M.Sc Mathematics (Two year program)

The overall structure of the program is that all courses of Part-I are compulsory. However in Part-II, the following three courses i) advanced Analysis ii) Mathematical Physics and III) Numerical Analysis are compulsory. The students will be required to choose three more courses from the list of optional.

Paper Pattern

The paper pattern will be as following

i) 40% will comprise of objective
ii) 60% will be subjective

Objective questions will compulsory

Regulation for M.Sc Mathematics Students

i) There shall be a total of 1200 marks for M.Sc (Mathematics)
ii) There shall be five papers in Part-I and six papers in Part-II. Each paper shall carry 100 marks.
iii) There shall be a Viva Voce Examination at the end of M.Sc Part-II examination carrying 100 marks.

The topics for Viva Voce Examination Shall be

i) Real Analysis
ii) Complex Analysis
iii) Algebra
iv) Mechanics (including Vector Analysis and Cartesian Tensors)
v) Topology and Functional Analysis

Course outline
M.Sc Mathematics (Annual System)
(for Private Candidate)

APPENDIX
(Detailed Outlines of Courses of Study)

M.Sc. Part-I Papers

Paper I: Real Analysis

Five questions to be attempted, selecting at least two questions from each section.

Section I (5/9)

The Real Number System

Ordered sets. Fields, The field of real, The extended real number system, Euclidean spaces.

Numerical Sequences and Series

Convergent sequences, Subsequences, Cauchy sequences, upper and lower limits, Series, Series of non-negative terms, the number, the root and ratio tests, power series.

Continuity The Limit of a function, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities.

Differentiation

The derivative of a real function, mean-value theorems, the continuity of derivatives. *Real-Valued Functions of Several Variables* Partial derivatives and differentiability, derivatives and differentials of composite functions. Change in the order of partial derivation, implicit functions, inverse functions, Jacobians, Maxima and minima (with and without side Conditions).

Section II (4/9)

The Riemann-Stieltjes Integrals

Definition and existence of the integral, properties of the integral, integration and differentiation, functions of bounded variation.

Sequences and Series of Functions

Uniform convergence, uniform convergence and continuity, uniform convergence and integration, unifrom convergence.

Improper integrals

Tests for convergence of improper integrals, infinite series and infinite integrals, Beta and Gamma functions and their properties.

Books Recommended

1. W.Rudin, Principles of Mathematical Analysis, McGraw-Hill 1976.
2. T.M. Apostol, Mathematical Analysis, Addison-Wesley, 1974.
3. W.Kaplan, Advanced Calculus, Addison-Wesley. 1952

Paper II: Algebra (Group Theory and Linear Algebra)

Five questions to be attempted, selecting at least two questions from each section.

Section I (4/9)

Group Theory

Cyclic groups, coset decomposition of a group, Lagrange's theorem and its consequences, conjugacy classes, centralisers and normalisers, normal subgroups, homomorphisms of groups. Cayley's theorem, Quotient groups, fundamental theorem of homomorphism, isomorphism theorems, endomorphisms and automorphisms of groups, Direct product of groups, Characteristic and fully invariant subgroups, simple groups (Definition and examples). Double cosets, Sylow theorems.

Section II (5/9)

Ring Theory (2/9)

Definition and example of rings, special classes of rings, Fields, Ideals, Ring homomorphisms, Quotient rings, prime and maximal ideals. Field of quotients.

Linear Algebra (3/9)

Vector spaces. Subspaces. Bases. Dimension of a vector space. Homomorphism of vector spaces, Quotient spaces, Dual spaces. Linear transformation and matrices. Rank and nullity of a linear transformation, characteristic equation, eigenvalues and eigenvectors, similar matrices, diagonalization of matrices. Orthogonal matrices and orthogonal transformations.

Books Recommended

1. J.J. Rottman, The Theory of Groups: An Introduction, Allyn & Bacon, Boston, 1965.
2. J.Rose, A Course on Group Theory, C.U.P. 1978.
3. I.N. Herstein, Topics in Algebra, Xerox Publishing Company, 1964.
4. G. Birkhoff and S. Maclane, A Survey of Modern Algebra, Macmillan, New York,1964.
5. I. Macdonald, The Theory of Groups, Oxford University Press, 1968.
6. P.M. Cohn, Algebra, Vol. I, London: John Wiley, 1974.
7. D. Burton, Abstract and Linear Algebra, Addison-Wesley publishing Co.
8. P.B. Battacharya, S.K. Jain and S.R. Nagpaul, Basic Abstract Algebra, C.U.P., 1986.
9. N. Jacobson, Basic Algebra, Vol. II Freeman, 1974.

Paper III: (Complex Analysis and Differential Geometry)

Five questions to be attempted selecting not more than two questions from each section.

Section I (3/9)

The concept of analytic functions

The complex number, points sets in the complex plane, functions of a complex variable, General properties of analytic rational functions. The nth power, polynomials rational functions, linear transformations. Basic properties of linear transformations, mapping for problems, stereographic projections, Mapping by rational functions of second order, The exponential and the logarithmic functions, the trigonometric functions, infinite series with complex terms, Power series, infinite products.

