

$$\{(0,1), (-2,-3)\}$$

Case.2

When both equations are quadratic

(a) When both equations are quadratic but involving only x^2 and y^2 terms.
such as Exp ① Questions. 1, 2, 3,

Solution Method

To solve such equations we put $x^2 = u$, $y^2 = v$ and solve it further by balancing coefficients of u or v .

Example.1

$$x^2 + y^2 = 25$$

$$2x^2 + 3y^2 = 66$$

Let $x^2 = u$ and $y^2 = v$ Then given equation becomes

$$u + v = 25 \rightarrow ①$$

$$2u + 3v = 66 \rightarrow ②$$

Multiplying eq. ① by 2 we get

$$2u + 2v = 50 \rightarrow ③$$

Subtracting eq. ③ from eq. ②

$$2u + 3v = 66$$

$$2u + 2v = 50$$

$$\underline{v = 16}$$

Putting value of v in ① we get

$$u + 16 = 25 \Rightarrow u = 25 - 16 = 9$$

If $u = 9$ then

$$x^2 = 9$$

$$x = \pm 3$$

$$\{(\pm 3, \pm 4)\}$$

$$\text{or } \{(3, \pm 4), (-3, \pm 4)\}$$

$$\text{or } \{(3, 4), (3, -4), (-3, 4), (-3, -4)\}$$

When both equations are quadratic one of them is homogeneous quadratic

Solution Method

In This type we factorize the homogeneous quadratic equation.

Homogeneous Equation:-

An equation whose degree of every term is 2 is called homogeneous quadratic equation. For Example

$$ax^2 + 2xxy + by^2 = 0$$

$$x^2 - 3xxy + 2y^2 = 0, 2x^2 - 7xxy + 3y^2 = 0$$

$$2x^2 + 5xxy - 3y^2 = 0, x^2 - 5xxy + 6y^2 = 0$$

are all homogeneous quadratic equations.

Questions of such type are

Exp ②, Q. 4, 5, 6

Example.2

$$x^2 - 3xxy + 2y^2 = 0 \rightarrow ①$$

$$2x^2 - 3xxy + y^2 = 24 \rightarrow ②$$

Since equation ① is homogeneous, factorizing it, $x^2 - xxy - 2xxy + 2y^2 = 0$

$$x(x-y) - 2y(x-y) = 0$$

$$(x-y)(x-2y) = 0$$

$$x-y = 0, x-2y = 0$$

$$x = y, x = 2y$$

If $x=y$ then from ②

$$2(y)^2 - 3y(y) + y^2 = 24$$

$$2y^2 - 3y^2 + y^2 = 24 = 0$$

$$3y^2 - 3y^2 - 24 = 0$$

Dividing by 3

$$y^2 - y - 8 = 0$$

Using

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-8)}}{2(1)}$$

$$y = \frac{1 \pm \sqrt{1+32}}{2}$$

$$y = \frac{1 \pm \sqrt{33}}{2}$$

If $x=2y$ then from ②

$$2(2y)^2 - 3(2y)y + y^2 = 24$$

$$8y^2 - 6y^2 + y^2 = 24 = 0$$

$$9y^2 - 6y^2 - 24 = 0$$

Dividing by 3

$$3y^2 - 2y - 8 = 0$$

$$3y^2 - 6y + 4y - 8 = 0$$

$$3y(y-2) + 4(y-2) = 0$$

$$(y-2)(3y+4) = 0$$

$$y-2 = 0, 3y+4 = 0$$

$$y=2, y = -\frac{4}{3}$$

If $y=2$ then

$$x = 2(2)$$

$$\begin{array}{l}
 y = \frac{1+\sqrt{33}}{2}, y = \frac{1-\sqrt{33}}{2} \\
 \text{If } y = \frac{1+\sqrt{33}}{2} \text{ then } x = 4 \\
 x = \frac{1+\sqrt{33}}{2} \\
 \text{If } y = \frac{1-\sqrt{33}}{2} \text{ then } x = 2(-\frac{4}{3}) \\
 x = -\frac{8}{3} \\
 \left\{ \left(\frac{1+\sqrt{33}}{2}, \frac{1+\sqrt{33}}{2} \right), \left(\frac{1-\sqrt{33}}{2}, \frac{1-\sqrt{33}}{2} \right), \right. \\
 \left. (4, 2), (-\frac{8}{3}, -\frac{4}{3}) \right\}
 \end{array}$$

(C) When both equations are quadratic but one of them involving xy in product form.

