UNIVERSITY OF THE PUNJAB

A/2011
Examination:- B.A./B.Sc.

Roll No. 23942

Subject: B Course of Mathematics — **TIME ALLOWED: 3 hrs.**
PAPER: A — **MAX. MARKS: 100**

Attempt SIX questions by selecting ONE question from Section-I, TWO questions from Section-II, TWO questions from Section-III, and ONE question from Section-IV.

Section-I

Q.1. a) Show that with usual notations $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ — 8,8

b) If $\vec{r} = (\text{Cos}\,nt)\hat{i} + (\text{Sin}\,nt)\hat{j}$, where "$n$" is a constant, then show that $\vec{r}\times\frac{d\vec{r}}{dt} = n\hat{k}$

Q.2. a) If $\vec{F}$ and $\vec{G}$ are vector point functions then prove that — 8,8

$$\nabla\times(\vec{F}\times\vec{G}) = (\vec{G}\cdot\nabla)\vec{F} - (\vec{F}\cdot\nabla)\vec{G} + \vec{F}(\nabla\cdot\vec{G}) - \vec{G}(\vec{\nabla}\cdot\vec{G})$$

b) If $\vec{F} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{(x^2+y^2+z^2)^{\frac{3}{2}}}$, then show that divergence of $\vec{F} = 0$

Section-II

Q.3. a) If two concurrent forces are represented by "$\lambda\overrightarrow{OA}$" and "$\mu\overrightarrow{OB}$", prove that their resultant is given by $(\lambda+\mu)\overrightarrow{OC}$, where "$C$" divides "$AB$" so that $AC:CB = \mu:\lambda$ — 9,8

b) Forces P, Q, R act along the sides BC, CA, AB of a triangle "ABC". Find the condition that their resultant is parallel to "BC" and determine its magnitude.

Q.4. a) Forces P, Q, R act along the sides BC, CA, AB respectively of a triangle "ABC". Prove that if — 8,9

$$P\,\text{Cos}\,A + Q\,\text{Cos}\,B + R\,\text{Cos}\,C = 0$$

then the line of action of the resultant passes through the circumcentre of the triangle.

b) AB and AC are similar uniform rods of length "a", smoothly joined at "A". BD is a weightless bar of length "b", smoothly joined at "B", and fastened at "D" to a smooth ring sliding on "AC". The system is hung on a small pin at A. Show that the rod "AC" makes with the vertical an angle $\tan^{-1}\frac{b}{a+\sqrt{a^2-b^2}}$.

Q.5. a) Find the centroid of the region bounded by the coordinate axes and the circle $x^2+y^2=a^2$, which lies in the first quadrant. — 9,8

b) Find the centre of gravity of uniform lamina forming a quadrant of an ellipse bounded by its semi-axes.

Q.6. a) A uniform rod of length "$2a$" and weight "W" rests with its middle point upon a rough horizontal cylinder whose axis is perpendicular to the rod. Show that the greatest weight that can be attached to one end of the rod without sliding it off the cylinder is $\frac{b\lambda}{a-b\lambda}W$, where "$b$" is the radius of the cylinder and "λ" the angle of friction. — 9,8

b) A uniform ladder rests with its upper end against smooth ertical wall and its foot on rough horizontal ground. Show that the force of friction at the ground is $\frac{W}{2}\tan\theta$, where "W" is the weight of the ladder and "θ" as its inclination with the vertical.

Section-III

Q.7. a) Find the radial and transverse components of the velocity of a particle moving along the curve $ax^2 + by^2 = 1$ at any time "t" if the polar angle $\theta = ct^2$ 9,8

b) A particle is projected vertically upwards with a velocity $\sqrt{2gh}$ and another is let fall from a height "h" at the same time. Find the height of the point where they meet eachother.

Q.8. a) Prove that the field of force determined by $\vec{F} = (y^2 - 2xyz^3)\hat{i} + (3 + 2xy - x^2z^3)\hat{j} + (6z^3 - 3x^2yz^2)\hat{k}$ is conservative and find its potential. 9,8

b) Find the equation of the parabola of safety for particles projected in a vertical plane with a given velocity "V_o" from a fixed point. Find also its focus and directrix.

Q.9. a) The maximum velocity that a particle executing simple harmonic motion of amplitude "a" attains is "v". If it is disturbed in such a way that its maximum velocity becomes "nv", find the change in the amplitude and the time period of motion. 9,8

b) A horizontal bar "OA" of length "a" is made to rotate with uniform angular velocity "ω" about a vertical axis throuh the end "O". If a particle is attached to "A" by a light string of length "l", the string makes an angle "θ" with the vertical when the motion is steady. Show that

$$\omega^2(a \operatorname{Cot}\theta + l \operatorname{Cos}\theta) = g$$

Q.10. a) What are the Aps and Apsidal distance? Prove that the orbit of a particle moving under a central force is necessarily a plane curve. 9,8

b) Find the law of force towards the pole when the path is the cardioid $r = a(1 - \cos\theta)$ and prove that, if "F" be the force at the apse and "v" the velocity, then

$$3v = 4aF$$

Section-IV

Q.11. a) A smooth sphere of mass "m" impinges on another of mass "M" at rest, the direction of motion making an angle of 45° with the line of centres at the moment of impact. Show that if $e = \frac{1}{2}$, the direction of motion of the sphere of mass "m" is turned through an angle $\tan^{-1}\left(\frac{3m}{4m+M}\right)$ 8,8

b) A ball "A", moving with velocity "u" impinges directly on an equal ball "B" moving with velocity "v" in the opposite direction. If "A" be brought to rest by the impact, show that $u:v = 1 + e : 1 - e$ where "e" is the co-efficient of restitution.

Q.12. a) Two smooth equal sphere, moving with velocity "u_1" and "u_2" in the direction of line joining their centers impinge directly. If the co-efficient of restitution be "$\frac{1}{2}$". Show that exactly half the energy is lost in collision if "u_1" and "u_2" are in the ratio $1 + \sqrt{2} : 1 - \sqrt{2}$ 8,8

b) Two equal balls of elasticity "e" impinges having before impact resolved velocities u_1, v_1 in the direction of common normal and u_2, v_2 perpendicular to them. If their motion after impact are at right angle prove that;

$$(u_1 + v_1)^2 + 4u_2v_2 = e^2(u_1 - v_1)^2$$

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