Section II (3/9)

Integration in the complex domain

Cauchy's theorem, Cauchy's integral formula and its applications, Laurent's expansion, isolated singularities of analytic functions, mapping by rational functions, the residue theorem and its applications, the residue theorem, definite integrals, partial fraction, expansion of cot 2z, the arguments principle and its application.

Analytic continuation

The principle of analytic continuation, the monodromy theorem, the inverse of a rational function, the reflection principle.

Section III (3/9)

Differential Geometry

SpacJe curves, arc length, tangent, normal and binormal, curvature and torsion of a curve. Tangent surface, Involutes and Evolutes. Existence theorem for a space curve. Helices, Curves on surfaces, surfaces of revolution, Helicoids. Families of curves, Developables, Developables associated with space curves. Developables associated with curves on surfaces, the second fundamental form. Principal curvatures, lines of curvature.

Book Recommended

1. W. Kaplan, Introduction to Analytic Functions, Addison-Wesley, 1966.
2. L.L. Pennissi, Introduction to Complex Variables, Holt Rinehart, 1976.
3. R.V. Churchill, Complex Variables and Applications, J.W. & Brown, 5 th Edition,1960.
4. J.E. Mersden, Basic Complex Analysis, W.H. Freeman & Co., San Francisco, 1973.
5. T.J. Wilmore, An Introduction to Differential Geometry, Oxford Calarendon Press,1966.
6. D. Laugwitz, Differential and Riemannian Geometry, Academic Press, New York.
7. C.E. Weatherburn, Differential Geometry, Cambridge University Press, 1927.

Paper IV: Mechanics

Five questions to be attempted, selecting at least two questions from each section.

Section I: Vector and Tensor Analysis (4/9)

A. Vector Calculus (2/9)

Gradient, divergence and curl of point functions, expansion formulas, curvilinear coordinates, line, surface and volume integrals, Gauss's, Green's and Stokes's theorems.

B. Cartesian Tensors (2/9)

Tensors of different ranks, Inner and outer products, contraction theorem, Kronecker tensor and Levi-Civita tensor, Applications to Vector Analysis.

Section II (5/9)

Mechanics

General Motion of a rigid body, Euler's theorem and Chasles' theorem, Euler's angles, Moments and products of inertia, inertia tensor, principal axes and principal moments of inertia, Kinetic energy and angular momentum of a rigid body. Momental ellipsoid and equimomental systems, Euler's dynamical equations and their solution in special cases. Heavy symmetrical top, equilibrium of a rigid body, General conditions of equilibrium, and deduction of conditions in special cases.

Books Recommended

1. F. Chorlton, A Text Book of Dynamics, CBS Publishers, 1995.
2. H. Heffrey, Cartesian Tensors, Cambridge University Press.
3. F.Chorlton, Vector and Tensor Methods, Ellis Horwood Publisher, Chichester,UK.1977.
4. Lunn, M., Classical Mechanics (Oxford).
5. Griffine, Theory of Classical Dynamics, C.U.P.

Paper V: Topology & Functional Analysis

Five questions to be attempted, selecting at least two questions from each section.

Section I (4/9) (Topology)

Definition, Open and closed sets, subspaces, neighbourhoods, limit points, closure of a set, comparison of different topologies, bases and sub-bases, first and second axiom of countability, separability, continuous functions and homeomorphisms, weak topologies, Finite product spaces. Separation axioms (T0, T1, T2), regular spaces, completely regular spaces, normal spaces, compact spaces, connected spaces.

Section II (5/9) (Functional Analysis)

Metric Spaces

Definition & examples, Open and closed sets, Convergences, Cauchy sequence and examples, completeness of a metric space, completeness proofs.

Banach spaces

Normed linear spaces, Banach spaces, Quotient spaces, continuous and bounded linear operators, linear functional, linear operator and functional on finite dimensional spaces.

Hilbert spaces

Inner product spaces, Hilbert spaces (definition and examples), Orthogonal complements, Orthonormal sets & sequences, conjugate spaces, representation of linear functional on Hilbert space, reflexive spaces.

Books Recommended

1. G.F. Simon, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, New York, 1963.
2. J. Willard, General Topology, Addison-Wesley Publishing Company, London.
3. E.. Kreyszig, Introduction to Functional Analysis with Applications, John Wiley and Sons, 1978.
4. W. Rudin, Functional Analysis, McGraw Hill Book Company, New York.
5. N. Dunford and J. Schwartz, Linear Operators (Part-I General Theory), Interscience Publishers, New York.

M.Sc. Part II Papers

Paper I: Advanced Analysis

Five questions to be attempted, selecting at least one question from each section.

Section I (2/9)

Advanced Set Theory

Equivalent sets, Countable and uncountable sets, The concept of cardinal number, addition and multiplication of cardinals, Cartesian products as sets of functions, addition and multiplication of ordinals, partially ordered sets, axiom of choice, statement of Zorn's lemma.

