UNIVERSITY OF THE PUNJAB

A/2009
Examination:- B.A./B.Sc.

Roll No. .....

Subject: Mathematics-B Course
PAPER: B

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt SIX question in all, selecting TWO question from Section-I & Section II each and ONE question from Section III & Section IV each.

Section-I

Q.1. a) Find the Four-Fourth roots of $-2\sqrt{3}+2i$ 9+8

b) If $\sin(\theta+i\varphi)=\cos\alpha+i\sin\alpha$, prove that $\cos^2\theta=\pm\sin\alpha$

Q.2. a) Find the sum of infinite series: 9+8

$$1+c\cos\theta+\frac{c^2}{2!}\cos 2\theta+\frac{c^3}{3!}\cos 3\theta+\cdots\cdots\cdots$$

b) Find the direction of Qibla of Badshahi Mosque Lahore

Latitude $=31^O\cdot 35'\cdot 4''\ N$

Longitude $=70^O\cdot 18'\cdot 7''\ E$

Latitude & Longitude of Khana-e-Kaba are $21^O\cdot 25'\ N$ & $39^O\cdot 49'\ E$ respectively.

Q.3. a) If $f(x,y)=x^2 arc\tan\left(\frac{y}{x}\right)-y^2 arc\tan\left(\frac{x}{y}\right)$ 9+8

Show that $\frac{\partial^2 f}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$

b) If $u=\ln\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$

Q.4. a) Examine for relative extrema 9+8

$$f(x,y)=\frac{1}{x}+xy-\frac{8}{y}$$

b) Show that the surfaces $z=16-x^2-y^2$ and $63z=x^2+y^2$ intersect orthogonally.

Section-II

Q.5. a) Use the "Comparison test" to investigate the convergence or divergence of the series $\sum_1^\infty \sin\left(\frac{\pi}{n}\right)$ 9+8

b) Apply appropriate test for converence or divergence of the seires $\sum_1^\infty\left(\frac{1}{1+n^3}\right)^n$

Q.6. a) Test the series $\sum_{1}^{\infty}(-1)^{n-1}\frac{n^2}{(n+2)!}$ for i) Absolute Convergence ii) Conditional Convergence iii) Divergence 9+8

b) Find the radius of convergence & interval of convergence of $\sum_{n=1}^{\infty} n^2(x-2)^n$

Q.7. a) The area in the first Quadrant bounded by $x = 2y^3 - y^4$ & the y-axis is revolved about the x-axis. Find the volume of the resulting solid. 9+8

b) Determine whether the integral $\int_0^2 \frac{x}{x^2-5x+6}dx$ converges or diverges. If converges, then evaluate.

Q.8. a) Evaluate $\int_2^4 \int_1^2 (x^2+y^2)dy\,dx$ 9+8

b) Use Spherical co-ordinate to evaluate $I = \iiint_S Z^2\,dx\,dy\,dz$; where S is the quarter $x^2+y^2+z^2 \le 1;$ $y \ge 0$; $Z \ge 0$

Section-III

Q.9. a) If G has three elements, where G is a group, show that it is an Abelian group. 8+8

b) Every subgroup of a cyclic group is cyclic.

Q.10. a) The set S_n of all permutations on a set X with n-elements is a group under the operation of composition of permutations. 8+8

b) Determine whether the permutation is even or odd

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 \end{pmatrix}$$

Section-IV

Q.11. a) Show that $d(x,y) = d\big((x_1,x_2),(y_1,y_2)\big) = ((x_1-y_1)^2 + (x_2-y_2)^2)^{\frac{1}{2}}$ $\forall x_1, x_2, y_1, y_2 \in \Re$ is a metric on $\Re^2$ 8+8

b) Let x, y be two points of $\Re^n$ or $\mathbb{C}^n$, then

$$\left[\sum_{k=1}^{n} |x_k + y_k|^2\right]^{1/2} \le \left[\sum_{k=1}^{n} |x_k|^2\right]^{1/2} + \left[\sum_{k=1}^{n} |y_k|^2\right]^{1/2}$$

Q.12. a) Prove that any open ball in a metric space is an open set. 8+8

b) If A & B are wo subsets of a metric space X, then show that

i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ ii) $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$