

Department of Mathematics

M. Phil. Admission test

Total Time: 2 H

Max Marks: 80

Note; Attempt any 8 questions all questions carry equal marks.

- Q. 1. (a) Define the following
(i) Order of Diff. Eq. (ii) Ordinary Diff. Eq. (iii) Degree of Diff. Eq. (iv) Linear Diff. Eq. (v) Trivial solution
(b) Solve the given differential equation
$$x \frac{dy}{dx} - y = x^3 e^{y/x}$$

- Q. 2. (a) Define the following
(i) Parallel axis theorem (ii) Perpendicular axis theorem
(iii) Chasles theorem (iv) Rigid bodies (v) Moment of inertia
(b) Find the moment of inertia of a uniform disc of radius a and mass m about a tangent line on the radius.

- Q. 3. (a) Define the following
(i) Harmonic functions (ii) Simply connected domain
(iii) Isolated singular point (iv) Pole of order m (v) Zeros of an analytic function
(b) Find the residues of the function $f(z) = \frac{z}{z^4 + 4}$ at its singular points.

- Q. 4. (a) Define the following
(i) Gage pressure (ii) Viscosity (iii) Vorticity vector
(iv) Newtonian and non Newtonian fluids (v) Fully developed flow
(b) A body weighs 1000 lbf when exposed to a standard earth gravity $g = 32.174 \text{ ft/s}^2$
(i) what is its mass in kg?
(ii) what will be the weight of this body in N if it is exposed to the moon's standard acceleration $g_{\text{moon}} = 1.62 \text{ m/s}^2$

- Q. 5. (a) Define the following
(i) Inertial frame of reference (ii) Einstein two postulates

(b) (i) $t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$ is called -----

(ii) $l = \frac{l_0}{\sqrt{1 - v^2/c^2}}$ is called -----

(iii) $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ is called -----

(c) Show that the Lorentz transformations leaves the expression $x^2 + y^2 + z^2 - ct^2$ invariant.

Q. 6 (a) Mark true or false

- (i) Let G be a group and $a, b \in G$, let there be an $x \in G$ such that $b = xax^{-1}$. Then a and b have same order.
- (ii) The group of order 4 is non abelian.
- (iii) A linear transformation T is surjective if and only if $\ker T = 0$
- (iv) Degree of symmetric group S_4 is 24.
- (v) Group of Quaternion has 8 elements.
- (vi) Centre of S_3 has 3 elements.
- (vii) Every vector space has a finite basis.
- (viii) A group of order 37 is non cyclic.

(b) Define the following.

- (i) Basis
- (ii) normalizer
- (iii) Factor group

(c) Find order of the permutations

- (i) (9675438)
- (ii) (123)(456789)
- (iii) (ab)

Q.7 (a) Find the units of $R(\sqrt{-5})$.

(b) Define the following concepts

- (i) Primitive root of m
- (ii) arithmetic function
- (iii) jacobi symbol
- (iv) Symmetric polynomial
- (v) Degree of polynomial.

Q. 8 (a) Define the following

- (i) Discrete Topology
- (ii) open base of a topology
- (iii) Open cover
- (iv) closure point

(b) (i) let $X = \{a, b, c\}$, then write the discrete and indiscrete topology on X .

(ii) Let $X = \{a, b, c, d\}$, $T = \{\emptyset, X, \{a\}, \{a,d\}, \{a,b,d\}\}$,
 $A = \{a,c,d\}$. Then write the interior of A .

Q. 9 (a) Define and give two examples of norm space and Banach space.

(b) Let C be convex in vector space X and $T: X \rightarrow Y$ be a linear transformation, then show that $T(C)$ is convex.

Q. 10 (a) State true or false

(i) Every Cauchy sequence is not convergent.

(ii) The subsequence of a convergent sequence is converge to one and same limit.

(iii) If f is continuous in $[a,b]$, then it is bounded on $[a,b]$.

(iv) The upper and lower integral is defined for every bounded function.

(v) \sqrt{x} is uniformly continuous on $(0, 1]$.

(vi) The sequence $\{ar^{n-1}\}$, $r > 1$ is not bounded above.

(b) Show that the equation $x^3 - 2x^2 - 3x + 1 = 0$ has a solution $c \in [-1, 1]$.

(c) Show that there is no real number p such that $p^2 = 2$