Solution Method

In this type we balance constant terms of both equations, then subtracting each other we get Homogeneous Quadratic Equation and solve as homogeneous case.

Questions of such type are

Exp ③, Q. 7, 8, 9, 10

Example.3

$$x^2 - y^2 = 5 \rightarrow ①$$

$$4x^2 - 3xy = 18 \rightarrow ②$$

To balance constant terms multiplying eq. ① by 18 and eq. ② by 5 we get

$$18x^2 - 18y^2 = 90 \rightarrow ③$$

$$20x^2 - 15xy = 90 \rightarrow ④$$

Subtracting eq. ③ from eq. ④

$$20x^2 - 15xy = 90$$

$$+ 18x^2 - 18y^2 = 90$$

$$\hline 2x^2 + 18y^2 - 15xy = 0$$

$$2x^2 - 15xy + 18y^2 = 0$$

Factorizing it, we get

$$2x^2 - 12xy + 3xy - 18y^2 = 0$$

$$2x(x-6y) - 3y(x-6y) = 0$$

$$(x-6y)(2x-3y) = 0$$

$$x-6y=0 \quad , \quad 2x-3y=0$$

$$x=6y \quad , \quad x=\frac{3}{2}y$$

If $x=6y$ then from ① If $x=\frac{3}{2}y$ then from ①

$$(6y)^2 - y^2 = 5$$

$$36y^2 - y^2 = 5$$

$$35y^2 = 5$$

$$y^2 = \frac{5}{35} \rightarrow y^2 = \frac{1}{7}$$

$$y = \pm \frac{1}{\sqrt{7}}$$

$$y = \frac{1}{\sqrt{7}}, y = -\frac{1}{\sqrt{7}}$$

$$\text{If } y = \frac{1}{\sqrt{7}} \text{ then}$$

$$x = 6(\frac{1}{\sqrt{7}}) = \frac{6}{\sqrt{7}}$$

$$\text{If } y = -\frac{1}{\sqrt{7}} \text{ then}$$

$$x = 6(-\frac{1}{\sqrt{7}}) = -\frac{6}{\sqrt{7}}$$

$$\left\{ (\frac{6}{\sqrt{7}}, \frac{1}{\sqrt{7}}), (-\frac{6}{\sqrt{7}}, -\frac{1}{\sqrt{7}}), (3, 2), (-3, -2) \right\}$$

