(54)

B.A/B. Sc 1st Annual Examination 2012

General Math Paper: B

Maximum Marks: 100 Time Allowed: 3 Hours

Note: Attempt any two questions from each section.

Section- I

Q.1. a. If $\underline{a}, \underline{b}$ are constant vectors, w is a constant and $\vec{\gamma}$ be a vector function of scalar variable t, given by $\vec{\gamma} = \cos wt\,\underline{a} + \sin wt\,\underline{b}$, then show that $\vec{\gamma} \times \frac{d\vec{\gamma}}{dt} = w\underline{a} \times \underline{b}$ (9)

b. Apply Cauchy's root test to determine whether the series converges or diverges $\sum_1^\infty \left(\frac{n}{1+n^3}\right)^n$ (8)

Q.2. a. Test the series for convergence or divergence $\sum_1^\infty \left(\frac{\ln(n+1)}{n^2}\right)$ (9)

b. If $a^{\alpha+i\beta} = (x+iy)^{p+q}$, $a > 0$ prove that $\alpha = \frac{1}{2}p\log_a(x^2+y^2) - q\tan^{-1}\frac{y}{x}\log_a^e$ (8)

Q.3. a. Prove that $[(a \times (b \times c)) \times d] \times e = (a.c)[e.bd - e.db] - (a.b)[e.cd - e.dc]$ (9)

b. Find the square of all fifth roots of $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ (8)

Q.4. a. Prove that $\nabla(\phi(r)) = \frac{\phi'(r)}{r}\vec{r}$ (9)

b. Evaluate the sum of the series $\cos^2\theta + \cos 2^2\theta + \cos^2 4\theta + \_\_\_\_\_ + \cos^2 n\theta$ (8)

Section- II

Q.5. a. If u, w are subspaces of a vector space v, then $(u + w)$ is a subspace of v containing u and w. Further $u + w$ is the smallest vector space containing both u and w. (8)

b. Prove that the product of matrices (8)

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix};\quad B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is the zero matrix when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

Q.6. a. For what value of λ, the equations (8)

$$\begin{aligned} (1-\lambda)x_1 + x_2 - x_3 &= 0 \\ x_1 - \lambda x_2 - 2x_3 &= 0 \\ x_1 + 2x_2 - \lambda x_3 &= 0 \end{aligned}$$

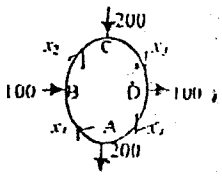
have nontrivial solutions. Find these solutions.

b. Prove that $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos(\alpha+\beta) \\ \cos\beta & \cos(\alpha+\beta) & 1 \end{vmatrix} = 0$ (8)

Q.7. a. The flow of traffic at Kalma Chowk on Ferozepur road Lahore is shown below. (8)

i. Solve the system

ii. Find the traffic flow when $x_4 = 300$.



b. For what value of α is the matrix $A = \begin{bmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{bmatrix}$ singular. (8)

Q.8. a. Find a basis and dimension of the subspace w of R^4 spanned by $(1,4,-1,3)$, $(2,1,-3,-1)$ and $(0,2,1,-5)$ (8)

b. Find the solution of system of linear equations (8)

$$\begin{aligned} 2x_1 - x_2 - x_3 &= 4 \\ 3x_1 + 4x_2 - 2x_3 &= 11 \\ 3x_1 - 2x_2 + 4x_3 &= 11 \end{aligned}$$

Section- III

Q.9. a. Solve the differential equation $\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$ (8)

b. Find the general solution $(D^3 - D^2 + D - 1)y = 4\sin x$ (9)

Q.10. a. Solve the differential equation $xp^2 - 2yp + 4x = 0$ (9)

b. Solve the differential equation $dy + \frac{y-\sin x}{x}dx = 0$ (8)

Q.11. a. Solve the differential equation by method of U.C. $y''' + y' = 2x^2 + 4\sin x$ (8)

b. Find the equation of orthogonal trajectory of curve $r = a(1 + \sin\theta)$ (9)

Q.12. a. Solve the differential equation $x^4\frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 1$ (9)

b. Solve $\sin x\frac{d^2y}{dx^2} - \cos x\frac{dy}{dx} + 2y\sin x = 0$ (8)