(27)

University of Sargodha

B.A/B.Sc 1st Annual Examination 2011

Pure Math **Paper: B**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Evaluate $\int \frac{dx}{(x^2-1)\sqrt{x^2+1}}$ (8)

b. Evaluate $\int_a^b x^k dx$ by definition. (9)

Q.2. a. Evaluate $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$ (8)

b. Find the asymptotes of the curve $y(x-y)^2 = x+y$ (9)

Q.3. a. Find the point of inflection of the curve $a^2y^2 = (a^2-x^2)x^2$ (8)

b. Find the length of the loop of the curve $3ay^2 = x(a-x)^2$ (9)

Q.4. a. Evaluate $\int_0^{2\pi}\int_0^{1-\cos\theta} \gamma^3 \cos^2\theta \, d\gamma \, d\theta$ (8)

b. Prove that intrinsic equation of the cycloid (9)

$$x = a(\theta + \sin\theta),\ y = a(1-\cos\theta) \quad is \quad S = 4a\sin\alpha$$

Section- II

Q.5. a. Let (X, d) be a metric space. Show that d_1, defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$ (8)

is also a metrix on X.

b. Let $(X, \mathcal{J})$ be a topological space and let O be a subset of X, then O is open if and only if it is a neighbourhood of each of its points. (8)

Q.6. a. Prove that co-finite topology on X is discrete if and only if X is finite. (8)

b. Prove that a subset U of a metric space X is open if and only if U is a union of open spheres. (8)

Q.7. a. Let $(X, \mathcal{J})$ be a topological space and let A be any subset of X. then A^o is an open set and it is largest open set contained in A. (8)

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b. Let $(X, \mathcal{T})$ be topological space and let A be any subset of X. then prove that (8)

$F_r(A) \cap A^\circ = \phi$

Q.8. a. Let (X, d) be a metric space and $\{F_\alpha : \alpha \in I\}$ be a family of closed sub sets of X. (8)

prove that i. $\cap\{F_\alpha : \alpha \in I\}$ is closed set ii. $F_1 \cup F_2 \cup F_3 \cup \ldots\ldots \cup F_n$ is closed ,

each $F_i, i = 1,2,3\ldots\ldots, n$ being closed.

b. Let $(X, \mathcal{T})$ be a topological space and let A, B be any subsets of X. Prove that (8)

$(A \cap B)^\circ = A^\circ \cap B^\circ$

Section- III

Q.9. a. If G be a cyclic group of order n generated by a. then prove that for each positive divisor d of n, there is a unique subgroup of G of order d. (8)

b. If the matrices A, B and C are conformable for the product, then prove that (9)

$A(BC) = (AB)C$

Q.10. a. (8)

Show that $\begin{vmatrix} 1 & w & w^2 & w^3 \\ 1 & w^2 & w^4 & w \\ 1 & w^3 & w & w^4 \\ 1 & w^4 & w^3 & w^2 \end{vmatrix} = 125,$ where w is a fifth root of 1.

b. Let G be a group such that $(ab)^n = a^n b^n$ for three consecutive natural numbers n and all $a, b \in G$. Show that G is abelian. (9)

Q.11. a. Let G be a group and H is a subgroup of G. then prove that the set (8)

$aHa^{-1} = \{aha^{-1} : h \in H\}$ is subgroup of G.

b. Find an equation defining the subspace W of R^3 spanned by $V_1 = (1, -3, 2)$, $V_2 = (-2, 1, 2)$, $V_3 = (-3, -1, 6)$ by expressing an arbitrary element $(x, y, z) \in R^3$ as a linear combination of V_1, V_2, V_3. (9)

Q.12. a. Examine the following homogenous system for nontrivial solution. (8)

$$x_1 - x_2 + 2x_3 + x_4 = 0$$
$$3x_1 + 2x_2 + x_4 = 0$$
$$4x_1 + x_2 + 2x_3 + 2x_4 = 0$$

b. Express the vector (2, -5, 3) in R^3 as a linear combination of the vectors (1, -3, 2), (2, -4, -1) and (1, -5, 7) (9)