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University of Sargodha

B.A/B.Sc 2nd Annual Examination 2006

Math General **Paper- B**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Attempt **six** questions in all, selecting **two** questions from section- I, **two** questions from section- II, **one** question from section- III and **one** question from section- IV.

Section- I

Q.1. (a) Analyze and graph the conic represented by the equation. (9)

$$4x^2 - 4xy + 7y^2 + 12x + 6y - 9 = 0$$

(b) If C P and CD are semi conjugate diameters of an ellipse with centre C, show that the eccentric angles of P and D differ by a right angle. (8)

Q.2. (a) Find the pedal equation of $r^m = a^m Cosm\theta$. (9)

(b) Show that the normal at any point of the curve (8)

$$x = aCos\theta + a\theta Sin\theta$$
$$y = aSin\theta - a\theta Cos\theta$$

Is at a constant distance from the origin.

Q.3. (a) If $\vec{r} = \vec{a}Sin\omega t + \vec{b}Cos\omega t + \frac{\vec{c}t}{\omega^2}Sin\omega t$ (9)

prove that $\frac{d^2\vec{r}}{dt^2} + \omega^2\vec{r} = \frac{2\vec{c}}{\omega}Cos\omega t,$

where $\vec{a}, \vec{b}, \vec{c}$ are constant vectors and 'ω' being a constant scalar.

(b) If $\vec{V} = \vec{a} \times \vec{r}$ prove that $\vec{a} = \frac{1}{2} curl\vec{V}$ where $\vec{a}$ is a constant vector. (8)

Q.4. (a) Prove that for any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ (9)

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$$

(b) Prove that (8)

i. $|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$

ii. $|\vec{a}.\vec{b}|^2 - |\vec{a} \times \vec{b}|^2 = a^2 b^2 Cos2\theta$

Section- II

Q.5. (a) Find equations of the straight line passing through the point P(2, 0,-2) and perpendicular to each of the straight lines (9)

$$\frac{x-3}{2} = \frac{y}{2} = \frac{z+1}{2} \quad \text{and} \quad \frac{x}{3} = \frac{y+1}{-1} = \frac{z+2}{2}$$

(b) Find an equation of the plane that passes through the points (3, 2, -1) and (1,-3,4) and contains a line parallel to $2\vec{i} - 4\vec{j} + 3\vec{k}$ (8)

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Q.6. (a) Find an equation of the sphere passing through the points (0,-2,-4); (2,-1,-1) and having its centre on the straight line $2x - 3y = 0 = 5y + 2z$ (9)

(b) Write the equation $x^2 - 9y^2 - 4z^2 - 6x + 18y + 16z + 20 = 0$ referred to new set of parallel axes with the origin at (3, 1, 2). (8)

Q.7. (a) Prove that $64(Cos^8\theta + Sin^8\theta) = Cos8\theta + 28Cos4\theta + 35$ (9)

(b) Separate $(\alpha + i\beta)^{p+iq}$ into real and imaginary parts. (8)

Q.8. (a) Evaluate the sum of Infinite series. (9)

$$Cos\theta - \frac{1}{2}Cos2\theta + \frac{1}{3}Cos3\theta - \frac{1}{4}Cos4\theta + \ldots\ldots$$

(b) In any spherical triangle ABC, show that (8)

$$2Cos\frac{a+b}{2}.Cos\frac{a-b}{2}Tan\frac{c}{2} = SinbCosA + SinaCosB$$

Section- III

Q.9. (a) Determine whether or not the set of vectors {(1,2,-1), (0,3,1), (1,-5,3)} is a basis for R^3. (8)

(b) Show that (8)

$$\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a-1)^3(a+3)$$

Q.10. (a) Show that (8)

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} R\ I_3.$$

(b) Find the solution of the system of linear equations (8)

$$2x_1 - x_2 - x_3 = 4$$
$$3x_1 + 4x_2 - 2x_3 = 11$$
$$3x_1 - 2x_2 + 4x_3 = 11$$

by Gauss-Jordan elimination Method.

Section- IV

Q.11. (a) Solve the initial value problem (8)

$$(2xCosy + 3x^2y)dx + (x^3 - x^2\sin y - y)dy = 0 \qquad y(0) = 2$$

(b) Solve $\frac{dy}{dx} = \frac{1}{e^y - x}$ (8)

Q.12. (a) Solve and find the singular solution (if any) $6p^2y^2 + 3px - y = 0$ (8)

(b) Find the General Solution of $(D^2 - 2D - 3)y = 2e^x - 10Sinx$ (8)

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