University of Sargodha

5261

B.A/B. Sc. 1st Annual Exam 2014.

Subject: Applied Math Paper: B

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Show by using vectors, the medians of a triangle are concurrent. (8)

b. Find the two unit vectors which make an angle of 60° with both vectors $[1,-1,0]$ and $[1,0,-1]$ (9)

Q.2. a. If f is a vector function of t, prove that $\frac{d}{dt}\left(\frac{f}{|f|}\right) = \frac{f'(f.f) - f(f.f')}{(f.f)^{\frac{3}{2}}}$ (8)

b. Prove that if a tensor is symmetric in one co-ordinate system, then it will be symmetric in every other co-ordinate system. (9)

Q.3. a. Forces $2\overrightarrow{BC}, \overrightarrow{CA}, \overrightarrow{BA}$ act along the sides of a triangle ABC. Show that their resultant is $6\overrightarrow{DE}$, where D bisects BC and E is a point on CA such that $CE = \frac{1}{3}CA$. (8)

b. Three forces P, Q, R act along the sides BC, CA, AB respectively of a triangle ABC. Prove that, if $PsecA + QsecB + RsecC = 0$, then the line of action of the resultant passes through the orthocentre of the triangle. (9)

Q.4. a. A triangular lamina ABC is suspended from a point 0 by light strings fastened to the points A, B and hangs so that the side BC is vertical. Prove that, if α, β are angles which AO, BO make with the vertical, then $2\cot\alpha - \cot\beta = 3\cot B$. (8)

b. A body of weight W is suspended by two equal threads AP, BQ, the points of support A, B are on the same level at a distance a apart and the threads are fastened to two points P, Q on the body so that PQ is horizontal and $PQ = b$. A couple G is applied about a vertical axis and the body is deflected through an angle θ with PQ at a depth h below AB. Show that $G = \frac{1}{4}W\left(\frac{ab}{h}\right)\sin\theta$ (9)

Section- II

Q.5. a. Find the centre of gravity of a semi-circular lamina of radius r when the density varies as the cube of the distance from the centre. (8)

b. Show that the centre of mass of a segment of a solid sphere of radius a, at a distance b from the centre of the sphere is at a distance $\frac{3}{4}\frac{(a+b)^2}{2a+b}$ (8)

Q.6. a. Find the force necessary just to support a heavy particle on an inclined plane of inclination $\alpha(\alpha > \lambda)$ (8)

b. A uniform semi-circular wire hangs on a rough peg, the line joining its extremities making an angle of 45° with the horizontal. If it is just on the point of slipping, find the co-efficient of friction between wire and peg. (8)

Q.7. a. Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed V when the particle is at $(0, b)$. (8)

b. State and prove the principle of virtual work done for single particle. (8)

Q.8. a. A particle projected vertically upwards is $t = 0$ with a velocity u, passes a point at a height h at $t = t_1$ and $t = t_2$. Show that $t_1 + t_2 = \frac{2u}{g}$ and $t_1 t_2 = \frac{2h}{g}$ (8)

b. A particle moving in a straight line starts with a velocity u and has acceleration v^3, where v is the velocity of the particle at time t. Find the velocity and the time as functions of the distance travelled by the particle. (8)

Section- III

Q.9. a. Find the least speed with which a particle must be projected so that it passes through two points P and Q at heights h_p and h_q. (8)

b. A cannon has maximum range R. prove that (a) the height reached is $\frac{1}{4}R$, and the time of flight is $\sqrt{\frac{2R}{g}}$. (9)

Q.10. a. Prove that when a particle moves under a central force, the areal velocity is constant. (9)

b. If a particle be describing an ellipse about a centre of force in the centre, show that the sum of the reciprocates of its angular velocities about the foci is constant. (8)

Q.11. a. Prove that the straight lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ intersect. Also find the point of intersection and the plane through them. (9)

b. Show that the shortest distance between the straight lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and the equation of the straight line perpendicular to both are $11x + 2y - 7z + 6 = 0 = 7x + y - 5z + 7$ (8)

Q.12. a. A sphere of radius K passes through the origin and meets the axes in A, B, C. Prove that centriod of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4K^2$ (8)

b. Find the direction of Qibla of the Badshahi Mosque, Lahore. (9)