UNIVERSITY OF THE PUNJAB

A/2011
Examination:- B.A./B.Sc.

Roll No. 023944

Subject: B Course of Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: B MAX. MARKS: 100

Attempt SIX questions in all, selecting TWO questions from Section I & II each and ONE question from Section III & IV each.

Section-I

Q.1. a) Find the six, 6-th roots of $(1+i)$ 9+8

b) If $\operatorname{Sin}(\theta + i\varphi) = \operatorname{Cos}\alpha + i\operatorname{Sin}\alpha$; prove that $\operatorname{Cos}^2\theta = \pm\operatorname{Sin}\alpha$

Q.2. a) Evaluate the sum of infinite series 9+8

$$1 + c\operatorname{Cos}\theta + \frac{c^2}{2!}\operatorname{Cos}2\theta + \frac{c^3}{3!}\operatorname{Cos}3\theta + \cdots$$

b) Find the direction of Qibla of Faisal Masjad Islamabad if;

Latitude $= 33^\circ \cdot 40'N$

Longitude $= 73^\circ \cdot 8'E$

The Latitude & Longitude of Khana-Kaba are $21^\circ \cdot 25'N$ & $39^\circ \cdot 49'E$ respectively.

Q.3. a) Let $f(x,y) = \begin{cases} x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right) & if \quad (x,y) \neq (0,0) \\ 0 & if \quad (x,y) = (0,0) \end{cases}$ 9+8

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$

b) If $U = \frac{\ln(x^2+y^2)}{x+y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

Q.4. a) Examine for relative extrema 9+8

$$f(x,y) = x^2 - xy + y^2 + 6x$$

b) Show that the sphere $x^2 + y^2 + z^2 = 18$ and the cone $x^2 + z^2 = (y-6)^2$ are tangent along their intersection.

Section-II

Q.5. a) Use limit comparison "Test" to investigate the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{\ln(n+1)}{n^2}$ 9+8

b) Apply appropriate 'Test' to determine convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{1^n + 2^n}{3^n}$

Q.6. a) Determine the value of x for which the series (i) converges aboslutely (ii) converges conditionally (iii) Diverges $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$ 9+8

b) Find radius of convergence & interval of convergence of $\sum_{n=2}^{\infty}\frac{x^n}{(\ln n)^n}$

Q.7. a) Find the volume generated by revolving the area in first Quadrant bounded by parabola $y^2 = 8x$ and its latus rectum about the x-axis. 9+8

b) Determine whether the integral $\int_0^3 \frac{dx}{x^2+2x-3}$ converges or diverges. If converges, then evaluate.

Q.8. a) Evaluate the integral $\int_1^2\int_0^3 (x+y)\,dx\,dy$ 9+8

b) Use spherical co-ordinates to evaluate $I = \iiint_S z^2\,dx\,dy\,dz$ where S is the quarter $x^2+y^2+z^2 \le 1,\quad y \ge 0\,;z \ge 0$

Section-III

Q.9. a) State and prove Lagranges' Theorem. 8+8

b) Show that the set $\{\bar{1},\bar{2},\bar{4},\bar{5},\bar{7},\bar{8}\}$ is a group under multiplication modulo 9. Find order of each element of S.

Q.10. a) Show that set S_n of all permuations on a set X with n-elements is a group under the operation of composition of permutations. 8+8

b) Determine whether the permutation $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7\\ 7 & 6 & 5 & 3 & 4 & 2 & 1\end{pmatrix}$ is even or odd

Section-IV

Q.11. a) Let (X,d) be a metric space, show that 'd_1' defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$ is a metric. 8+8

b) Let x, y be two points of R^n or C^n then show that

$$\left\{\sum_{k=1}^{n}|x_k+y_k|^2\right\}^{1/2} \le \left\{\sum_{k=1}^{n}|x_k|^2\right\}^{1/2} + \left\{\sum_{k=1}^{n}|y_k|^2\right\}^{1/2}$$

Q.12. a) Prove that any open ball in a metric space is an open set. 8+8

b) If A, B are two subsets of a metric space X; then show that

i) $\overline{A\cup B} = \bar{A}\cup\bar{B}$ ii) $\overline{A\cap B} \subseteq \bar{A}\cap\bar{B}$

We are very thankful to Muhammad Imran Siddiqui for sharing this paper.