University of Sargodha

B.A/B. Sc. 1st Annual Exam 2017.

Subject: A Course of Math **Paper: A**

Total Marks: 3 Hours **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. (a) Differentiate $y=(\tan x)^{\cot x}+(\cot x)^{\tan x}$ with respect to x. (8)

(b) If $U=f(x,y)$ is a homogenous function of degree n, prove that (9)

$$x^2 f_{xx}+2xy\, f_{xy}+y^2 f_{yy}=n(n-1)f$$

Q.2. (a) Show that the sphere $x^2+y^2+z^2=18$ and the cone $x^2+z^2=(y-6)^2$ are tangent along their intersection. (8)

(b) Use L'Hospital's rule to prove that $\lim_{x\to\infty}\left[\frac{a^{\frac{1}{x}}+b^{\frac{1}{x}}}{2}\right]^x=\sqrt{ab}, a>0, b>0$ (9)

Q.3. (a) Prove that $\left||a|-|b|\right|\leq|a-b|$ for every $a,b\in R$ (8)

(b) If $y=e^{m\,arc\sin x}$ (9)

show that $(1-x^2)y^{(n+2)}-(2n+1)xy^{n+1}-(n^2+m^2)y^{(n)}=0$

find the value of $y^{(n)}$ at $x=0$.

Q.4. (a) Use mean value theorem to show that $\frac{1}{6}<\sqrt{27}-5<\frac{1}{5}$. Also approximate $\sqrt{168}$ by the mean value theorem. (8)

(b) Find the values of a and b so that the function f is continuous and differentiable at $x=1$ where $f(x)=\begin{cases}x^3 & if \quad x<1\\ ax+b & if \quad x\geq 1\end{cases}$ (9)

Section- II

Q.5. (a) Find the locus of the middle points of a system of parallel chords of the parabola $y^2=4ax$. (8)

(b) Determine K so that the vectors $(1,-1,K-1),(2,K,-4),(0,2+K,-8)$ in R^3 are linearly dependent. (8)

Q.6. (a) Show that in any conic the sum of the reciprocals of the segments of any focal chord is constant. (8)

(b) Determine whether or not the given set of vectors is a basis for R^3 $\{(1,2,-1),(0,3,1),(1,-5,3)\}$ (8)

Q.7. (a) Find the pedal equation of $r^m=a^m\cos m\theta$ (8)

(b) Find an equation (or equations) of the subspace W of R^3 spanned by the set of vectors $\{(1,-2,1),(-2,0,3),(3,-2,-2)\}$ (8)

Q.8. (a) If $x=a\cos g(t),\ y=b\sin g(t)$ Prove that $xy^2\frac{d^2y}{dx^2}=b^2\frac{dy}{dx}$ (8)

(b) Is $W=\{(x,y,z): x,y,z\in R, 2x+3y-4z=0\}$ is a subspace of R^3. (8)

Section- III

Q.9. (a) If $\tan(\alpha+i\beta)=x+iy$, show that $x^2+y^2+2x\cot 2\alpha=1$ and $x^2+y^2-2y\coth 2\beta=-1$ (9)

(b) Test the series $\sum_2^\infty\frac{1}{n(\ln n)^p}$ for convergence or divergence (8)

Q.10. (a) Prove analytically that the complex numbers z_1, z_2 (9)

$$\left||z_1|-|z_2|\right|\leq|z_1+z_2|\leq|z_1|+|z_2|$$

(b) Test for convergence the series $\sum_1^\infty\frac{1}{n}$ (8)

Q.11. (a) Prove that $64(\cos^8\theta+\sin^8\theta)=\cos 8\theta+28\cos 4\theta+35$ (9)

(b) Test the series $\sum_0^\infty(-1)^n\left[\frac{\pi}{2}-arc\tan n\right]$ for (8)

(i) absolute convergence (ii) conditional convergence (iii) divergence.

Q.12. (a) Evaluate the sum of infinite series (9)

$$\cos\theta-\frac{1}{2}\cos 2\theta+\frac{1}{3}\cos 3\theta-\frac{1}{4}\cos 4\theta+---$$

(b) Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^\infty\frac{n!\,x^n}{(2n)!}$ (8)