Section II (5/9)

Lebesgue Measure

Introduction, outer measure, Measurable sets and Lebesgue measure, A non-measurable set, Measurable functions, the Lebesgue Integral and the Riemann integral, the Lebesgue integral of a bounded function over a set of finite measure. The integral of a non-negative function. The general Lebesgue integral. Convergence in measure.

Section III (2/9)

Ordinary Differential Equations

Hypergeometric function F(a,b,c,l) and its evaluation. Solution in series of Bessel differential equation. Expression for Jn(x) when n is half odd integer, recurrence formulas for Jn(x). Series solution of Legendre differential equation. Rodrigues formula forpolynomial Pn(x). Generating function for Pn(x), recurrence relations and the orthogonality of Pn(x) functions.

Books Recommended

1. A.A. Fraenkal, Abstract Set Theory, North-Holland Publishing, Amsterdam, 1966.
2. Patrick Suppes, Axiomatic Set Theory, Dover Publications, Inc., New York, 1972.
3. P.R. Halmos, Naive Set Theory, New York, Van Nostrand, 1960.
4. B. Rotman & G.T. Kneebone, The Theory of Sets and Transfinite Numbers, old bourne, London.
5. P.R. Halmos, Measure Theory, Von Nostrand, New York, 1950.
6. W. Rudin, Real and Complex Analysis, McGraw Hill, New York, 1966.
7. R.G. Bartle, Theory of Integration.
8. H.L. Royden, Real Analysis, Prentice-Hall, 1997.
9. E.D. Rainville, Special Functions, Macmillan and Co.
10. N.N. Lebedev, Special Functions and their Applications, Prentice-Hall.

Part-II: Methods of Mathematical Physics

At least one question to be selected from each section, five questions in all.

Section I

Partial Differential Equations of Mathematical Physics (2/9)

Formation and classification of partial differential equations. Methods of separation of variables for solving elliptic, parabolic and hyperbolic equations. Eigenfunction expansions.

Section II

Sturm-Liouville System and Green's Functions (2/9)

Some properties of Sturm-Liouville equations. Sturm-Liouville systems. Regular, periodic and singular Sturm-Liouville systems. Properties of Sturm-Liouville Systems. Green's function method. Green's function in one and two dimensions.

Integral Equations (1/9)

Formulation and classification of integral equations. Degenerate Kernels, Method of successive approximations.

Section III

Integral Transforms and their Applications (2/9)

Definition and properties of Laplace transforms. Inversion and convolution theorems. Application of Laplace transforms to differential equations. Definition and properties of Fourier transforms. Fourier integrals and convolution theorem. Applications to boundary value problems.

Section IV

Variational Methods (2/9)

Euler-Lagrange equations when integrand involves one, two, three and n variables; Special cases of Euler-Lagranges equations. Necessary conditions for existence of an extremum of a functional, constrained maxima and minima.

Books Recommended

1. E.L. Butkov, Mathematical Physics, Addison-Wesley, 1973.
2. H. Sagan, Boundary and Eigenvalue Problems in Mathematical Physics.
3. R.P. Kanwal, Linear Integral Equations, Academic Press, 1971.
4. Tyn Myint-U: & L. Denbnath, Partial Differential Equations, Elsevier Science Pub.1987.
5. G. Arfken, Mathematical Methods for Physics, Academic Press, 1985.
6. I. Stakgold, Boundary Value Problems of Mathematical Physics, Vol. II, Macmillan, 1968.

Paper III: Numerical Analysis

Five questions to be attempted, selecting at the most two questions from each section.

Section I (3/9)

Linear and Non-Linear Equations

Numerical methods for nonlinear equations. Regula-falsi method. Newton's method. Iterative method. Rate and conditions of convergence for iterative and Newton's methods. Gaussian elimination method. Triangular decomposition (Cholesky) method and its various forms. Jacobi, Gauss-Seidel and iterative methods for solving system of linear equations. Ill-conditioned system and condition number. Error estimates and convergence criteria for system of linear equations. Power and Raleigh method for finding eigenvalues and eigenvectors.

Section II (3/9)

Interpolation and Integration

Various methods including Aitkins and Lagrange interpolation, error estimate formulae for interpolation and its applications, Numerical differentiation, trapezoidal, Simpson and quadrature formulae for evaluating integrals with error estimates.

Section III (3/9)

Difference and Differential Equations

Formulation of difference equations, solution of linear (homogeneous and inhomogeneous) difference equations with constant coefficients, the Euler and the modified Euler method. Runge-Kutta methods and predictor-corrector type methods for solving initial value problems along with convergence and instability criteria. Finite difference, collocation and variational method for boundary value problems.

Books Recommended

1. C. Gerald, Applied Numerical Analysis, Addision-Wesley Publishing Company,1978.
2. A. Balfour & W.T. Beveridge, Basic Numerical Analysis with Fortran, Heinemann Educational Books Ltd., 1977.
3. Shan and Kuo, Computer Applications of Numerical Methods, Addision-Wesley, National Book Foundation, Islamabad, 1972.