EXERCISE.4.9

Q.1

$$2x^2 = 6 + 3y^2$$

$$\text{or } 2x^2 - 3y^2 = 6$$

$$\text{and } 3x^2 - 5y^2 = 7$$

Put $x^2 = u$, $y^2 = v$ thus we get

$$2u - 3v = 6 \rightarrow ①$$

$$3u - 5v = 7 \rightarrow ②$$

To balance coefficient of u multiplying eq. ① by 3 and eq. ② by 2 we get

$$6u - 9v = 18 \rightarrow ③$$

$$6u - 10v = 14 \rightarrow ④$$

Subtracting eq. ③ from eq. ④

$$6u - 10v = 14$$

$$\underline{-6u + 9v = 18}$$

$$\hline -v = -4 \rightarrow v = 4$$

Putting value of v in ① we get

$$24 - 3(4) = 6$$

$$2u - 12 = 6 \rightarrow 2u = 6 + 12$$

$$2u = 18 \rightarrow u = 9$$

If $u = 9$ then

$$x^2 = 9 \rightarrow x = \pm 3$$

$$\{(\pm 3, \pm 2)\}$$

$$\text{or } \{(3, \pm 2), (-3, \pm 2)\}$$

$$\text{or } \{(3, 2), (3, -2), (-3, 2), (-3, -2)\}$$

If $v = 4$ then

$$y^2 = 4 \rightarrow y = \pm 2$$

and $x^2 - 13 = -2y^2$ or $x^2 - 2y^2 = 13$

Put $x^2 = u$ and $y^2 = v$ we get

$$2u - 5v = 8 \rightarrow ①$$

$$u + 2v = 13 \rightarrow ②$$

To balance coefficient of u multiply

eq. ② by 2 we get

$$2u + 4v = 26 \rightarrow ③$$

Subtracting eq. ① from eq. ③

$$2u + 4v = 26$$

$$2u - 5v = -8$$

$$\underline{9v = 18} \rightarrow v = 2$$

put $v = 2$ in eq. ② we get

$$u + 2(2) = 13 \rightarrow u = 13 - 4 \rightarrow u = 9$$

If $u = 9$ then

$$x^2 = 9$$

$$x = \pm 3$$

$$y^2 = 2$$

$$y = \pm \sqrt{2}$$

$$\{(\pm 3, \pm \sqrt{2})\}$$

Q.2

$$8x^2 = y^2$$

$$\text{or } 8x^2 - y^2 = 0$$

$$\text{and } x^2 + 2y^2 = 19$$

Put $x^2 = u$ and $y^2 = v$ we get

$$8u - v = 0 \rightarrow ①$$

$$u + 2v = 19 \rightarrow ②$$

To balance coefficient of v multiply eq. ① by 2 we get

$$16u - 2v = 0 \rightarrow ③$$

Adding ② and ③

$$u + 2v = 19$$

$$16u - 2v = 0$$

$$\underline{17u = 19}$$

$\rightarrow u = \frac{19}{17}$, Putting value of u in ① we get

$$8\left(\frac{19}{17}\right) - v = 0 \rightarrow v = \frac{152}{17}$$

If $u = \frac{19}{17}$ then

$$x^2 = \frac{19}{17}$$

$$x = \pm \sqrt{\frac{19}{17}}$$

$$\{(\pm \sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}})\}$$

If $v = \frac{152}{17}$ then

$$y^2 = \frac{152}{17}$$

$$y = \pm \sqrt{\frac{152}{17}}$$

$$y = \pm \sqrt{\frac{4 \times 38}{17}}$$

$$y = \pm 2\sqrt{\frac{38}{17}}$$

Q.3 $2x^2 - 8 = 5y^2$ or $2x^2 - 5y^2 = 8$

Q.4 $x^2 - 5xy + 6y^2 = 0 \rightarrow ①$

$$x^2 + y^2 = 45 \rightarrow ②$$

Factorizing homogeneous equation we get

$$x^2 - 2xy - 3xy + 6y^2 = 0$$

$$x(x-2y) - 3y(x-2y) = 0$$

$$(x-2y)(x-3y) = 0$$

$$x-2y=0, x-3y=0$$

$$x=2y, x=3y$$

If $x = 2y$ then from ② If $x = 3y$ then from ②

$$(2y)^2 + y^2 = 45$$

$$4y^2 + y^2 = 45$$

$$5y^2 = 45$$

$$y^2 = 9 \rightarrow y = \pm 3$$

$$y = 3, y = -3$$

$$y = 3, y =$$

$$\begin{aligned} x &= 2(-3) \\ x &= -6 \\ \{(6,3),(-6,-3),\left(\frac{9}{\sqrt{2}},\frac{3}{\sqrt{2}}\right),\left(\frac{-9}{\sqrt{2}},\frac{-3}{\sqrt{2}}\right)\} \end{aligned}$$

