DEPARTMENT OF MATHEMATICS
UNIVERSITY OF SARGODHA

ADMISSION TEST M.PHIL/PH.D

Max Marks: 100 **Date:** 16-09-11 **Time:** 2 Hours

Instructions:

- *Attempt any 10 questions.*
- *Read all questions very carefully before answering them.*
- *Write only solutions of the problems on answer sheet.*

Question 1. Fill in the blanks.

(i) In English engineering system of units, unit of temperature is ....

(ii) In British Gravitational System of units, unit of mass is ....

(iii) In SI system force is a .... dimension.

(iv) 1N = ... $Kg.m/s^2$.

(v) In FMLtT, 1slug = ... lbm.

(vi) Write equation of continuity for steady flow in integral form: ....

(vii) Write x component of momentum equation in integral form in absence of body forces: ....

(viii) For incompressible flow .... is constant.

(ix) For frictionless flow .... is zero.

(x) For steady flow the law of conservation of mass in different form is ....

Question 2. Solve the following differential equations:

(a) $\dfrac{dy}{dx} = \dfrac{1}{x + y^2}$.

(b) $y'' - y = \dfrac{1}{x}$.

Question 3(a). If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that maps $(1, 1)$ to $(3, 4)$ and $(-1, 1)$ to $(2, 2)$, what's the image of $(3, 1)$? (5)

(b). Let G and H be two groups and $f : G \to H$ be isomorphic. Then prove that $f^{-1} : H \to G$ is isomorphic. (5)

Question 4(a). Fill in the blanks. (4)

(i) A function of bounded variation is expressible as the .... of two monotonically increasing function.

(ii) Suppose f is a real differentiable function on some interval $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$ then there exist a point $x \in (a, b)$ such that ....

(iii) If $\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n =$.................

(iv) For every real number x there is a set E of rational number such that ....

(b). Let $\{x_n\}$ and $\{y_n\}$ be two real sequences such that (6)

(a) $x_n \leq y_n$ for all n;

(b) $\{x_n\}$ is increasing;

(c) $\{y_n\}$ is decreasing.

Show that $\{x_n\}$ and $\{y_n\}$ are convergent.

Question 5(a). Define the following. (2+2=4)

(i) Radius of gyration

(ii) Perpendicular axis theorem

(b). If $\psi(x, y, z)$ be a scalar point function then derive the expression for the curl of the gradient of ψ. (6)

Question 6(a). If A be any subset of a discrete topological space X, show that the derived set A' of A is empty. (4)

(b). Show that (6)

(i) A convergent sequence is bounded.

(ii) If $x_n \to x$ and $y_n \to y$, then $d(x_n, y_n) \to d(x, y)$.

Question 7(a). State and prove Newton-Rephson method.

(b). Find the zero of the $f(x) = x^3 - 5$ in the interval $[0, 3]$ by regular-falsi method and perform 3-iteration.

Question 8(a). Fill in the blanks. (4)

(i) The Lebesgue outer measure of an interval is equal to its ....

(ii) If $\{A_n\}_1^\infty$ is a decreasing sequence in a measure space $(X, \mathcal{A}, \mu)$, then $\lim_{n\to\infty} A_n =$ ....

(iii) A measure space in which every subset of a null set is measurable is called ....

(iv) For an increasing sequence $\{E_n\}_1^\infty$ in a measure space $(X, \mathcal{A}, \mu)$, we $\mu(\lim_{n\to\infty} E_n) =$ ....

(b). Answer the following short questions: (6)

(i) Let μ^* be an outer measure on a set X. If $E \in P(X)$ with $\mu^*(E) = 0$, then show that every subset E_0 of E is μ^*-measurable.

(ii) Let G be any open set in $\mathbb{R}$. Let $(X, \mathcal{A})$ be a measurable space and f an $\mathcal{A}-$measurable function on a set $D \in \mathcal{A}$, then show that $f^{-1}G \in \mathcal{A}$

(iii) Show that in a measure space the sum of two measurable functions is measurable.

Question 9(a). Evaluate $\text{Log}(z-i)$ by expressing it in the form $u+iv$ and prove that Cauchy Riemann equations are satisfied where $z = x + iy$.

(b). Calculate the Residues of $f(z) = \dfrac{z^3}{(z-1)(z-2)(z-3)}$ at $z = 1, 2, 3$, and at $z = \infty$.

Question 10(a). Show that the polynomial $x^2 - 2$ is irreducible over $\mathbb{Q}$, ring of rational numbers. (3)

(b). Describe all ring homomorphisms of $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$. (3)

(c). Define the following: (4)

(i) Zero divisor

(ii) Characteristic of a ring R

(iii) Irreducible polynomial in $F(x)$, where F is a field.

(iv) Principle ideal domain.

Question 11(a). Show that group of Quaternion is Nilpotent. (3)

(b). Find the upper central series of D_8, Dihedral group of order 8. (3)

(c). Define the following (4)

(i) Commutator subgroup

(ii) Sylow p-subgroup

(iii) Special linear group

(iv) Write down all elements of order 3 and 4 in S_4, symmetric group of degree 4.

Question 12(a). Fill in the blanks. (1+1+2=4)

(i) The transformation matrix A is called orthogonal if ....

(ii) The mass of a stable nucleus is always .... the sum of the masses of its constituent particles.

(iii) The length of plate form in rest frame is 65m. Find the velocity of an observer in a rocket so that the observed length is half of its original length.

(b). Prove that the composition of two Lorentz transformations is a Lorentz transformation.

Good Luck