OPTIONAL PAPERS

Paper (IV-VI) option (i): Mathematical Statistics
Five questions to be attempted, selecting at least two questions from each section.
Section I (4/9)
Probability
The postulates of probability and some elementary theorems, addition and multiplication rules, Baye's rule, probability functions, probability distributions (discrete, uniform, Bernoulli, binomial, hypergeometric, geometric, negative binomial, Poisson). Probability densities, the uniform, exponential, gamma, beta and normal distributions, change of variable.
Section II (5/9)
Mathematical Expectation
Moments, moment generating functions, moments of binomial, hypergeometric,
Poisson,
gamma, beta and normal distributions.
Sums of Random Variables
Convolutions, moment generating functions, the distribution of the mean, differences between means, differences between proportions, the distribution of the mean for finite populations.
Sampling Distributions
The distribution of x-bar, the chi-squared distribution and the distribution of s-squared, the *F* distribution, the *t* distribution.
Regression and Correlation
Linear regression, the methods of least squares, correlation analysis.
Books Recommended
1. J.E. Freund, Mathematical Statistics, Prentice-Hall Inc., 1992.
2. Hogg & Craig, Introduction to Mathematical Statistics, Collier Macmillan, 1958.
3. Mood, Greyill & Boes, Introduction to the Theory of Statistics, McGraw Hill.

Paper (IV-VI) option (ii): Computer Applications
The evaluation of this paper will consist of two parts:
1. Written examination: 50 marks
2. Practical examination: 50 marks
(The practical examination includes 10 marks for the notebook containing details of work done in the Computer Laboratory and 10 marks for the oral examination). It will involve writing and running programmes on computational projects. It will also include familiarity with the use of Mathematical Recipes subroutines (and MATHEMATICA in calculus and graphing of functions).

Course Outline for the Written Examination
Five questions to be attempted, selecting at least two questions from each section.
Section I (4/9)
Computer Orientation
General introduction to digital computers, their classes and working. Concepts of low-level and high-level computer languages, an algorithm and a programme. Problem-solving process using digital computers including use of flow-charts.
Programming in FORTAN: (Fortran 90, 95)

Arithmetic expressions, Assignment statements, I/O statements including the use of I, F, E, H and X specifications. Computed IF statements, computed Go To-statement, Logical expressions and logical IF-statements, Nested Do-loops, Do-WHILE loop. Subscripted variables and arrays, DIMENSION statements, Implied Do-loops, Data statement, COMMON and EQUIVALENT statements, SUBROUTINE subprogrammes and FUNCTION subprogrammes.

Section II (5/9)

Computational projects in Fortran

a) Bisection method, Regula falsi method, Newton-Raphson method for solving non-linear equations.

b) Gaussion elimination with different pivoting strategies, Jacobi and Gauss—Seidel iterative methods for systems of simultaneous linear equations.

c) Trapezoidal rule, Simpson's rule and Gaussian method of numerical integration.

d) Modified and improved Euler's methods; Predictor-Corrector methods for finding the numerical solution of IVP's involving ODE's.

Note: Practical examination will be of two hours duration in which one or more computational projects will be examined.

Books Recommended

1. M.L. Abell and J.P. Braselton, Mathematica Handbook, New Yourk, 1992.
2. T.J. Akai, Applied Numerical Methods, J. Wiley, 1994.
3. J.H. Mathews, Numerical Methods for Computer Science, Engineering and Mathematics, Prentice-Hall, 1987.

Paper (IV-VI) option (iii): Group Theory

Five questions to be attempted, selecting at least two questions from each section.

Section I (5/9)

Characteristic and fully invariant subgroups, normal products of groups, Holomorph of a group, permutation groups, cyclic permutations and orbits, the alternating group, generators of the symmetric and alternating groups, simplicity of A, n > 5. The stabilizer subgroups. n = series in groups. Zassenhaus lemma, normal series and their refinements,composition series, principal or chief series.

Section II (4/9)

Solvable groups, definition and examples, theorems on solvable groups. Nilpotent groups, characterisation of finite nilpotent groups, upper and lower central series, the Frattini subgroups, free groups, basic theorems, definition and examples of free products of groups, linear groups, types of linear groups. Representation of linear groups, group algebras and representation modules.

Books Recommended

1. I.D. Macdonald, The Theory of Groups, Oxford, Clarendon Press, 1975.
2. H.Marshall, The Theory of Groups, Macmillan, 1967.
3. M.Burrow, Representation Theory of Finite Groups, New York: Academic Press,1965.
4. W.Magnus, A. Karrass & D. Solitar, Combinatorial Group Theory: Presentation of Groups in Terms of Generators and Relations, New York, John Wiley, 1966.

Paper (IV-VI) option (iv): Rings and Modules

Five questions to be attempted, selecting at least two questions from each section.

