(44) 188

Subject: Mathematics General **B.A/B.Sc.** **Time Allowed: 3 Hours**
Paper: A **II-A/05** **Maximum Marks: 100**

Note: Attempt **six** questions, selecting **two** questions from part- I, **two** questions from part- II, **one** question from part- III and **one** question from part- IV.

Part- I

Q.1. (a) Solve the inequality $x^{-2} - 4x^{-1} + 4 > 0$ (5)

(b) Let $f(x) = x^2$ and $g(x) = \begin{cases} -4 & if \quad x \le 0 \\ |x-4| & if \quad x > 0 \end{cases}$ Determine whether fog and gof are continuous at $x = 0$. (5)

(c) Find $y^{(n)}(0)$ if $y = \left(x + \sqrt{1+x^2}\right)^m$ (7)

Q.2. (a) Prove that $\frac{x}{x+1} < \ln(x+1) < x$ for all $x > 0$ (5)

(b) Use M.V.T. to prove that $|Sinx - Siny| \le |x - y|$ for all $x, y \in R$. (5)

(c) Determine a, b, c, d, e such that $\lim_{x\to 0} \frac{\cos ax + bx^3 + cx^2 + dx + e}{x^4} = \frac{2}{3}$ (7)

Q.3. (a) Oil spilled from a tanker spreads in a circle whose radius increases at the rate of 2 ft/sec. How fast is the area increasing when the radius of a circle is 40 feet? (8)

(b) Find the intervals in which the curve $y = 3x^3 - 40x^3 + 3x - 20$ faces (9)

i. Upward ii. Down ward. Also find the points of inflection.

Q.4. (a) Find Asymptotes of the curve $(x - y + 1)(x - y - 2)(x + y) = 8x - 1$ (8)

(b) Show that for the parabola $y = ax^2 + bx + c$, the radius of curvature f is minimum at its vertex. (9)

Part- II

Q.5. (a) Evaluate $\int \frac{x^2+1}{(x+1)^2} e^x dx$ (5)

(b) $\int \frac{x^4}{x^4 + 2x^2 + 1} dx$ (6)

(c) $\int \frac{1}{1 + Sinx + Cosx} dx$ (6)

Q.6. (a) Prove that $\int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}$ (8)

45

(b) Show that $\int \sec^{2n+1} x \, dx = \dfrac{\sec^{2n-1} x \tan x}{2n} + \left(1 - \dfrac{1}{2n}\right) \int \sec^{2n-1} x \, dx$ (9)

Q.7. (a) Sketch the curve $r = a\sin 2\theta$. (8)

(b) Find area of region bounded by the cardioid $r = a(1+\cos\theta)$ (9)

Q.8. (a) Find the length of one arch of the cycloid $x = a(\theta - \sin\theta),\ y = a(1-\cos\theta)$ (8)

(b) Find the surface area of a sphere of radius r. (9)

Part- III

Q.9. Discuss the convergence and divergence of series. (5,5,6)

(a) $\sum_{0}^{\infty} \dfrac{2^n + n}{(n+1)!}$ (b) $\sum_{2}^{\infty} \dfrac{1}{n(\ell_{n} n)^p}$ (c) $\sum_{1}^{\infty} \dfrac{\sqrt{n}}{2^n}$

Q.10. (a) State the alternating series test. ( )

(b) Test the series. $\sum_{1}^{\infty} \dfrac{(-1)^{n-1} 2\sinh n}{e^{2n}}$ ( )

For the

(i) Absolute convergence (ii) Conditional convergence

(iii) Divergence

(c) Find the interval of convergence and radius of convergence of the series. (7)

$$\sum_{0}^{\infty} \frac{x^n}{(2n)!}$$

Part- IV

Q.11. (a) If $U = \ln\left(\dfrac{x^2+y^2}{x+y}\right)$ Prove that $x\dfrac{\partial U}{\partial x} + y\dfrac{\partial U}{\partial y} = 1$. (8)

(b) Let (8)

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & if \quad (x,y) \neq (0,0) \\ 0 & if \quad (x,y) = (0,0) \end{cases}$$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

Q.12. (a) Find three positive numbers whose sum is 48 and whose product is as large as possible. (8)

(b) If $U = \sqrt{x+2y}$ and x changes from 3 to 2.98 while y changes from 0.5 to 0.51 find an approximate value for the change in U. (8)
