13**A/2004**BSc

B- COURSE OF MATHEMATICS TIME ALLOWED: 3 hours

PAPER: **B** MAX. MARKS: 100

Attempt **SIX** questions in all, selecting TWO questions from section-I, TWO questions from section-II, ONE question from section -III and ONE question from section-IV.

SECTION – I

1. a) Prove that $\left[\dfrac{1+Sinx+iCosx}{1+Sinx-iCosx}\right]^n = Cos\, n\left(\dfrac{\pi}{2}-x\right)+i\, Sin\, n\left(\dfrac{\pi}{2}-x\right)$. 9

b) Find the sum of the series $1 + x\,Cos\theta + x^2\,Cos\,2\theta + \ldots + x^n\,Cos\,n\theta$. 8

2. a) Separate $Sin^{-1}(Cos\theta + i\,Sin\theta)$ into real and imaginary parts. 9

b) Find the direction of Qibla at Peshawar given that: 8

Latitude of Peshawar	= 34° 1′ N
Longitude of Peshawar	= 71° 40′ E
Latitude of KHANA-e-KABA	= 21° 25.2′ N
Longitude of KHANA-e-KABA	= 39° 49.2′ E

3. a) If $U = f(x, y)$ is a homogeneous function of degree n, then prove that $x^2 f_{xx} + 2xy\, f_{xy} + y^2 f_{yy} = n(n-1)\, f$. 9

b) Examine the continuity of $f(x,y)=\begin{cases}\dfrac{x^2y}{x^4+y^2} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$ 8

at (0, 0). Do $f_x(0, 0)$ and $f_y(0, 0)$ exists?

4. a) Examine the function $f(x, y) = 2x^2 - 4x + xy^2 - 1$ for relative extrema. 9

b) The dimensions of a box are measured to be 10 inches, 12 inches and 15 inches and the measurements are correct to 0.02 inch. Find the maximum error if the volume of the box is calculated from given measurements. Also find the percentage error. 8

SECTION – II

5. a) State Ratio Test for the convergence of an infinite series and test the convergence of the series $\sum_1^{\infty}\dfrac{2^{2n-1}}{(2n-1)!}$. 9

b) Find the values of x for which the series $\sum_1^{\infty}\dfrac{(-1)^n\, x^n}{3^n\,(n+1)!}$ 8

i) Converges absolutely ii) Converges conditionally
iii) Diverges

6. a) Find radius and interval of convergence of the series $\sum_{n=1}^{\infty}\dfrac{n(x-1)^n}{2n(3n-1)}$. 9

b) Determine whether the series $\sum_1^{\infty}\dfrac{1}{9n^2+3n-2}$ converges or diverges. Find the sum if it converges. 8

7. a) Find the volume of the solid that remains after boring a hole of radius 'a' through the center of a sphere of radius r > a. 9

b) Evaluate the improper integral $\int_2^{\infty}\dfrac{dx}{x(\ell n\, x)^3}$. 8

8. a) Evaluate $\int_0^4 \int_0^{\sqrt{4y-y^2}} (x^2+y^2)\,dx\,dy$ by changing into polar coordinates. 9

b) Evaluate $\iiint_S (x+1)\,dz\,dy\,dx$ s : $0 \le x \le 1$, $y = 0$, $y = x$ and $z = -y^2$ & $z = y^2$. 8

SECTION – III

9. a) Let G be a group. Show that G is abelian if and only if $(ab)^2=a^2b^2$ for all a, b∈ G. 8

b) Prove that the union H∪K of two subgroups H and K of G is a subgroup of G if and only if $H \subset K$ or $K \subset H$. 8

10. a) Prove that the order and index of a subgroup of a finite group divide the order of the group. 8

b) Prove that the order of a cyclic permutation of length m is m. 8

SECTION – IV

11. a) Let A be a non-empty sub – set of a metric space (X, d). Show that $|d(x, A) - d(y, A)| \le d(x, y)$ for all x, y ∈X. 8

b) Define an open set in a metric space and prove that any open ball in a metric space is an open set. 8

12. a) Let (X, d) be a metric space and A, B be sub sets of X. Prove that: 8

i) $A^\circ \cup B^\circ \subset (A \cup B)^\circ$ ii) $(A \cap B)^\circ = A^\circ \cap B^\circ$

b) If A is any sub-set of a metric space (X, d), then prove that closure $\bar{A}$ of A is the smallest closed super set of A. 8