Section I (5/9)

Rings

Construction of new rings, Direct sums, polynomial rings, Matrix rings. Divisors, Units and associates, Unique factorisation domains. Principal ideal domains and Euclidean domains. Field Extensions. Algebraic and transcendental elements. Degree of extension. Algebraic extensions. Reducible and Irreducible polynomials. Roots of polynomials.

Section II (4/9)

Modules

Definition and examples, submodules, Homomorphisms and quotient modules. Direct Sums of modules. Finitely generated modules, Torsion Modules, Free modules. Basis, rank and endomorphisms of free modules. Matrices over rings and their connection with the basis of a free module. A module as the direct sum of a free and a torsion module.

Books Recommended

1. I.N. Herstein, Topics in Algebra, Xerox Publishing Company Mass, 1972.
2. B.Hartley & T.O. Hauvkes, Rings, Modules and Linear Algebra, Chapmann and Hall Ltd., London, 1970.
3. R.B.J.T. Allenly, Rings, Fields and Groups, An Introduction to Abstract Algebra, Edward Arnold, 1985.

Paper (IV-VI) option (v): Number Theory

Five questions to be attempted, selecting at least two questions from each section.

Section I (5/9)

Analytic Number Theory

Congruences

Elementary properties, Residue classes and Euler's function. Linear congruences and congruences of higher degree. Congruences with prime moduli. The theorems of Fermat, Euler and Wilson.

12.*Primitive roots and indices*

Integers belonging to a given exponent, composite moduli, Indices.

Quadratic Residues

Composite moduli, Legendre symbol, Law of quadratic reciprocity, the Jacobi symbol.

Number-Theoretic Functions

Mobius function, the function [x], the symbols O, O and – and their basic properties.

Diophantine Equations

Equations and Fermat's conjecture for n=2, n=4.

Section II (4/9)

Algebraic Number Theory

Algebraic numbers and integers, Units and Primes in R[v] ideals. Arithmetic of ideals congruences. The norm of an ideal. Prime ideals. Units of algebraic number field.

Applications to Rational Number Theory

Equivalence and class number. Cyclotomic field Kp' Fermat's equation. Kummer's theorem, the equation, $x^2 + 2 - y^3$, pure cubic fields. Distribution of primes and Riemann's zeta function, the prime number theorem.

Book Recommended

W.J. Leveque, Topics in Number Theory, Vols. I and II, Addison-Wesley Publishing Co., 1956.

Paper (IV-VI) option (vi): Fluid Mechanics

Five questions to be attempted, selecting at most two questions from each section.

Section I (3/9)

Introduction

Fluid and continuity hypotheses: Surface and Body forces, Stress at a point, Viscosity, Newton's viscosity law; Viscous and inviscid flows, Laminar and Turbulent flows, Compressible and Incompressible flows; Lagrangian and Eulerian descriptions; Local; Connective and total rates of change; Conservation of mass.

Section II (3/9)

Inviscid Fluids

Irrotational motions, Boundary conditions, streamlines, vortex lines and vortex sheets, Kelvin's minimum energy theorem, Conservation of Linear momentum, Bernoulli's theorem and its applications, Circulations, Rate of change of circulation (Kelvin's theorem), axially symmetric motion, Stokes's stream function, Two-dimensional motion, Stream function, Complex potential and some potential flows, Sources, sinks and doublets, Circle theorem, Method of images, Blassius theorem, aerofoil and the theorem of Kutta and Joukowski, vortex motion, Karman's vortex street.

Section III (3/9)

Viscous Fluids

Constitutive equations, Navier-Stokes's equations, exact solutions of Navier-Stokes's equations: Steady unidirectional flow, Poiseuille flow, Couette flow, unsteady unidirectional flow, sudden motion of a plane boundary in a fluid at rest, Flow due to an oscillatory boundary. Equations of motion relative to a rotating system, Ekman flow, Dynamical similarity and the Reynold's number, Boundary layer concept and its governing equations, Flow over a flat plat (Blassius solution); Reynold's equations of turbulent motion.

Books Recommended

1. H. Schlichting, Boundary-Layer Theory, McGraw Hill.
2. Yith Chia-Shun, Fluid Mechanics, McGraw Hill, 1974.
3. I.L. Distsworth, Fluid Mechanics, McGraw Hill.
4. I.G. Curie, Fundamentals of Mechanics of Fluids, McGraw Hill, 1974.
5. R.W. Fox & A.T. McDonald, Introduction to Fluid Mechanics, John Wiley & sons.

Paper (IV-VI) option (vii): Quantum Mechanics

Four questions to be attempted, selecting at least one question from each section.

Section I (2/7)

Inadequacy of classical mechanics

Black body radiation, Photoelectric effect, Compton effect, Bohr's theory of atomic structure, Wave-particle duality, the de Broglie postulate.

The Uncertainty Principle

Uncertainty of position and momentum, statement and proof of the uncertainty principle. Energy-time uncertainty. Eigenvalues and eigenfunctions, Operators and eigenfunctions, Linear Operators, Operator formalism in Quantum Mechanics, Orthonormal systems, Hermitian operators and their properties, Simultaneous eigenfunctions. Parity operators, postulates of quantum mechanics, the Schrodinger wave equation.

