025334

A/2001BA/BSc
NEW COURSE

B- COURSE OF MATHEMATICS
PAPER: B

TIME ALLOWED: 3 hours
MAX MARKS: 100

Attempt SIX questions in all selecting TWO questions from section (I), TWO questions from section (II), ONE from section (III) and ONE from section (IV)

SECTION – I

1. a) Prove that $\left(\dfrac{1+\sin x+i\cos x}{1+\sin x-i\cos x}\right)^n = \cos n\left(\dfrac{\pi}{2}-x\right)+i\sin n\left(\dfrac{\pi}{2}-x\right)$ 8

   b) Sum the series to infinity. 9
   $\cos\theta-\frac{1}{2}\cos 2\theta+\frac{1}{3}\cos 3\theta-\frac{1}{4}\cos 4\theta+\ldots\ldots\ldots$

2. a) Prove that $\tan^{-1}\left(\dfrac{x+iy}{x-iy}\right)=\dfrac{\pi}{4}+\dfrac{i}{2}\ln\dfrac{x+y}{x-y}$ if $x>y>0$ 8

   b) Find the direction of Qibla of SHAH FAISAL MASJID, Islamabad, given that 9
   Latitude of Islamabad $= 33^\circ\text{-}40'$ N
   Longitude of Islamabad $= 73^\circ\text{-}8'$ E

3. a) Examine the continuity of $\begin{cases}\dfrac{x^2y}{x^4+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ 8
   at (0,0)
   Do $f_x(0,0)$ and $f_y(0,0)$ exit?

   b) If: $f(x,y)=x^2\tan^{-1}\left(\dfrac{y}{x}\right)-y^2\tan^{-1}\left(\dfrac{x}{y}\right)$ 8
   show that: $\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{x^2-y^2}{x^2+y^2}$

4. a) If u = f(x,y) is a homogeneous function of degree n, then prove that: $x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=n(n-1)f.$ 8

   b) Examine $f(x,y) = 2x^2-4x+xy^2-1$ for relative extrema. 8

SECTION – II

5. a) Prove that the series $\sum_{n=1}^{\infty}\dfrac{1}{n^p}$ is divergent when $p\le 1$ and convergent when $p>1$ 8

   b) Use alternating series test to check the series 9
   $$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{n+4}{n^2+n}\right)$$
   for convergence or divergence.

6. a) State Cauchy's root test for convergence or divergence of infinite series apply, this test on $\sum_{n=1}^{\infty}\dfrac{2^n}{n^3}$ to check convergence or divergence. 8

P. T. O.

b) Find the radius of convergence and the interval of convergence of the power series. $\sum_{n=0}^{\infty} \cdot \frac{(-1)^{n+1}}{(n+1)^2} \frac{(x+1)^{2n}}{5^n}$. 9

7. a) Evaluate $\iint_D \frac{x^2}{(x^2+y^2)^2} \cdot dA$ where D is the region in the first quadrant enclosed by the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$, $0<a<b$ 8

b) Find the volume of a right pyramid whose height is h and has a square base with each side of length a. 8

8. a) Evaluate the improper integral $\int_0^1 x \ \ln x \ dx$ 8

b) Evaluate $\iiint_S 15x^2 z^2 \, dx \, dy \, dz$ 8

where S is bounded by $x^2+y^2=1$ and $x^2+z^2=1$

SECTION – III

9. a) Let $(G,\cdot)$ be a group. Such that $(ab)^n = a^n b^n$ for three consecutive natural numbers and all a, b in G. Show that G is abelian. 8

b) The Union HUK of two subgroups H and K of a group G is a subgroup of G if and only if either. $H \subset K$ or $K \subset H$. 8

10. a) Let H and K be two finite sub groups of a group G. Whose order are relatively prime, prove that $H \cap K = \{e\}$ 8

b) Define a transposition and determine whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 \end{pmatrix}$ is even or odd 8

SECTION – IV

11. a) Let (X,d) be a metric space and let $d'(X \times X) \to R$ be given by 8

$$d'(x_1, x_2) = \frac{d(x_1, x_2)}{1 + d(x_1, x_2)}$$

prove that d′ is a metric

b) Prove that open sphere in a metric space is an open set. 8

12. a) Define the following: 8

i) Closed sphere in a metric space X
ii) Open set in a metric space X
iii) Closure of a set A in X
iv) Limit point of a set A in X

b) If A and B are two subsets of a metric space X, then prove that. 8

i) $\text{Int}(A) \cap \text{Int}(B) = \text{Int}(A \cap B)$

ii) $\text{Int}(A) \cup \text{Int}(B) \subseteq \text{Int}(A \cup B)$