Q.5 $12x^2 - 25xy + 12y^2 = 0 \rightarrow ①$
 $4x^2 + 7y^2 = 148 \rightarrow ②$

Factorizing homogeneous equation we get

$$\begin{aligned} 12x^2 - 16xy - 9xy + 12y^2 &= 0 \\ 4x(3x - 4y) - 3y(3x - 4y) &= 0 \\ (3x - 4y)(4x - 3y) &= 0 \\ 3x - 4y &= 0, \quad 4x - 3y = 0 \\ 3x &= 4y, \quad 4x = 3y \\ x = \frac{4}{3}y &\quad x = \frac{3}{4}y \end{aligned}$$

If $x = \frac{4}{3}y$ then from ②

$$\begin{aligned} 4\left(\frac{4}{3}y\right)^2 + 7y^2 &= 148 \\ \frac{64}{9}y^2 + 7y^2 &= 148 \\ 64y^2 + 63y^2 &= 1332 \\ 127y^2 &= 1332 \\ y^2 &= \frac{1332}{127} \\ y &= \pm \sqrt{\frac{1332}{127}} \end{aligned}$$

$$y = \pm \sqrt{\frac{4 \times 333}{127}}$$

$$y = \pm 2\sqrt{\frac{333}{127}}$$

$$y = 2\sqrt{\frac{333}{127}}, y = -2\sqrt{\frac{333}{127}}$$

If $y = 2\sqrt{\frac{333}{127}}$ then

$$\begin{aligned} x &= \frac{4}{3}\left(2\sqrt{\frac{333}{127}}\right) \\ &= \frac{8}{3}\sqrt{\frac{333}{127}} \end{aligned}$$

If $y = -2\sqrt{\frac{333}{127}}$ then

$$x = \frac{4}{3}\left(-2\sqrt{\frac{333}{127}}\right)$$

$$x = -\frac{8}{3}\sqrt{\frac{333}{127}}$$

If $y = \frac{-3}{\sqrt{2}}$ then

$$x = 3\left(\frac{-3}{\sqrt{2}}\right) = \frac{-9}{\sqrt{2}}$$

$$\left\{ \left(\frac{8}{3}\sqrt{\frac{333}{127}}, 2\sqrt{\frac{333}{127}}\right), \left(\frac{-8}{3}\sqrt{\frac{333}{127}}, -2\sqrt{\frac{333}{127}}\right), (3, 4), (-3, -4) \right\}$$

Q.6

$$\begin{aligned} 12x^2 - 11xy + 2y^2 &= 0 \rightarrow ① \\ 2x^2 + 7xy &= 60 \rightarrow ② \end{aligned}$$

Factorizing homogeneous equation we get

$$12x^2 - 8xy - 3xy + 2y^2 = 0$$

$$4x(3x - 2y) - y(3x - 2y) = 0$$

$$(3x - 2y)(4x - y) = 0$$

$$3x - 2y = 0, \quad 4x - y = 0$$

$$3x = 2y, 4x = y \Rightarrow x = \frac{2}{3}y, x = \frac{1}{4}y$$

If $x = \frac{2}{3}y$ then from ②

If $x = \frac{1}{4}y$ then from ②

$$2\left(\frac{2}{3}y\right)^2 + 7\left(\frac{2}{3}y\right)y = 60$$

$$\frac{8}{9}y^2 + \frac{14}{3}y^2 = 60$$

$$8y^2 + 42y^2 = 540$$

$$50y^2 = 540$$

$$y^2 = \frac{540}{50} \Rightarrow y = \frac{54}{5}$$

$$y = \pm \sqrt{\frac{54}{5}}$$

$$y = \pm \sqrt{\frac{9 \times 6}{5}}$$

$$y = \pm 3\sqrt{\frac{6}{5}}$$

$$y = 3\sqrt{\frac{6}{5}}, y = -3\sqrt{\frac{6}{5}}$$

If $y = 3\sqrt{\frac{6}{5}}$ then

$$x = \frac{2}{3}(3\sqrt{\frac{6}{5}}) = 2\sqrt{\frac{6}{5}}$$

If $y = -3\sqrt{\frac{6}{5}}$ then

$$x = \frac{2}{3}(-3\sqrt{\frac{6}{5}})$$

$$x = -2\sqrt{\frac{6}{5}}$$

$$\left\{ \left(2\sqrt{\frac{6}{5}}, 3\sqrt{\frac{6}{5}}\right), \left(-2\sqrt{\frac{6}{5}}, -3\sqrt{\frac{6}{5}}\right) \right.$$

