University of Sargodha

B.A/B. Sc. 1st Annual Exam 2019.

Subject: A Course of Math **Paper: B**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Attempt any two questions from each section.

Section-I

Q.1. a. Evaluate $\int \frac{\sec x}{1+\operatorname{cosec} x}dx$ (8)

b. Evaluate by definition $\int \sin^2 x dx$. (9)

Q.2. a. Show that $\int_0^\pi x\ln(\sin x)\mathrm{d}x = \pi$. (8)

b. Find the asymptote of the curve $r^n = a^n \operatorname{cosec} n\theta$. (9)

Q.3. a. Calculate the area of the region bounded by the loop of the curve $y^2(a+x) = x^2(a-x)$. (8)

b. Find the length of the arc of parabola $y^2 = 4ax$ cut off by a straight line $3y = 8x$. (9)

Q.4. a. Prove that radius of the curvature at the point $(2a, 2a)$ on the curve $x^2 y = a(x^2+y^2)$ is $2a$. (8)

b. Evaluate $\int_2^4 \int_y^{8-y} y dx dy$. (9)

Section-II

Q.5. a. If (X, d) is a metric space, $|d(x,z)-d(y,z)| \le d(x,y)\ \forall x,y,z \in X$. (8)

b. Let X be topological space. Then prove that $\overline{A\cup B} = \overline{A}\cup\overline{B}$ where A and B are subset of X. (8)

Q.6. a. Let X be a metric space, then intersection of any finite number of open sets in X is open. (8)

b. If a, b are nonzero integers such that $(a,b) = d$, then $\left(\frac{a}{b}, \frac{b}{d}\right) = 1$. (8)

Q.7. a. State and prove Euclidian Theorem. (8)

b. Prove that Union as well as intersection of two closed sets is closed in topological space. (8)

Q.8. a. Prove that a subset A of X is open, iff A is neighborhood of each of its points in metric space. (8)

b. Find limit points of the subset of a discrete metric space. (8)

Section-III

Q.9. a. If G is an abelian group, show that $(ab)^n = a^n b^n\ \forall a,b \in G$ (8)

b. Define Periodic matrix and show that $A = \begin{bmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{bmatrix}$ is periodic of period 2. (9)

Q.10. a. Find all subgroup of the cyclic group of order "60" generated by "a". (8)

b. Show that the system $2x-y+3z=a;\ 3x+y-5z=b;\ -5x-5y+21z=c;$ is inconsistent if $c \ne 2a-3b$ (9)

Q.11. a. State and Prove Lagrange Theorem. (8)

b. If A is invertible and $AB = 0$ then Show that $B = 0$. (9)

Q.12. a. If A and B are 3×3 matrices such that $\det(A^2B^3) = 108$ and $\det(A^3B^2) = 72$ find $\det(2A)$ and $\det(B^{-1})$. (8)

b. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$. (9)