



UNIVERSITY OF THE PUNJAB

A/2010

Examination:- B.A./B.Sc.

Roll No.

Subject: Mathematics-B Course
PAPER: B

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt SIX questions in all, selecting TWO questions from Section I & II each and ONE question from Section III & IV each.

Section-I

Q.1. a) Solve the equation $x^7 + x^4 + x^3 + 1 = 0$ 9+8

b) Show that $-3 - 4i = 5 e^{i(\pi + \tan^{-1} \frac{4}{3})}$

Q.2. a) Prove that $\tan^{-1}(\cos \theta + i \sin \theta) = \pm \frac{\pi}{4} + \frac{i}{4} \ln \left(\frac{1 + \sin \theta}{1 - \sin \theta} \right)$ 9+8

b) Find the Direction of Qibla of Peshawar where

Latitude = $34^{\circ}1'N$; Longitude = $71^{\circ}40'E$

Latitude ϕ_o Longitude λ_o of Khana-Kaaba are

$\phi_o = 21^{\circ} \cdot 25.2'N$; $\lambda_o = 39^{\circ} \cdot 49.2'E$

Q.3. a) If $U = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$; then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ 9+8

b) Verify that $f_{xy} = f_{yx}$ if $f(x, y) = x \sin xy + y \cos xy$

Q.4. a) Examine for relative extrema 9+8

$$f(x, y) = 2x^2 + xy^2 - 4x - 1$$

b) Find the point on $x^2 - 2y^2 - 4z^2 = 16$ at which the tangent plane is parallel to the plane $4x - 2y + 4z = 5$

Section-II

Q.5. a) Use the "Comparison test" to investigate the convergence or divergence of 9+8

the Series $\sum_1^{\infty} \frac{2}{\sqrt{n} + 1}$

b) Use appropriate test for investigating convergence or divergence of the

Series $\sum_1^{\infty} \frac{\sqrt{n}}{2^n}$

Q.6. a) Test the Series for i) Absolute Convergence 9+8

ii) Conditional Convergence iii) Divergence

$$\sum_1^{\infty} \frac{\sin \sqrt{n}}{\sqrt{n^3 + 1}}$$

- b) Find the radius of Convergence and interval of Convergence of the series

$$\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{n^2}$$

- Q.7. a) Determine whether the integral converges or diverges $\int_0^e x^2 \cdot \ln x \cdot dx$ 9+8

Evaluate if converges.

- b) Find the volume of the solid generated by revolving the area enclosed by $y = 2x$ and $y = x^2$ about the y-axis.

- Q.8. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (y + y^3) dy dx$ 9+8

- b) Evaluate $\iiint_S 15x^2z^2 dx dy dz$; S bounded by $x^2 + y^2 = 1$; $x^2 + z^2 = 1$

Section-III

- Q.9. a) State and prove Lagranges' Theorem. 8+8

- b) Show that the set $S = \{\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{7}, \bar{8}\}$ is a group under multiplication modulo 9. Find order of each element of S .

- Q.10. a) Prove that every permutation of degree, n can be written as a product of cyclic permutation acting on mutually disjoint sets. 8+8

- b) Find order of Permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$

Section-IV

- Q.11. a) Let (X, d) be a metric space if $d_1(x, y) = \min\{1, d(x, y)\}$. Prove that d_1 is a metric space. 8+8

- b) Let x, y be two points of R^n or C^n then show that

$$\left[\sum_{k=1}^n |x_k + y_k|^2 \right]^{1/2} \leq \left[\sum_{k=1}^n |x_k|^2 \right]^{1/2} + \left[\sum_{k=1}^n |y_k|^2 \right]^{1/2}$$

- Q.12. a) Let (X, d) be a discrete metric space then prove that every sub set of X is open. 8+8

- b) A sub set O of a metric space in (X, d) is closed if and only if it contains all of its limit points.