Section II (3/7)

Motion in one dimension

Step potential, Potential barrier, Potential well, oscillator, Motion in three dimensions, angular momentum, commulation relations of between components of angular momentum, and their representation in spherical polar coordinates, simultaneous eigenfunctions of L_Z and L^2 . Spherically symmetric potential and the hydrogen atom.

Section III (2/7)

Scattering Theory

The scattering cross-section, scattering amplitude, scattering equation. Born approximation, partial wave analysis.

Perturbation Theory
Time independent perturbation of non-degenerate and degenerate cases. Time-dependent perturbations.
Identical particles
Symmetric and antisymmetric eigenfunctions. The Pauli exclusion principle.
Books Recommended
1. J.G. Taylor, Quantum Mechanics, George Allen and Unwin, 1970.
2. T.L. Powell & B. Crasemann, Quantum Mechanics, Addison-Wesley, 1961.
3. E. Merzbacker, Quantum Mechanics, John Wiley & Sons, 1961.
4. R.M. Eisberg, Fundamentals of Modern Physics, John Wiley & Sons.
5. H. Muirhead, The Physics of Elementary Particles, Pergamon Press, 1965.
6. R. Dicke, R & J.P. Witke, Quantum Mechanics, Addision-Wesley.

Paper (IV-VI) option (Viii): Special Relativity and Analytical Dynamics
Note: Five questions to be attempted, selecting at least one question from section I.
Section I (3/9)
Special Relativity
Fundamental concepts, The Lorentz transformation, Time dilation and Lorentz-Fitzgerald contraction, Transformation of velocities, four-velocity and four-acceleration. Relativistic dynamics, relativistic equations of motion, relativistic mass, linear momentum, four-force, relativistic kinetic energy, four momentum.
Section II (6/9)
Analytical Dynamics
a) Generalized coordinates, Holonomic and non-holonomic systems. D' Alembert's principle, d-delta rule.
b) Lagrange's Theory of Holonomic Systems.
(i) Equations of Lagrange, Generalization of Lagrange equations, Quasi-coordinates and Lagrange equations in quasi-coordinates. (ii) First integrals of Lagrange equations of motion, Energy integral and Whittaker's equations, ignorable coordinates and Routhian function, Noether's theorem.
c) Lagrange's Theory of Non-Holonomic Systems.
Equations of Lagrange for non-holonomic systems with and without Lagrange multipliers, Chaplygin's equations.
d) Hamilton's Theory.
(i) Hamilton's principle, Generalized momenta and phase space, Hamilton's equations. (ii) Canonical transformations, Generating functions, the Lagrange and Poisson brackets, Bilinear covariants, infinitesimal exact transformations. (iii) Hamilton-Jacobi theorem.
Books Recommended
1. M.Saleem and M.Rafique, Special Relativity, Ellis Horwood, 1992.
2. Rosser, Special Relativity, 1972.
3. W. Ringler, Introduction to Special Relativity, Oxford, 1982.
4. D.T. Greenwood, Classical Dynamics, Prentice-Hall, Inc., 1965.
5. H.Goldstein, Classical Mechanics, Addison-Wesley;, 1964.
6. L.A. Pars, Treatise of Analytical Dynamics, Heninemann Press, London, 1965.

Paper (IV-VI) option (ix): Electromagnetic Theory

Five question to be attempted, selecting at least one question from each section

Section I (3/9)

Electrostatics

Coulomb's law. Electric Field and potential. Lines of force and equipotential surfaces. Gauss's law and deductions. Conductors and condensers. Dipoles. Dielectrics. Polarisation and apparent charges. Electric displacement. Energy of the field. Minimum energy.

Magnetostatic Field

The Magnetostatic Law of Force. Magnetic Doublets. Magnetic shells. Force on Magnetic doublets. Magnetic induction. Para and dia magnetism.

Section II (3/9)

Steady and Slowly Varying Currents

Electric current. Linear conductors. Conductivity. Resistance, Kirchhoff's laws. Heat production. Current density vector. Magnetic field of straight and circular current. Magntic flux. Vector potential. Forces on a circuit in magnetic field. Magnetic field energy. Law of electromagnetic induction coefficients of self and mutual induction. Alternating current and simple I.C.R. circuits in series and parallel. Power factor.

Section III (3/9)

Potential Problems

The equations of electromagnetism

Maxwell's equations in free space and material media. Solution of Maxwell's equations. Plane electromagnetic waves in homogeneous and isotropic media. Reflection and Refraction of plane waves, Wave guides. Laplace's equation in plane, polar and cylindrical coordinates. Simple introduction to the Legendre polynomials. Method of Images; Images in a plane, Images with spheres and cylinders.

