quad \text{--- (iii)}$$

Now - ing (i) and (ii)

$$S_9 + S_7 = 203$$

$$S_9 - S_7 = 49$$

$$\frac{2S_7}{2} = 154$$

$$\Rightarrow S_7 = 77$$

$$\Rightarrow \frac{7}{2} [2a_1 + (7-1)d] = 77$$

$$\Rightarrow 7(2a_1 + 6d) = 154$$

$$\Rightarrow 14a_1 + 42d = 154$$

$$\Rightarrow 14(a_1 + 3d) = 154$$

$$\Rightarrow a_1 + 3d = 11 \quad \text{--- (iv)}$$

Subtracting (iii) & (iv)

$$a_1 + 4d = 14$$

$$a_1 + 3d = 11$$

$$d = 3$$

putting in (iii)

$$a_1 + 4(3) = 14$$

$$\Rightarrow a_1 + 12 = 14 \Rightarrow a_1 = 14 - 12$$

$$\Rightarrow a_1 = 2$$

Now

$$a_2 = a_1 + d = 2 + 3 = 5$$

$$a_3 = a_1 + 2d = 2 + 2(3) = 8$$

$$a_4 = a_1 + 3d = 2 + 3(3) = 11$$

Thus the required series is

$$2 + 5 + 8 + 11 + \dots$$

Q No 13

Since

$$\frac{S_9}{S_7} = \frac{18}{11}$$

$$\Rightarrow 11S_9 = 18S_7$$

$$\Rightarrow 11 \cdot \frac{9}{2} [2a_1 + (9-1)d] = 18 \cdot \frac{7}{2} [2a_1 + (7-1)d]$$

$$\Rightarrow \frac{99}{2} [2a_1 + 8d] = 63 [2a_1 + 6d]$$

$$\Rightarrow 99a_1 + 396d = 126a_1 + 378d$$

$$\Rightarrow 99a_1 - 126a_1 = 378d - 396d$$

$$\Rightarrow -27a_1 = -18d$$

$$\Rightarrow a_1 = \frac{-18}{-27} d$$

$$\Rightarrow a_1 = \frac{2}{3} d \quad \text{--- (i)}$$

$$\text{also } a_7 = 20$$

$$\Rightarrow a_1 + 6d = 20$$

putting value of a_1 in above

$$\frac{2}{3} d + 6d = 20$$

$$\Rightarrow \frac{20}{3} d = 20$$

$$\Rightarrow d = \frac{3}{20} \cdot 20 \Rightarrow d = 3$$

putting in (i)

$$a_1 = \frac{2}{3} (3) \Rightarrow a_1 = 2$$

Now

$$a_2 = a_1 + d = 2 + 3 = 5$$

$$a_3 = a_1 + 2d = 2 + 2(3) = 8$$

$$a_4 = a_1 + 3d = 2 + 3(3) = 11$$

Thus the required series is

$$2 + 5 + 8 + 11 + \dots$$

Q No 4 Let the numbers in A.P
are $a-d$, a , $a+d$.
By given condition

$$a-d + a + a+d = 24$$

$$\Rightarrow 3a = 24 \Rightarrow a = 8$$

also by given condition

$$(a-d) \cdot a \cdot (a+d) = 440$$

$$\Rightarrow a(a^2 - d^2) = 440$$

putting $a=8$ in above

$$8((8)^2 - d^2) = 440$$

$$\Rightarrow 8(64 - d^2) = 440$$

$$\Rightarrow 512 - 8d^2 = 440$$

$$\Rightarrow 512 - 440 = 8d^2$$

$$\Rightarrow 8d^2 = 72 \Rightarrow d^2 = \frac{72}{8}$$

$$\Rightarrow d^2 = 9 \Rightarrow d = \pm 3$$

When $a=8$, $d=3$

$$a-d = 8-3 = 5$$

$$a = 8$$

$$a+d = 8+3 = 11$$

When $a=8$, $d=-3$

$$a-d = 8-(-3) = 8+3 = 11$$

$$a = 8$$

$$a+d = 8+(-3) = 8-3 = 5$$

Hence $5, 8, 11$ OR $11, 8, 5$ are the required numbers.

Q No 15 Consider four numbers $a-3d$, $a-d$, $a+d$, $a+3d$ are in A.P then

$$a-3d + a-d + a+d + a+3d = 32$$

$$\Rightarrow 4a = 32 \Rightarrow a = 8$$

also

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276$$

$$\Rightarrow a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2$$

$$+ a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 276$$

$$\Rightarrow 4a^2 + 20d^2 = 276$$

put $a=8$ in above

$$4(8)^2 + 20d^2 = 276$$

$$\Rightarrow 256 + 20d^2 = 276$$

$$\Rightarrow 20d^2 = 276 - 256$$

$$\Rightarrow 20d^2 = 20$$

$$\Rightarrow d^2 = 1 \Rightarrow d = \pm 1$$

When $a=8$, $d=1$

$$a-3d = 8-3(1) = 8-3 = 5$$

$$a-d = 8-(1) = 8-1 = 7$$

$$a+d = 8+(1) = 8+1 = 9$$

$$a+3d = 8+3(1) = 8+3 = 11$$

When $a=8$, $d=-1$

$$a-3d = 8-3(-1) = 8+3 = 11$$

$$a-d = 8-(-1) = 8+1 = 9$$

$$a+d = 8+(-1) = 8-1 = 7$$

$$a+3d = 8+3(-1) = 8-3 = 5$$

hence $5, 7, 9, 11$ OR $11, 9, 7, 5$

are required numbers.

