University of Sargodha

B.A/B. Sc. 1st Annual Exam 2017.

Subject: B Course of Math **Paper: A**

Total Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section–1

Q.1. a. Prove that derivative of a vector $\vec{a}$ of constant magnitude is orthogonal to $\vec{a}$. (8).

b. If r= xi +yj+zk then, show that div (grad r^m) $= m(m+1)\, r^{m-2}$. (9).

Q.2. a. Differentiate, $\frac{i+tj+t^2k}{|i+tj+t^2k|}$. (8).

b. Show that for vector fuction $f(t)$, $\frac{d}{dt}(f \cdot f' \times f'') = f' \cdot f'' \times f'''$ (9).

Q.3. a. If the forces of magnitude P and Q acting at an angle θ be interchanged in position, show that the resultant turns through an angle ϕ where $\tan\frac{\phi}{2} = \frac{P-Q}{P+Q}\tan\frac{\theta}{2}$ (8).

b. Forces 3,4 and 7 units act along the sides AB, CB, CA of an equilateral triangle. Taking A as origen and CA as x-axis, Find the magnitude and line of action of resultant. (9).

Q.4. a. If forces $p\vec{AB}$, $q\vec{CB}$, $r\vec{CD}$, $s\vec{AD}$ acting along the sides of a plane quadrilateral, show that $pr = qs$. (9).

b.Forces 1,2,3,5,P,Q units act along the lines AB, BC,CD, DA, AC and BD respectively of a square ABCD of side a. Find the magnitude of P and Q for the system to reduce to a couple find also the moment of couple. (8).

Section–2

Q.5.a. AB and AC are similar uniform rods of length a smoothly joined at A. BA is a weightless bar of length b smoothly joined at B and fastened at D to a smooth ring sliding on AC, the system is hung on a small smooth pin at A. Show that the rod AC makes with the vertical an angle $\tan^{-1}\left[\frac{b}{a+\sqrt{a^2-b^2}}\right]$. (8).

b. Find the C.G of a plate of uniform density forming a quadrant of an ellipse bounded by its semi-axes. (8).

Q.6. a. Find the c.m of a hallow right circular cone of semi- vertical angle α and hight h. (8).

b. Find the least force to drag a partical up the rough inclined plane. (8).

Q.7. a. The smallest force which can support a body of weight W on a smooth inclined plane, is of magnitude P, Show that the horizontal force necessary the same body on the same plane is of magnitude $\frac{PW}{\sqrt{W^2-P^2}}$. (8).

b. Arhombus ABCD is formed of four equal uniform rods freely jointed together and suspended form A. It is kept in position by a light rod joining the middle points of BC and CD; prove that if T be the thrust in this rod and W the weight of the rhombus, then $T = w\tan\frac{A}{2}$. (8)

Q.8. a. A uniform rod of length 2a and weight w rests with its middle point upon a rough horizontal cylinder whose axis is perpendicular to the rod . show that the greatest weight that can be attached to one end of the rod without sliding it off the cylinder is $\frac{b\lambda}{a-b\lambda}$w, where b is the radius of the cylinder and λ the angle of friction. (8).

b. A heavy elatic string whose natural length is $2\pi a$, is placed round a smooth cone whose axis is vertical and whose semi-vertical angle has measure α. If W be the weight and λ the modulus of the string, prove that it will be equilibrium when in the form of a circle of radius $a(1+\frac{W}{2\pi\lambda}\cot\alpha)$. (8).

Section–3

Q.9.a. Find the tagetial and normal components of velocity and acceleration. (8)

b. A particle moves in a plane such that the squre of its tangential velocity is proportional to its narmal acceleration, show that it discribes a circle. (9)

Q.10.a. A particle moves in a straight line OAB with simple hormonic motion. it is at rest at A and B and $OA = a$ and $OB = b$, its velocity is v when it is the middle point of AB. Show that the period is $\frac{\pi(b-a)}{v}$ (8).

b. Discuss the motionof a particle moving in a straight line if it starts from rest at a distance a from a point O and moves with an acceleration equal to μ times its distance from O. (9).

Q.11.a. Find the rang of projectile on an inclined plane. (9)

b. A projectile is launched at an angle α from a cliff of hight H above sea level . If it falls into the sea at a distance D from the base of the cliff, prove that themaximum hight above is $H + \frac{D^2\tan^2\alpha}{4(H+D\tan\alpha}$. (8).

Q. 12. a. Find the differential equation of the orbit in polar co-ordinates. (8)

b. Find the law of force when the particle describes the curve $r^n = A\cos n\theta + B\sin n\theta$. (9).