Books Recommended

1. Ferraro, Electromagnetic Theory, Athlone Press, 1954.
2. J.R. Reitz & Milford, Foundations of Electromagnetic Theory, Addison-Wesley, 1967.
3. Pugh & Pugh, Electricity and Magnetism.
4. J.D. Jackson, Classical Electrodynamics, John Wiley, 1963.

Paper (IV-VI) option (x): Operations Research

Five questions to be attempted, selecting at least one question from each section.

Section I (3/9)

Linear Programming Formulations and Graphical Solution. Simplex Method. M-Technique
and Two-phase Technique. Special Cases. Sensitivity Analysis. The Dual Problem. Primal-Dual Relationships. Dual Simplex Method. Sensitivity and Postoptimal Analysis.

Section II (3/9)

Transportation Model. North-West Corner, Least-Cost and Vogel's Approximations Methods. The Method of Multipliers. The Assignment model. The Transhipment Model. Network Minimization. Shortest-Route Algorithms for Acyclic Networks. Maximal-flow problem. Matrix Definition of LP Problem. Revised Simplex Method. Bounded Variables. Decomposition Algorithm. Parametric Linear Programming.

Section III (3/9)

Applications of Integer Programming. Cutting-plane Algorithms. Branch-and-Bound Method. Zero-one Implicit Enumeration. Elements of Dynamic Programming. Problem of Dimensionality. Solution of Linear Programmes by Dynamic Programming.

Books Recommended

1. Hamdy A. Taha, Operations Research-An Introduction, Macmillan Publishing Company Inc., New York, 1987.
2. B.E. Gillett, Introduction to Operations Research, Tata McGraw Hill Publishing Company Ltd., New Delhi.

3. F.S. Hillier & G.J. Liebraman, Operations Research, CBS Publishers and Distributors, New Delhi, 1974.
4. C.M. Harvey, Operations Research, North Holland, New Delhi, 1979.

Papers (IV-VI) option (xi) Differential Equations and Dynamical Systems

Attempt any five questions.

Systems of linear differential equations

Linear systems, solution matrix, fundamental solution matrix, phase space analysis, autonomous systems, definition of stability, stability for linear and almost linear systems, basic concepts of Liapunov's method.

Dynamical Systems

What is a dynamical system? Phase space, dynamical equations, flows, iterated maps, examples, Reduction to first order autonomous systems. Return maps. Examples. Fixed points and periodic orbits. Notions of stability, invariant sets and attractors. Examples of attractors, strange attractors and chaos, introduction to symbolic coding. Linear systems and their behaviour. Linearizing nonlinear systems. Linear stability. Floquet theory. Co-ordinate changes and conjugacies between dynamical systems. Hartman-Grossman theorem (without proof). Stable and unstable manifolds for fixed points and periodic orbits. Introduction to bifurcations, the distinction between local and global bifurcations. Introduction to symbolic dynamics, the shift map and the Smale horseshoe. Local bifurcation theory: the saddle node, transcritical, period doubling and Hopf bifurcations.

Books Recommended

1. Richard E. Williamson, Introduction to Differential Equations and Dynamical Systems, Mcgraw Hill, international edition, 1997.
2. V.I. Arnol'd, Ordinary Differential Equations, Springer, 1988.
3. V.I. Arnol'd, Geometrial Methods in the Theory of Ordinary Differential Equations, Springer, 1988.
4. D.K. Arrowsmith and C.M. Place, Introduction to Dynamical Systems, Cambridge University Press, 1990.
5. P.G. Drazin, Nonlinear Systems, Cambridge University Press, 1992.
6. P.A. Glendinning, Stability, Instability and Chaos, Cambridge University Press, 1994.
7. R. Grimshaw, Nonlinear Ordinary Differential Equations, CRC Press, 1991.
8. M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and Linear Algebra, Academic Press, 1974.
9. D.W. Jordan and P. Smith, Nonlinear Ordinary Differential Equations, Oxford University Press, 1987.
10. S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer, 1990.
11. R. Devaney, An Introduction to Chaotic Dynamical Systems, Addison-Wesley, 1989.
12. J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer, 1983.

13. A. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems, Cambridge University Press, 1995.
14. C.Robinson, Dynamical Systems: Stability, Symbolic Dynamics and Chaos, CRC Press,1995.

Paper (IV-VI) option (xii): Waves and Solutions

Attempt any five questions.

Weakly dispersive nonlinear waves. Water waves in a rectangular channel and the Korteweg-de Vries (KdV) equation. Solitary waves and their interactions. Conservation laws: mass, momentum, energy and others. The modified KdV equations and the Miura transformation to the KdV. Derivation of the KdV, mKdV and nonlinear Schrodinger equations as models of physica systems. The Lax pair representation for the KdV, scattering theory and the inverse transform, leading to solution of the initial value problem.

