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# University of Sargodha

**B.A/B. Sc 1st Annual Examination 2012**

**Applied Math** **Paper: B**



**Maximum Marks: 100** **Time Allowed: 3 Hours**

**Note:** **Attempt any two questions from each section.**

## Section- I

**Q.1.** a. Prove by using vectors that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ (8)

b. Prove that the necessary and sufficient condition for a vector $\underline{a}$ to have a constant magnitude is $\underline{a}.\frac{d\underline{a}}{dt} = 0$. (9)

**Q.2.** a. Show by using vectors, that the medians of the triangle are concurrent. (8)

b. Prove that if $\phi$ is scalar and $A_{ij}$ is a second order tensor, then $C_{ij} = \phi A_{ij}$ is also a second order tensor. (9)

**Q.3.** a. State and prove the $(\lambda-\mu)$ –theorem. (8)

b. A system of forces acts on a plane in the form of an equilateral triangle of side $2a$. The moments of forces about three vertices are $G_1, G_2, G_3$ respectively. Find the magnitude of the resultant. (9)

**Q.4.** a. The smallest force which can support a body of weight $w$ on a smooth inclined plane, is of magnitude $P$; show that the horizontal force necessary to support the same body on the plane is of magnitude $\frac{PW}{\sqrt{W^2-P^2}}$ (8)

b. A triangular lamina $ABC$, right angled at A, rests with its plane vertical, and with the sides $AC$, $AB$ supported by smooth fixed pegs $D$, $E$ in a horizontal line. Prove that the inclination $\theta$ of $AC$ to the horizontal is given by $AC\cos\theta - AB\sin\theta = 3DE\cos 2\theta$ (9)

## Section- II

**Q.5.** a. Two uniform solid spheres, composed of the same material and whose diameters are 6 in. and 12 in. respectively, are firmly united. Find the c.m of the combined body. (8)

b. A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and on a smooth peg at a distance b from the wall. Show that, in the position of equilibrium, the beam is inclined to the wall at angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$. (8)

**Q.6.** a. A uniform ladder rests in limiting equilibrium with one end on a rough horizontal plane, and the other against a smooth vertical wall. A man ascends the ladder. Show that he cannot go more than half way up. (8)

b. The least force which will move a weight up an inclined plane is of magnitude P. Show that the least force, acting parallel to the plane, which will move the weight upwards is $P\sqrt{1+\mu^2}$ where $\mu$ is the co-efficient of friction. (8)

**Q.7.** a. Find the position of the centriod of a quadrant of an elliptic lamina. (8)

b. A particle moves in a plane in such a way that at any time $t$, its distance from a fixed point $O$ is $r = at + bt^2$ and the line connecting $O$ and $P$ makes angle $\theta = ct^{\frac{3}{2}}$ with a fixed line $OA$. Find the radial and transverse components of velocity and acceleration of the particle at $t = 1$. (8)

**Q.8.** a. A particle is projected vertically upwards. After a time $t$, anther particle is sent up from the same point with the same velocity and meets the first at height $h$ during the downward flight of the first. Find the velocity of projection. (8)

b. A point describes simple harmonic motion in such a way that its velocity and acceleration at point $P$ are $u$ and $f$ respectively and the corresponding quantities at another point $Q$ are $v$ and $g$. Find the distance $PQ$. (8)

## Section- III

**Q.9.** a. Prove that the speed required to project a particle form a height h to fall a horizontal distance $a$ from the point of projection is at least $\sqrt{g\left(\sqrt{a^2+h^2}-h\right)}$ (8)

b. A projectile having horizontal range $r$, reaches a maximum height $H$. Prove that it must have been launched with (a) an initial speed equal to $\sqrt{\frac{g(R^2+16H^2)}{8H}}$ and (b) at an angle with the horizontal given by $Sin^{-1}\left(\frac{4H}{\sqrt{R^2+16H^2}}\right)$. (9)

**Q.10.** a. A particle describes the curve $r^n = A\cos n\theta + B\sin n\theta$ under the force $F$ to the pole, show that $F \propto \frac{1}{r^{2n+3}}$. (8)

b. If a particle be describing an ellipse about a centre of force in the center, show that the sum of the reciprocates of its angular velocities about foci is constant. (9)

**Q.11.** a. Find the condition that two straight lines $\frac{x-\alpha}{c_1} = \frac{y-\beta}{c_2} = \frac{z-\gamma}{c_3}$ and $\frac{x-\alpha'}{d_1} = \frac{y-\beta'}{d_2} = \frac{z-\gamma'}{d_3}$ may be coplanar. Also find an equation of the plane containing them. (8)

b. Show that the shortest distance between the straight lines. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and equations of the straight line perpendicular to both are $11x + 2y - 7z + 6 = 0 = 7x + y - 5z + 7$ (9)

**Q.12.** a. A sphere of radius K passes through the origin and meets the axes in $A$, $B$, $C$. Prove that the centriod of the triangle $ABC$ lies on the sphere $9(x^2+y^2+z^2) = 4k^2$ (8)

b. Find the direction of Qibla of the Badshai Mosque, Lahore, latitude $= 31°35.4'$ and longtitude $= 74°18.7'E$. (9)

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