

Note: Attempt any two questions from each section.

Section- I

Q.1. a. Discuss the continuity of the function (9)

$$f(x) = \begin{cases} \frac{x^2}{a} - a & \text{if } 0 < x < a \\ 0 & \text{if } x = 0 \\ a - \frac{a^2}{x} & \text{if } x > a \end{cases} \quad \text{at } x = a$$

b. Evaluate $\lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \ln(1-x)}{x \tan^2 x}$ (8)

Q.2. a. Find $\frac{dy}{dx}$, if $y = x^x e^x \sin x (\ln x)$. (9)

b. Find a root of the equation $x \sin x + \cos x = 0$ with $x_0 = \pi = 3.1416$ by Newton Raphson method. (8)

Q.3. a. State and prove Cauchy's mean value theorem. (9)

b. Given $x^2 y'' + xy' + y = 0$ show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$ (8)

Q.4. a. If $u = \arctan \frac{x^3 + y^3}{x - y}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ (9)

b. Find the percentage error in the area of an ellipse when an error of 1% is made in measuring each of major and minor axes of the ellipse. (8)

Section- II

Q.5. a. Determine whether the given equation represents two straight lines. If so find the equations. $6x^2 - 17xy - 3y^2 + 22x + 10y - 8 = 0$ (8)

b. Show that pedal equation of the asteroid $x = a \cos^3 \theta, y = a \sin^3 \theta$ is $r^2 = a^2 - 3p^2$ (8)

Q.6. a. Identify and graph the given polar equation $r = \frac{10}{2 + 3 \cos \theta}$ (8)

b. Find relative extrema for $f(x) = \frac{\ln x}{x}$ (8)

Q.7. a. Find the asymptotes of $2x^3 - x^2 y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$ (8)

b. Prove that for the Cardioid $r = a(1 + \cos \theta)$ $\frac{P^2}{r}$ is constant. (8)

- Q.8. a. Prove that the evolute of the hyperbola $2xy = a^2$ is $(x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$ (8)
- b. Maximize $z = 10x_1 + 11x_2$ with the condition (8)
- $$3x_1 + 4x_2 \leq 9$$
- $$5x_1 + 2x_2 \leq 8$$
- $$x_1 - 2x_2 \leq 1$$
- $$x_1 \geq 0, x_2 \geq 0$$

Section- III

- Q.9. a. Use Trapezoidal rule to approximate the integral $\int_0^2 \frac{dx}{1+x^3}$ with $n = 4$ (9)
- b. Show that $\int_0^{\frac{\pi}{2}} \sin 2x \ln(\tan x) dx = 0$ (8)
- Q.10. a. Evaluate $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ (9)
- b. Evaluate $\int \frac{dx}{x^4+1}$ (8)
- Q.11. a. Find the length of the cycloid $x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$ (9)
- b. Find the area of the region bounded by the loop of the curve $ay^2 = x^2(a-x)$ (8)
- Q.12. a. Find an equation of the plane through the points (1, 0, 1) and (2, 2, 1) and perpendicular to the plane $x - y - z + 4 = 0$ (9)
- b. Find a symmetric form for the line $x + y + z + 1 = 0$; $4x + y - 2z + 2 = 0$ (8)