Books Recommended

1. P.G. Drazin, Nonlinear Systems, Cambridge University Press, 1992.
2. R. Grimshaw, Nonlinear Ordinary Differential Equations, CRC Press, 1991.
3. D.W. Jordan and P. Smith, Nonlinear Ordinary Differential Equations, Oxford University Press, 1987.
4. P.G. Drazin, Solitons, Cambridge University Press, 1983.
5. P.G. Drazin & R.S. Johnson, Solitons: An Introduction, Cambridge University Press, 1989, 1990.
6. P.K. Dodd, J.C. Eilbeck, J.D. Gibbon and H.C. Morris, Solitons and Nonlinear Wave Equations, Academic Press, 1982.

Paper (IV-VI) option (xiii) Solid Mechanics

Five questions to be attempted, selecting at least two questions from each section.

Section I: Elasticity (5/9)

Analysis of stress and strain, Generalized Hook's law. Differential equations of equilibrium in terms of stress and in terms of displacements. Boundary conditions. Compatibility equations. Plane stress. Plane strain. Stress function. Two-dimensional problems in rectangular and polar co-ordinates. Torsion problems.

Section II: Elastodynamics (4/9)

Equations of wave propagation in elastic solids, Primary and secondary waves. Reflection and transmission at plane boundaries. Surface waves: Love waves and Raleigh waves. Dispersion relations. Geophysical applications.

Books Recommended

1. S.P. Timoshonko, & J.N. Goodier, Theory of Elasticity, McGraw-Hill, 1970.
2. I.S. Sokolnikoff, Mathematical Theory of Elasticity, McGraw Hill, 1956.
3. W.Prager, Introduction to Mechanics of Continua, Gim and Co., 1961.
4. J.D. Achenbach, Wave Propagation in Elastic Solids, North-Holland Publishing Company, Amsterdam, 1973.
5. W.M. Ewing, W.S. Jardetsky, and F. Press, Elastic Waves in Layered Media, McGraw Hill.
6. K.F. Graff, Wave Motion in Elastic Solids, Clarendon Press, 1975.
7. J. Jaffreys, The Earth, Fifth edition, Cambridge University Press, 1970.
8. K.E. Bullen, An Introduction to the Theory of Seismology, Cambridge University Press, 1965.

Paper (IV-VI) option (xiv): Theory of Optimization

Attempt any five questions.
Introduction to optimization, Single variable optimization. Multivariable optimization. Linear Programming Problem. Geometry of linear programming problems. Solution of system of linear simultaneous equations. Simplex method, Revised simplex method. Duality in linear programming. Decomposition Principle. Transportation problem. Unimodel function. Elimination methods, Unrestricted search. Exhaustive search. Dichotomous search. Fibonacci method. Quadratic interpolation method, Cubic interpolation method. Direct root method. Direct search method. Random search methods. Univariate method. Descent methods. Steepest descent method. Conjugate gradient method. Quasi Newton methods. Cutting plane method. Methods of feasible directions. Penalty function method, Dynamic programming.

Books Recommended

1. B.S. Gottfried & W. Joel, Introduction to Optimization Theory, Prentice-Hall, 1973.
2. S.S. Rao, Optimization Theory and Applications.
3. M.J. Fryer, Optimization Theory: Applications in Operations Research and Economics.
4. K.V. Mital, Optimization Methods in Operations Research and Systems Analysis, Second Edition, 1983.
5. R.K. Sudaram, A First Course in Optimization Theory, Cambridge University Press, 1996.

Paper (IV-VI) option (xv): Theory of Approximation and Splines

Five questions to be attempted, selecting at least two questions from each section.

Section I: (4/9)

Euclidean Geometry

Basic concepts of Euclidean Geometry, Scalar and Vector functions, Barycentric Coordinates, Convex Hull, Matrices of Affine Maps: Translation, Rotation, Scaling. Reflection and Shear.

Approximation using Polynomials

Linear Interpolation, Least squares polynomial curve fitting, Lagrange's Method. Hermite's Methods, Divided Differences Methods.

Section II (5/9)

Parametric Curves (Scalar and Vector Case)

Algebraic Form. Hermite Form. Control Point Form, Bernstein Bezier Form, Matrix Forms of Parametric Curves, Algorithms to compute B.B. Form, Convexhull Property, Affine invariance property, variation diminishing property, Rational Quadratic Form, Rational Cubic Form.

Spline Functions

Splines, Cubic Splines, End Conditions of Cubic Splines: Clamped conditions, Natural conditions, 2^{nd} Derivative conditions, Periodic conditions, Not a knot conditions, General
Splines: Natural Splines, Periodic Splines, Truncated Power Function, Representation of spline in terms of truncated power functions, Odd degree interpolating splines.

Books Recommended

1. Curves and Surfaces for Computer Aided Geometric Design A Practical Guide, Academic Press. Inc. by Gerald Farin.
2. Computational Geometry for Design and Manufacture, Ellis Horwood by I.D. Faux.

3. An Introduction to Spline for use in Computer Graphics and Geometric Modeling, Morgan Kaufmann Publisher, Inc. By Richard H. Bartels.
4. A Practical Guide to Splines, Springer Verlag 1978 by Carl de Boor.
5. Spline Functions: Basic Theory, John Wiley 1981 by Schumaker.