

Note: Attempt any two questions from each section.

Section- I

- Q.1. (a) Solve D.E. $(e^x + 1)ydy = (y + 1)e^x dx$ (8)
(b) Solve the initial value problem $\frac{dy}{dx} = \frac{2x}{y+x^2y}$ $y(0) = -2$ (9)
- Q.2. (a) Solve D.E. $dx + \left(\frac{x}{y} - \sin y\right) dy = 0$ (8)
(b) Solve D.E. $(x + 2y^3)\frac{dy}{dx} = y$ (9)
- Q.3. (a) Find an equation of orthogonal trajectories of the curve $\gamma^2 = a \sin 2\theta$ (8)
(b) Solve $P^2 + P - 6 = 0$ (9)
- Q.4. (a) Find singular solution of $x^3 P^2 + x^2 y P + a^3 = 0$ (8)
(b) Solve $(x^2 D^2 - 2xD + 2)y = x^3$ where $D = \frac{d}{dx}$ (9)

Section- II

- Q.5. (a) Compute i. $\mathcal{L}\{t^3 - 1\}e^{-2t}$ ii. $\mathcal{L}\{e^{-t} \sin 2t\}$ (8)
(b) Compute inverse Laplace transform $\frac{1}{(s-1)(s^2+4)}$ (8)
- Q.6. (a) Use the Newton-Raphson method to approximate up to four places $\sin x = 1 - x$ with $x_0 = 0$ (8)
(b) Use the trapezoidal rule to approximate the integral $\int_1^4 \frac{dx}{x} = \ln 4$ with $n = 3$ (8)
- Q.7. (a) Use Simpson's rule to approximate the integral $\int_{-1}^2 x^5 dx$ with $n = 10$ (8)
Also find bound error.
(b) Evaluate $\mathcal{L}^{-1}\left\{\tan^{-1} \frac{a}{s}\right\}$ (8)
- Q.8. (a) Solve $\frac{d^2 y}{dt^2} - 2\frac{dy}{dt} + y = te^t$ I.C. $y(0) = 0$ $y'(0) = 0$ by method of Laplace transform. (8)
(b) Use trapezoidal rule to approximate $\int_1^2 \ln x dx$ with $n = 4$ (8)

Section- III

- Q.9. (a) A straight line makes angles of measure $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$ (8)
(b) Find an equation of the plane through the points $P(4, -1, 2), Q(-3, -2, -1), R(7, -1, 3)$ (9)
- Q.10. (a) Show that the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is perpendicular to the plane $4x + 8y + 12z + 19 = 0$ (8)
(b) Find an equation of the cone whose directrix is $y^2 = x, z = 4$ and vertex is at $A(0, 2, 0)$ (9)
- Q.11. (a) An equation in cylindrical coordinates is $r^2 \cos 2\theta = z$ Transform the equation into rectangular coordinates. (8)
(b) Find the direction of Qibla of the Badshahi Mosque, Lahore, Latitude = $31^\circ 35.4' N$ and Longitude = $74^\circ 18.7' E$ (9)
- Q.12. (a) Find the eigen values and eigen vectors $A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$ (8)
(b) Find a real orthogonal matrix P which $P^{-1}AP$ is diagonal matrix where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (9)