Q No. 16

Do yourself

Consider $a-2d$, $a-d$, a , $a+d$ and $a+2d$ as five numbers in A.P

Q No 17

Since $a_6 + a_8 = 40$

$$\Rightarrow a_1 + 5d + a_1 + 7d = 40$$

$$\Rightarrow 2a_1 + 12d = 40$$

$$\Rightarrow 2(a_1 + 6d) = 40$$

$$\Rightarrow a_1 + 6d = 20 \quad (i)$$

Also

$$a_4 \cdot a_7 = 220$$

$$\Rightarrow (a_1 + 3d)(a_1 + 6d) = 220$$

$$\Rightarrow (a_1 + 3d)(20) = 220 \quad \text{from (i)}$$

$$\Rightarrow a_1 + 3d = \frac{220}{20}$$

$$\Rightarrow a_1 + 3d = 11 \quad (ii)$$

Subtracting (i) & (ii)

$$a_1 + 6d = 20$$

$$a_1 + 3d = 11$$

$$3d = 9 \Rightarrow d = 3$$

putting in (ii)

$$a_1 + 3(3) = 11$$

$$a_1 + 9 = 11 \Rightarrow a_1 = 11 - 9$$

$$\Rightarrow a_1 = 2$$

Now

$$a_2 = a_1 + d = 2 + 3 = 5$$

$$a_3 = a_1 + 2d = 2 + 2(3) = 8$$

$$a_4 = a_1 + 3d = 2 + 3(3) = 11$$

Thus the required A.P. is

$$2, 5, 8, 11, \dots$$

Q No. 18

Since a^2, b^2, c^2 are in A.P.

therefore

$$b^2 - a^2 = c^2 - b^2$$

$$\Rightarrow (b-a)(b+a) = (c-b)(c+b) \quad (\text{i})$$

Now to show $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. Consider

$$d = \frac{1}{c+a} - \frac{1}{b+c}$$

$$= \frac{b+c - c-a}{(c+a)(b+c)}$$

$$\Rightarrow d = \frac{b-a}{(c+a)(b+c)} \quad (\text{ii}).$$

Also

$$d = \frac{1}{a+b} - \frac{1}{c+a}$$

$$= \frac{c+a - a-b}{(a+b)(c+a)}$$

$$= \frac{c-b}{(a+b)(c+a)}$$

$$\text{From eq. (i)} \quad \frac{(b-a)(b+a)}{(c+b)} = c-b$$

putting in above

$$d = \frac{(b-a)(b+a)}{(c+b)(c+a)}$$

$$= \frac{(b-a)(b+a)}{(c+b)(c+a)(c+b)}$$

$$= \frac{b-a}{(c+b)(c+a)}$$

$$\Rightarrow d = \frac{b-a}{(c+a)(b+c)} \quad (\text{iii})$$

from (ii) and (iii)

$$d = d$$

$$\text{hence } \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$$

are in A.P.

Q No. 8 (Ex 6.3).

Let $A_1, A_2, A_3, \dots, A_n$ be

the n A.M.s between a & b .

then $a, A_1, A_2, A_3, \dots, A_n, b$ are in A.P.

Here $a_1 = a$

$$a_{n+2} = b$$

$$\Rightarrow a_1 + (n+2-1)d = b$$

$$\Rightarrow a_1 + (n+1)d = b$$

$$\Rightarrow (n+1)d = b - a$$

$$\Rightarrow d = \frac{b-a}{n+1}$$

Now

$$A_1 = a_2 = a_1 + d = a + \frac{b-a}{n+1}$$

$$A_2 = a_3 = a_1 + 2d = a + 2 \left(\frac{b-a}{n+1} \right)$$

$$A_3 = a_4 = a_1 + 3d = a + 3 \left(\frac{b-a}{n+1} \right)$$

$$A_n = a_{n+1} = a_1 + nd = a + n \left(\frac{b-a}{n+1} \right)$$

P.T.O

Now Sum of n A.Ms

$$= A_1 + A_2 + A_3 + \dots + A_n$$

$$= a + \frac{b-a}{n+1} + a + 2\left(\frac{b-a}{n+1}\right) + a + 3\left(\frac{b-a}{n+1}\right)$$

$$+ \dots + a + n\left(\frac{b-a}{n+1}\right)$$

$$= (a+a+a+\dots+a) + \frac{b-a}{n+1} + 2\left(\frac{b-a}{n+1}\right)$$

$$+ 3\left(\frac{b-a}{n+1}\right) + \dots + n\left(\frac{b-a}{n+1}\right)$$

$$= na + \frac{b-a}{n+1}(1+2+3+\dots+n)$$

$$a_1 = 1, d = 2-1 = 1, n = n$$

$$= na + \frac{b-a}{n+1}\left[\frac{n^2}{2} + (n-1)(1)\right]$$

$$= na + \frac{b-a}{n+1}\left(\frac{n}{2}(2+n-1)\right)$$

$$= na + \frac{b-a}{(n+1)}\left(\frac{n}{2}(n+1)\right)$$

$$= na + (b-a)\left(\frac{n}{2}\right)$$

$$= n\left(a + \frac{b-a}{2}\right)$$

$$= n\left(\frac{2a+b-a}{2}\right)$$

$$= n\left(\frac{a+b}{2}\right)$$

$$= n(A.Ms \text{ between } a \text{ & } b)$$

Hence sum of n A.Ms between
a & b is n times their A.Ms -
proved

End



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