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University of Sargodha

B.A / B.Sc 2nd Annual Exam 2010

Math General **Paper: A**

Time Allowed: 3 Hours **Maximum Marks: 100**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Let $f(x)=\begin{cases} x+2 & if \quad x\le -1 \\ ax^2 & if \quad x>-1 \end{cases}$ (9)

b. Evaluate $\lim_{x\to 1}\left\{\frac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}\right\}$ (8)

Q.2. a. Find $f'(x)$ *if* $f(x)=-\frac{\cos x}{2\sin^2 x}+\frac{1}{2}\tan\left(\frac{x}{2}\right)$ (9)

b. If $y=(\sin^{-1}x)^2$ then find $y^{(n)}(0)$ *i.e;* $y^{(n)}$ at $x=0$ (8)

Q.3. a. Find $x^2 f_{xx}-y^2 f_{yy}$ for $f(x,y)=\sin xy$ (9)

b. A boy is flying kite in a wind that is blowing it east at a rate of 50ft./min. He has already let out 200 feet of string and the kite is flying 100 feet above his hand. How fast must he let out the string at this moment to keep the kite flying with same speed and altitude? (8)

Q.4. a. Use Newton-Raphson method to approximate, upto four places of decimal, one root of $x^3-3x-3=0$ *at* $x_0=2$. (9)

b. If $V=p^m$ where $p^2=x^2+y^2+z^2$, show that (8)

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=m(m+1)p^{m-2}$$

Section- II

Q.5. a. Show that in any conic semi-latus rectum is the harmonic mean between segments of a focal chord. (8)

b. Show that the locus of the point of intersection of tangent at two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\sec^2\lambda$ where 2λ is the difference of the eccentric angles of the two points. (8)

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Q.6. a. Find pedal equation of $\frac{\ell}{r} = 1 + e\cos\theta$ (8)

b. Find equation of Tangent and Normal at $\theta = \pi/2$ to the cycloid (8)

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

Q.7. a. Find asymptotes of $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$ (8)

b. Find the point on the straight line $2x - 7y + 5 = 0$ which is closest to the origin. (8)

Q.8. a. Find position and nature of the singular points of the curve (8)

$$(2y + x + 1)^2 - 4(1 - x)^5 = 0$$

b. Find pedal equation of $p^2(a^2 + b^2 - r^2) = a^2b^2$ (8)

Section- III

Q.9. a. Calculate $\int_1^{10} \frac{dx}{(x-2)^{2/3}}$ (9)

b. Evaluate $\int \frac{dx}{1 + \sin x + \cos x}$ (8)

Q.10. a. Show that the shortest distance between the lines $x + a = 2y = -12z$ and $x = y + 2a = 6(z - a)$ is $2a$. (9)

b. If a, b, c are intercepts of a plane on the coordinate axes and r is the distance of the origin from the plane, prove that (8)

$$\frac{1}{r^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Q.11. a. Use trapezoidal rule to evaluate $\int_0^2 \frac{dx}{\sqrt{1 + x^2}}$ with $n = 4$ (9)

b. Evaluate $\int \frac{dx}{1 + x^4}$ (8)

Q.12. a. Find area of the region bounded by the loop of the curve $ay^2 = x^2(a - x)$ (9)

b. Show that length of arc of the curve $r\theta = a$ *from* $r = a$ *to* $r = 2a$ is (8)

$$a\left\{\sqrt{5} - \sqrt{2} + \ln\left(\frac{2 + \sqrt{8}}{1 + \sqrt{5}}\right)\right\}$$