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University of Sargodha

B.A/B.Sc 1st Annual Examination 2007

Math General (New Course) Paper- B

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Attempt any two questions from each section.

Section- I

Q.1. (a) If $|\underline{a}| = |\underline{b}| = |\underline{a}+\underline{b}|$, find the angle between the vectors $\underline{a}$ and $\underline{b}$. (8)

(b) Find the scalar function φ such that (9)

$$\underline{\nabla}\varphi = (y^2 - 2xyz^3)\underline{i} + (3 + 2xy - x^2z^3)\underline{j} + (6z^3 - 3x^2yz^2)\underline{k}$$

Also show that $\underline{\nabla} \times \underline{\nabla}\varphi = 0$

Q.2. (a) Prove that if a positive term series: $\sum_{1}^{\infty} a_n$ Converges, then the series $\sum_{1}^{\infty}\sqrt{a_n a_{n+1}}$ converges. (9)

(b) Show that $(-1+i)^{i+\sqrt{3}} = e^x(Cosy + iSiny)$ where $x = \frac{\sqrt{3}}{2}\ln 2 - \frac{3\pi}{4} - 2n\pi$ and $y = \frac{1}{2}\ln 2 + \frac{3\sqrt{3}}{4}\pi + 2\sqrt{3}n\pi$, where n is an integer. (8)

Q.3. (a) If z_1 and z_2 are any complex numbers, then show that $||z_1| - |z_2|| \le |z_1 + z_2|$ (9)

(b) State Cauchy's Root test for examining the convergence or divergence of an infinite series. Use the test for convergence divergence of $\sum_{1}^{\infty}\frac{n^n}{n!}$. (8)

Q.4. (a) Find the sum of infinite series: (8)

$$1 + \frac{1}{2}Cos2\theta - \frac{1}{2.4}Cos4\theta + \frac{1.3}{2.4.6}Cos6\theta ----+---$$

(b) If $\underline{f}(t) = a\cos t\,\underline{i} + a\sin t\,\underline{j} + bt\,\underline{k}$ prove that (9)

i. $\left|\underline{f}'(t)\right|^2 = a^2 + b^2$ ii. $\left|\underline{f}'(t) \times \underline{f}''(t)\right|^2 = a^2(a^2+b^2)$

Section- II

Q.5. (a) For a Skew symmetric matrix A, prove that $(A^n)^t = (-1)^n A^n$ for any positive integer n. Deduce that if n is even then A^n is symmetric and if n is odd, then A^n is Skew symmetric. (8)

(b) Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ (8)

Q.6. (a) Reduce the following matrix into Echelon form and hence determine its rank. (8)

$$\begin{bmatrix} 1 & 2 & 2 & 3 & 4 \\ 2 & 1 & 3 & 2 & 1 \\ 0 & 2 & 1 & 1 & 3 \\ 3 & 1 & 3 & 4 & 2 \end{bmatrix}$$

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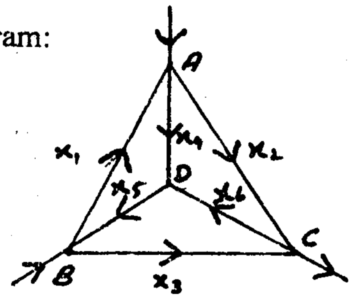
Sect. I Q. 1-4 2
— II " 5-8 "
— III " 9-12 2

(6)

(b) For what value of λ, the following system has Non-trivial solution. (8)

$$(3-\lambda)x_1 - x_2 + x_3 = 0$$
$$x_1 - (1-\lambda)x_2 + x_3 = 0$$
$$x_1 - x_2 + (1-\lambda)x_3 = 0$$

Q.7. (a) The flow through a network is as shown in the diagram: (8)



Solve the system. If $x_3 = 100, x_5 = 50, x_6 = 50$ Find the flow.

(b) Let V be the set of all ordered pairs of real numbers. Check whether V is a vector space or not over R (Set of real number) with respect to the indicated operations. $(a,b)+(c,d) = (a+c, b+d)$ and $k(a,b) = (ka, 0)$ (8)

Q.8. (a) Let $T: R^3 \to R^4$ be the linear Transformation defined by $T(x_1, x_2, x_3) = (x_2 + x_3, x_1 + x_2, x_1, x_1 - x_2)$. Find the matrix of T w.r.t. the base $B = \{v_1, v_2, v_3\}$ for R^3 and the base $F = \{w_1, w_2, w_3, w_4\}$ for R^4 where (8)
$v_1 = (1,1,0)$, $v_2 = (1,0,-1)$, $v_3 = (0,1,0)$
$w_1 = (1,-1,0,0)$, $w_2 = (1,0,1,0)$, $w_3 = (0,1,0,0)$, $w_4 = (0,0,1,1)$

(b) Prove that (8)

$$\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + dab \\ 1 & d & d^2 & d^3 + abc \end{vmatrix} = 0$$

Section- III

Q.9. (a) Find the differential equation of which the function: $x^2 + y^2 + 2gx + 2fy + c = 0$ is a solution of it. (8)

(b) Solve: $\dfrac{dy}{dx} = Sin(x+y) + Cos(x+y)$ (9)

Q.10. (a) Solve: $\dfrac{dy}{dx} + \dfrac{y}{2x} = \dfrac{x}{y^3}, y(1) = 2$ (8)

(b) Solve: $p^3 - 4xyp + 8y^2 = 0$ where $p = \dfrac{dy}{dx}$ (9)

Q.11. (a) Solve: $(D^3 + D^2 + 2D)y = x^2 + e^{-x}$ where $D = \dfrac{d}{dx}$ (8)

(b) Solve by the Method of Undetermined Coefficients: (9)

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = x^2 + 3Sinx$$

Q.12. (a) By the Method of Variation of parameters, find the solution of $\dfrac{d^2y}{dx^2} + y = Sec^3 x$ (9)

(b) Solve: $x^2 \dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} - 6y = 10x^2$ given that $y(1) = 1,\ y'(1) = 6$ (8)