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University of Sargodha

B.A / B.Sc 1st Annual Examination 2008.

Math General (New Course) **Paper: B**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1	(a) If $\vec{a}$ and $\vec{b}$ are any vectors, then show that $\lvert\vec{a}\rvert\,\vec{b}+\lvert\vec{b}\rvert\,\vec{a}$ is perpendicular to $\lvert\vec{a}\rvert\,\vec{b}-\lvert\vec{b}\rvert\,\vec{a}$. (b) Find the condition for absolute convergence ,radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{n^2}$
Q.2	(a) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(2n+1)(3^n+1)}{4^n+1}$ (b)Find the sum of the series $\cos^2\theta+\cos^2 2\theta+\cos^2 3\theta+\cdots$ to n terms
Q.3	(a)Prove that $\tan^{-1}(\cos\theta+i\sin\theta)=\pm\frac{\pi}{4}+\frac{i}{4}\ln\frac{1+\sin\theta}{1-\sin\theta}$ (b) If $\hat{u}$ (t) is a unit vector , then show that $\hat{u}\cdot\left(\hat{u}+\frac{d^2\hat{u}}{dt^2}\right)+\left(\frac{d\hat{u}}{dt}\right)^2=1$
Q.4	(a) Evaluate $\nabla\times\left(\frac{\vec{r}}{r^2}\right)$, where $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ and $r=\lvert\vec{r}\rvert$ (b) If z_1, z_2 are complex numbers , show that $\lvert z_1+z_2\rvert^2+\lvert z_1-z_2\rvert^2=2(\lvert z_1\rvert^2+\lvert z_2\rvert^2)$
Q.5	**Section II** (a) Show that $\begin{vmatrix}1 & \omega & \omega^2 & \omega^3\\ 1 & \omega^2 & \omega^4 & \omega\\ 1 & \omega^3 & \omega & \omega^4\\ 1 & \omega^4 & \omega^3 & \omega^2\end{vmatrix}=125$, where ω is a fifth root of unity (b) Find a basis and the dimension of R(T) and N(T) where $T:R^3\rightarrow R^4$ is defined by $T(x_1,x_2,x_3)=(2x_1+x_3,\ 4x_1+x_2,\ x_1+x_3,\ x_3-4x_2)$ Where $N(T)=\{u\in U: T(u)=0\}$ is the Null space and $R(T)=\{v\in V:$ there exist $u\in U$ with $T(u)=v\}$ is the range of T

P.T.O

Q- No | **Questions** | **Marks**

Q.6

Q.6 (a) For what value of λ the following homogeneous equations **(8)**

$$\begin{aligned}(1-\lambda)x_1 + x_2 + x_3 &= 0\\ x_1 - \lambda x_2 + x_3 &= 0\\ x_1 - x_2 + (1-\lambda)x_3 &= 0\end{aligned}$$

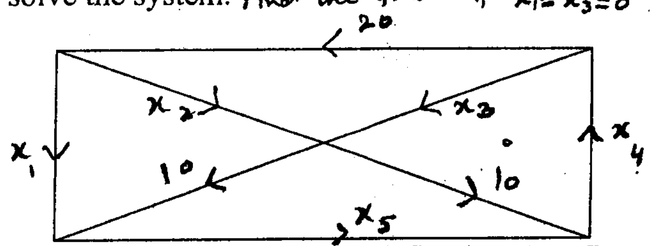
have nontrivial solutions? Find these solutions.

(b) Show that **(8)**

$$\begin{vmatrix} x & a & a & a & a\\ a & x & a & a & a\\ a & a & x & a & a\\ a & a & a & x & a\\ a & a & a & a & x\end{vmatrix} = 0$$

Q.7

(a) Set up a system of linear equations to represent the network shown in the diagram and solve the system. Find the flow if $x_1 = x_3 = 0$ **(8)**



(b) Let V be vector space of all functions defined on R to R. **(8)**
Check whether the vectors 2, $4\sin^2 x$, $\cos^2 x$ are linearly independent in V

Q.8

(a) Find the inverse of the matrix **(8)**

$$\begin{bmatrix} -1 & 2 & 3\\ 2 & 1 & 0\\ 4 & -2 & 5\end{bmatrix} \text{ over R}$$

(b) If 35,282 , 44,759 , 58,916 , 80,652 , and 92,469 are all multiples of 13 , show that **(8)**

$$\begin{vmatrix} 3 & 5 & 2 & 8 & 2\\ 4 & 4 & 7 & 5 & 9\\ 5 & 8 & 9 & 1 & 6\\ 8 & 0 & 6 & 5 & 2\\ 9 & 2 & 4 & 6 & 9\end{vmatrix}$$

is also a multiple of 13 .

Section III

Q.9

(a) Solve $e^x\,dx + (e^x \cot y + 2y \operatorname{cosec} y)\,dy = 0$ **(8)**

(b) Solve. $(D^4 + D^2)\,y = 3x^2 + 6\sin x - 2\cos x$ **(9)**

Q.10

(a) Solve: $\dfrac{dy}{dx} = \dfrac{2y - x + 5}{2x - y - 4}$ **(8)**

(b) Solve: $\dfrac{d^2y}{dx^2} + y = \sec^3 x$ **(9)**

Q.11

(a) Solve : $y'' - 8y' + 15y = 9xe^{2x}$, $y(0) = 5$, $y'(0) = 10$ **(8)**

(b) Use the method of undetermined coefficients to solve the differential equation **(9)**

$y'' - 3y' + 2y = 2x^2 + 2xe^x$

Q.12

(a) Solve : $p^2 y + (x - y)p - x = 0$ **(8)**

(b) Solve $x(2 + x)\dfrac{dy}{dx} + 2(1 + x)y = 1 + 3x^2$, $y(-1) = 1$ **(9)**

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