$$\left. (\sqrt{2}, 4\sqrt{2}), (-\sqrt{2}, -4\sqrt{2}) \right\}$$

$$\text{Q.7} \quad x^2 - y^2 = 16 \rightarrow ①$$

$$xy = 15 \rightarrow ②$$

To balance constant terms multiply eq. ① by 15 and eq. ② by 16 we get

$$15x^2 - 15y^2 = 240 \rightarrow ③$$

$$16xy = 240 \rightarrow ④$$

Subtracting eq. ④ from eq. ③

$$15x^2 - 15y^2 - 16xy = 0$$

$$15x^2 - 16xy - 15y^2 = 0$$

Factorizing

$$15x^2 - 25xy + 9xy - 15y^2 = 0$$

$$5x(3x - 5y) + 3y(3x - 5y) = 0$$

$$(3x - 5y)(5x + 3y) = 0$$

$$3x - 5y = 0, \quad 5x + 3y = 0$$

$$3x = 5y, \quad 5x = -3y$$

$$x = \frac{5}{3}y, \quad x = -\frac{3}{5}y$$

If $x = \frac{5}{3}y$ then from ②

$$(\frac{5}{3}y)y = 15$$

$$\frac{5}{3}y^2 = 15$$

$$y^2 = 15 \times \frac{3}{5}$$

$$y^2 = 9 \rightarrow y = \pm 3$$

$$y = 3, y = -3$$

If $y = 3$ then

$$x = \frac{5}{3}(3) = 5$$

If $y = -3$ then

$$x = \frac{5}{3}(-3)$$

$$x = -5$$

$$\{(5, 3), (-5, -3), (-3, 5), (3, -5)\}$$

$$\text{Q.8} \quad x^2 + xy = 9 \rightarrow ①$$

$$x^2 - y^2 = 2 \rightarrow ②$$

To balance constant term multiply eq.

① by 2 and eq. ② by 9 we get

$$2x^2 + 2xy = 18 \rightarrow ③$$

$$9x^2 - 9y^2 = 18 \rightarrow ④$$

Subtracting eq. ③ from ④

$$9x^2 - 9y^2 = 18$$

$$\underline{2x^2 + 2xy = 18}$$

$$\underline{7x^2 - 9y^2 - 2xy = 0}$$

$$7x^2 - 2xy - 9y^2 = 0$$

$$7x^2 + 7xy - 9xy - 9y^2 = 0$$

$$7x(x+y) - 9y(x+y) = 0$$

$$(x+y)(7x - 9y) = 0$$

$$x+y = 0, \quad 7x - 9y = 0$$

$$x = -y, \quad x = \frac{9}{7}y$$

If $x = -y$ then from ②

$$y^2 - y^2 = 2$$

$$0 = 2$$

which is impossible

If $x = \frac{9}{7}y$ then from ②

$$(\frac{9}{7}y)^2 - y^2 = 2$$

$$\frac{81}{49}y^2 - y^2 = 2$$

$$81y^2 - 49y^2 = 98$$

$$32y^2 = 98$$

$$y^2 = \frac{98}{32} = \frac{49}{16}$$

$$y = \pm \frac{7}{4}$$

$$y = \frac{7}{4}, y = -\frac{7}{4}$$

If $y = \frac{7}{4}$ then $x = \frac{9}{7}(\frac{7}{4}) = \frac{9}{4}$

If $y = -\frac{7}{4}$ then $x = \frac{9}{7}(-\frac{7}{4}) = -\frac{9}{4}$

$$\left\{ \left(\frac{9}{4}, \frac{7}{4} \right), \left(\frac{9}{4}, -\frac{7}{4} \right) \right\}$$

