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University of Sargodha

B.A / B.Sc 2nd Annual Exam 2010

Applied Math **Paper: B**

Time Allowed: 3 Hours **Maximum Marks: 100**

Note: **Attempt six questions in all, selecting two questions from each section.**

Section- I

Q.1. a. Prove that in a quadrilateral the straight lines joining the mid points of the opposite sides bisect each other. (8)

b. Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at point (2, 0, 3) in the direction of $2\underline{i} - \underline{j}$. (9)

Q.2. a. Show that the necessary and sufficient condition for a vector function f of a scalar variable t to have constant magnitude is $\underline{f}.\frac{d\underline{f}}{dt} = 0$ (8)

b. Prove that if a tensor is anti-symmetric in one co-ordinate system, then it will be anti-symmetric in every other co-ordinate system. (9)

Q.3. a. Three forces P, Q, R acting at a point, are in equilibrium, and the angle between P and Q is double the angle between P and R. Prove that $R^2 = Q\,(Q - P)$ (8)

b. If forces $\ell\overrightarrow{AB}, m\overrightarrow{BC}, \ell\overrightarrow{CD}, m\overrightarrow{DA}$ acting along the sides of a quadrilateral are equivalent to a couple, show that either $\ell = m$ or $ABCD$ is a parallelogram. (9)

Q.4. a. A uniform square lamina of side $2a$ rests in a vertical plane with two of its sides in contact with two smooth pegs distant b apart, and in the same horizontal line. Show that, if $\frac{a}{\sqrt{2}} < b < a$, a non symmetrical position of equilibrium is possible in which $b(\sin\theta + \cos\theta) = a$ where θ is inclination of a side of the square to the horizontal. (8)

b. A triangular lamina ABC, right angled at A , rests with its plane vertical, and with the side AB, AC supported by smooth fixed pegs D,E in a horizontal line. Prove that the inclination θ of AC to the horizontal is given by $AC\cos\theta - AB\sin\theta = 3DE\cos 2\theta$. (9)

Section- II

Q.5. a. Weights of 1, 2, 3, 4 lb. are placed at the corners A, B, C,D of a square of side 8 inches. Find the distances of the centre of gravity of the set from AB and AD. (8)

b. A hexagon ABCDEF, consisting of six equal heavy rods, of weight W, freely jointed together, hangs in a vertical plane with AB horizontal, and the frame is kept in the form of a regular hexagon by a light rod connecting the mid points of CD and EF. Show that the thrust in the light rod is $2\sqrt{3}\,W$. (8)

Q.6. a. A rod, 4 ft. long, rests on a rough floor against the smooth edge of a table of height 3ft. if the rod is on the point of sliding when inclined at an angle of 60° to the horizontal, find the coefficient of friction. (8)

b. A uniform ladder rests in limiting equilibrium with one end on a rough horizontal plane, and the other against a smooth vertical wall. A man ascends the ladder. Show that he cannot go more than half way up. (8)

Q.7. a. Find the centroid of the surface formed by the revolution of the cardiod $r = a(1 + \cos\theta)$ about the initial line. (8)

b. Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed v when the particle is at (0, b). (8)

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Q.8. a. A particle moving along a straight line starts from rest and is accelerated uniformly till it attains a velocity v. The motion is then retarded and the particle comes to rest after traversing a total distance X. if the acceleration is f, find the retardation and the total time taken by the particle from rest to rest. (8)

b. A particle describes simple harmonic motion with frequency N. if the greatest velocity is v, find the amplitude and the maximum value of acceleration. (8)
Also show that the velocity v at a distance x from the centre of motion is given by $v = 2\pi N\sqrt{a^2 - x^2}$ where a is the amplitude.

Section- III

Q.9. a. Find the equation of parabola of safety of a projectile. Find its focus and directrix. (8)

b. From a gun placed on a horizontal plane, which can fire a shell with speed $\sqrt{2gH}$, it is required to throw a shell over a wall of height h, and the elevation of the gun cannot exceed $\alpha < 45°$. Show that this will be possible only when $h < H\sin^2\alpha$, and that, if this condition be satisfied, the gun must be fired from within a strip of the plane whose breadth is $4\cos\alpha\sqrt{H(H\sin^2\alpha - h)}$. (9)

Q.10. a. A particle of unit mass describes an ellipse under the action of central force Mr. Show that the normal component of acceleration at any instant is $\dfrac{abM^{\frac{3}{2}}}{v}$, where v is the velocity at that instant and a, b the semi-axes of the ellipse. (8)

b. A particle moves under a central repulsive force $\dfrac{\mu}{r^3}$ and is projected from an apse at a distance a with velocity v. show that the equation to the path is $r\cos p\theta = a$, and that the angle θ described in time t is $\dfrac{1}{p}\tan^{-1}\dfrac{pvt}{a}$ where $p^2 = \dfrac{\mu + a^2v^2}{a^2v^2}$. (9)

Q.11. a. If a, b, c are the intercepts of a plane on co-ordinate axes and r is the distance of the origin from the plane, prove that $\dfrac{1}{r^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2}$. (8)

b. Show that the shortest distance between the straight lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$ is $\dfrac{1}{\sqrt{6}}$ and equations of the straight line perpendicular to both are $11x + 2y - 7z + 6 = 0 = 7x + y - 5z + 7$ (9)

Q.12. a. Find an equation of the sphere for which the circle (8)

$$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y + 4z - 8 = 0$$

is a great circle.

b. Find the direction of Qibla for Peshawar with given data: (9)

Place	Latitude	Longitude
Khana-e-Kaaba	$21°25.2'N$	$39°49.2'E$
Peshawar	$34°1'N$	$71°40'E$