Q.9

$$y^2 - 7 = 2xy \text{ or } y^2 - 2xy = 7 \rightarrow ①$$

$$2x^2 + 3 = xy \text{ or } 2x^2 - xy = -3 \rightarrow ②$$

To balance coefficients multiply eq. ① by 3 and eq. ② by 7 we get

$$3y^2 - 6xy = 21 \rightarrow ③$$

$$14x^2 - 7xy = -21 \longrightarrow ④$$

Adding. eq. ③ and eq. ④

$$3y^2 - 6xy = 21$$

$$14x^2 - 7xy = -21$$

$$\underline{14x^2 + 3y^2 - 13xy = 0}$$

$$14x^2 - 13xy + 3y^2 = 0$$

$$14x^2 - 7xy - 6xy + 3y^2 = 0$$

$$7x(2x-y) - 3y(2x-y) = 0$$

$$(2x-y)(7x-3y) = 0$$

$$2x-y = 0, 7x-3y = 0$$

$$x = \frac{1}{2}y, x = \frac{3}{7}y$$

If $x = \frac{1}{2}y$ then from ①

$$y^2 - 2\left(\frac{1}{2}y\right)y = 7$$

$$y^2 - y^2 = 7$$

$$0 = 7$$

which is impossible

If $x = \frac{3}{7}y$ then from ①

$$y^2 - 2\left(\frac{3}{7}y\right)y = 7$$

$$y^2 - \frac{6}{7}y^2 = 7$$

$$7y^2 - 6y^2 = 49$$

$$y^2 = 49$$

$$y = \pm 7$$

$$y = 7, y = -7$$

$$\text{If } y = 7 \text{ then } x = \frac{3}{7}(7) \rightarrow x = 3$$

$$\text{If } y = -7 \text{ then } x = \frac{3}{7}(-7) \rightarrow x = -3$$

$$\{(3, 7), (-3, -7)\}$$

Q.10

$$x^2 + y^2 = 5 \longrightarrow ①$$

$$xy = 2 \longrightarrow ②$$

To balance coefficients multiply eq.

① by 2 and ② by 5 we get

$$2x^2 + 2y^2 = 10 \longrightarrow ③$$

$$5xy = 10 \longrightarrow ④$$

Subtracting eq. ④ from eq. ③

$$2x^2 + 2y^2 - 5xy = 0$$

$$2x^2 - 5xy + 2y^2 = 0$$

$$2x^2 - xy - 4xy + 2y^2 = 0$$

$$x(2x-y) - 2y(2x-y) = 0$$

$$(x-2y)(2x-y) = 0$$

$$x-2y = 0, 2x-y = 0$$

$$x = 2y, x = \frac{1}{2}y$$

If $x = 2y$ then from ② If $x = \frac{1}{2}y$ then from ②

$$(2y)y = 2$$

$$2y^2 = 2$$

$$y^2 = 1 \rightarrow y = \pm 1$$

$$y = 1, y = -1$$

If $y = 1$ then

$$x = 2(1) = 2$$

If $y = -1$ then

$$x = 2(-1) = -2$$

$$\{(2, 1), (-2, -1), (1, 2), (-1, -2)\}$$

$$\left(\frac{1}{2}y\right)y = 2$$

$$y^2 = 4$$

$$y = \pm 2$$

$$y = 2, y = -2$$

If $y = 2$ then

$$x = \frac{1}{2}(+2) = +1$$

If $y = -2$ then

$$x = \frac{1}{2}(-2) = -1$$

Theoretical Problems on Quadratic Equation:

To solve theoretical problems we should keep following steps in mind:-

(i) Read the problem carefully.

(ii) Suppose the unknown quantities.

as x, y, z C.L.C

(iii) Translate the problem in to symbols.

For example if we are given

(a) 5 is greater than x , we write it as
 $5 > x$

(b) x is greater than y , we write it as
 $x > y$

(c) x is greater than z by 5, we write
it as $x - z = 5$

(d) 5 is less than x , we write it as
 $x < 5$

Example.1 Let x be the one part then $12-x$ will be another part.